Question 1) A heavy nucleus N, at rest, undergoes fission N → P + Q, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are V_P and V_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

(A) $E_P + E_Q = c^2\delta$

(B)

$E_p = \left(\frac{M_P}{M_P+M_Q}\right)c^2\delta$

(C)

$\frac{V_P}{V_Q} = \frac{M_Q}{M_P}$

(D) The magnitude of momentum for P as well as Q is

$c\sqrt{2\mu\delta}$

, where

$\mu = \frac{M_P M_Q}{M_P+M_Q}$

Answer: (A,C,D)

Solution:

The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu=M_PM_Q/(MP+MQ)$

Given:

N→P+Q

Energy released in this fission will be,

$(M_N-M_P-M_Q)c^2=\delta c^2$

This will be distributed kinetic energy of P and Q.

$E_P + E_Q = \delta c^2$ ------(1)

By conservation of momentum,

$M_PV_P = M_QV_Q$

So, $v_P/v_Q=M_Q/M_P$ ...(2)

Kinetic energy be written as $KE=p^2/2M$

So equation (1), became,

$\Rightarrow p^2/2M_P+p^2/2M_Q=\delta c^2$

$\Rightarrow p^2/2=M_PM_Q/(M_P+M_Q)\delta c^2$ ......(3)

$\therefore E_P=p^2/2M_P=M_Q/(M_P+M_Q)\delta c^2$

Similarly, $E_Q=M_P/(M_P+M_Q)\delta c^2$

From eq. (3), momentum of P or Q will be,

$p=\sqrt{2M_PM_Q/(M_P+M_Q)\delta c^2}=c\sqrt{2\mu\delta}$

Where, $\mu=M_PM_Q/(M_P+M_Q)$

Hence, (A), (C) and (D) are the correct options

Question 2) A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with a probability of 60% and beta-decay with a probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is

(A) 50

(B) 75

(C) 350

(D) 525

Answer: (D) 525

Solution:

$t_{1/2}$ = 20 min

In 60 min, no. of half-life = 3

= 1000 - 1000$(1/2)^3$ = 875

The number of alpha-decays of Q in the first one hour = 875 x 0.6 = 525

Question 3) Which of the following statement(s) is(are) correct about the spectrum of a hydrogen atom?

(A) The ratio of the longest wavelength to the shortest wavelength in the Balmer series is 9/5

(B) There is an overlap between the wavelength ranges of the Balmer and Paschen series

(C) The wavelengths of the Lyman series are given by $(1 + 1/m^2)\lambda_0$ where λ_0 is the shortest wavelength of Lyman series and m is an integer

(D) The wavelength ranges of Lyman and Balmer series do not overlap

Answer (A, D)

Solution:

For Balmer series :

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

n = 3,4,5 −−

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$
$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{4}\right]$$
$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{9}{5}$$

For Layman Series

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$$

n = 2,3,4 −−

$1/\lambda_{min} = R$

$$\Rightarrow \lambda = \lambda_0 n^2/(n^2 - 1)$$

Question 4) For an element decaying through simultaneous reaction, the half-life for the respective decaying path is 1400 s and 700 s. Find the time taken when the number of atoms becomes $N_0/3$ in the element sample. (N_0 is the initial number of atoms in sample)

(A) (1400/5) In 3

(B) (1400/3) In 3

(C) (1400/3)ln 2

(D) (700/3)ln 2

Solution: (b) (1400/3) In 3

$N = N_0 e^{t/\tau}$

$N_0/3 = N_0 e^{t/\tau}$

Taking natural log on both sides,

$$ln\left(\frac{1}{3}\right) = \left(\frac{-t}{\tau}\right) lne$$
$$-ln3 = \left(\frac{-t}{\frac{1400}{3}}\right) \times 1$$

Therefore, t = (1400/3) ln3

Question 5) The ratio of the mass densities of nuclei ^{40}Ca and ^{16}O is close to

(A) 1

(B) 0.1

(C) 5

(D) 2

Answer: (a) 1

Solution:

Nuclear density is independent of atomic number. The mass density of all nuclei is the same. So the ratio is 1.

Question 6) For the uranium nucleus how does its mass vary with volume?

(A) $m \propto V$

(B) $m \propto 1/V$

(C) $m \propto \sqrt{V}$

(D) $m \propto V^2$

Answer: (A) $m \propto V$

Solution:

We know that the radius of the nucleus

$R = R_0A^{1/3}$

where A is the mass number

$R^3 = R_0^3A$

Volume, $V = (4/3)\pi R^3$

$= (4/3)\pi R_0^3A$

Therefore, mass $\propto$ volume

Question 7) Find the Binding energy per nucleon for $^{120}_{50}Sn$. Mass of proton $m_p = 1.00783$ U, mass of neutron $m_n = 1.00867$ U and mass of tin nucleus $m_{Sn} = 119.902199$ U. (take 1U = 931 MeV)

(A) 7.5 MeV

(B) 9.0 MeV

(C) 8.0 MeV

(D) 8.5 MeV

Answer: (D) 8.5 MeV

Solution:

Mass defect, $\Delta m = (50m_p + 70m_n) - m_{sn}$

= (50 x 1.00783 + 70 x 1.00867) - (119.902199)

= (50.3915 + 70.6069) - (119.902199)

= (120.9984) - (119.902199)

= 1.096201

Binding energy = $(\Delta m)c^2$

= (Δm) x 931 = 1.096201 x 931 = 1020.56

Binding energy/Nucleon = 1020.56/120 = 8.5 MeV

Question 8) Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be (velocity of light, c = 3 x 10^8 m/s).

(A) 0.96 gm

(B) 0.8 gm

(C) 4 x 10^{-2} gm

(D) 6.6 x 10^{-5} gm

Answer: (C) 4 x 10^{-2} gm

Solution:

Energy converted in one hour = p x t

= 10^9 x 3600

= 36 x 10^{11} J

$M = E/c^2$

= 36 x 10^{11}/ (3 x $10^8)^2$

= 4 x 10^{-5} kg

= 4×10^{-2} gm

Question 9) Fast neutrons can easily be slowed down by

(A) the use of lead shielding

(B) passing them through water

(C) elastic collisions with heavy nuclei

(D) applying a strong electric field

Answer: (B) passing them through water

Solutions:

Fast neutrons can easily be slowed down by passing them through the water. In nuclear reactors, heavy water is used as a moderator.

Question 10) During a nuclear fusion reaction

(A) a heavy nucleus breaks into two fragments by itself

(B) a light nucleus bombarded by thermal neutrons breaks up

(C) a heavy nucleus bombarded by thermal neutrons breaks up

(D) two light nuclei combine to give a heavier nucleus and possibly other products.

Answer: (D) two light nuclei combine to give a heavier nucleus and possibly other products.

Solution:

In a nuclear fusion reaction, two light nuclei combine to give a heavier nucleus and possibly other products and a huge amount of energy.