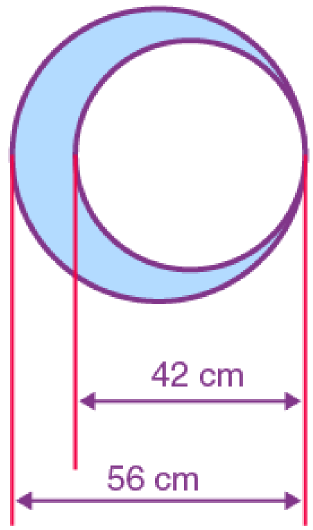
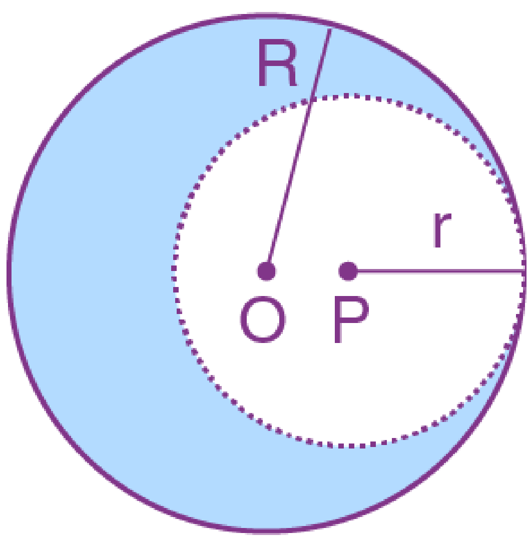
Question 1) A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in the figure. Find the position of the centre of mass of the remaining portion**



Answer: Centre of mass lies at a distance of 9 cm from the origin towards left

Solution:



Let σ be the mass per unit area of the uniform plate

Mass of the whole disc = σ x πR^2

Mass of the portion removed = σ x πr^2

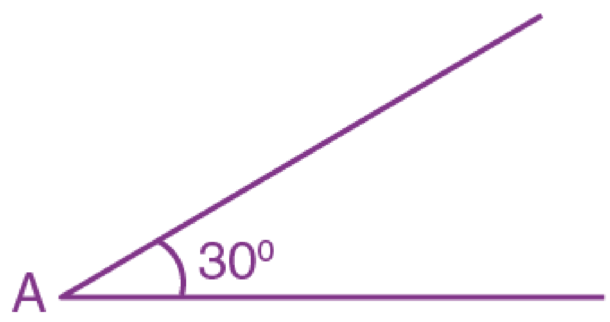
R = 28 cm, r = 21 cm, OP = 7 cm

Position of the centre of mass

$$x = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$
$$x = \frac{\sigma \times \pi R^2 \times 0 - \sigma \times \pi r^2 \times 7}{\sigma \pi R^2 - \sigma \pi r^2}$$
$$x = \frac{-(21)^2 \times 7}{(28)^2 - (21)^2} = -9cm$$

The centre of mass lies at a distance of 9 cm from the origin towards left

Question 2) A solid sphere of mass 2 kg radius 0.5m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30^0 with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A?



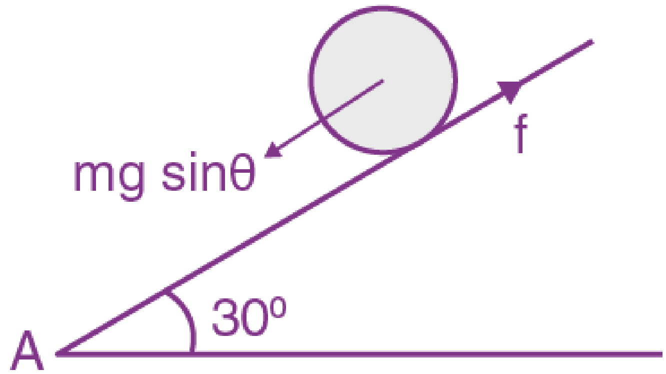
(A) 0.80 s

(B) 0.60 s

(C) 0.52 s

(D) 0.57 s

Answer (D)



$a = \frac{gsin\theta}{1+c}$

For a solid sphere, c = ⅖

$a = \frac{9.8sin30^0}{1+\frac{2}{5}}$

a = 3.5 m/sec^2

Time of ascent is given by

v = u + at

0 = 1 - 3.5 t

$t = \frac{1}{3.5}sec$

Time of decent

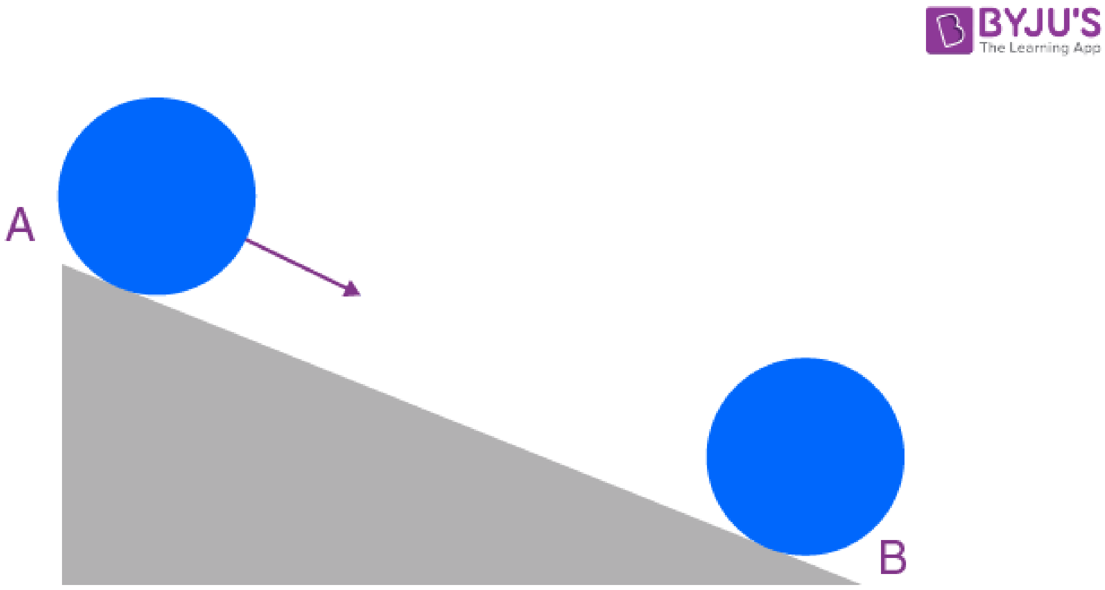
$t = \frac{1}{3.5}sec$

(due to symmetry of motion)

Total time, $T = \frac{2}{3.5}sec$ = 0.57 sec

Question 3) When a disc slides on a smooth inclined surface from rest, the time taken to move from A to B is t_1. When the disc performs pure rolling from rest then the time taken to move from A to B is t_2. If

$\frac{t_1}{t_2} = \sqrt{\frac{3}{x}}$ **find x.**



(A) 2

(B) 1

(C) 5

(D) 7

Answer: (A) 2

Solution:

When disc slides $a_1 = g\sin\theta$

$S = ut_1 + \frac{1}{2}a_1t_1^2 = \frac{1}{2}gsin\theta t_1^2$
............(1)

When disc do pure rolling

$a_2 = \frac{gsin\theta}{1+\frac{k^2}{R^2}} = \frac{gsin\theta}{1+1/2} = \frac{2}{3}gsin\theta$

So,

$S = ut_2 + \frac{1}{2}a_2t_2^2 = \frac{1}{2} \times \frac{2}{3}gsin\theta t_2^2$
---(2)

From (1) and (2)

$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$
So, x = 2

Question 4) A wheel rotating with an angular speed of 600 rpm is given a constant acceleration of 1800 rpm^2 for 10 sec. The number of revolutions revolved by the wheel is

(A) 125

(B) 100

(C) 75

(D) 50

Answer: (A)

Solution:

$$\omega_o = 600\,rpm$$
$$\alpha = 1800\,rpm^2$$
$$t = 10\,sec = \frac{1}{6}\,min$$
$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$
$$\theta = 600 \times \frac{10}{60} + \frac{1}{2} \times 1800 \times \frac{1}{36}$$
$$\theta = 100 + 25 = 125 \text{ revolutions}$$

Question 5) For a body in pure rolling, its rotational kinetic energy is 1/2 times of its translational kinetic energy. The body should be?

(A) Solid cylinder

(B) Ring

(C) Solid sphere

(D) Hollow sphere

Answer: (A)

Solution:

Given,

Rotational K.E = (½) Translational K.E

$(\frac{1}{2})I\omega^2 = (\frac{1}{2}) \times (\frac{1}{2})mv^2$

In pure rolling, $v = R\omega$

$(\frac{1}{2})I\omega^2 = (1/4)\, mR^2\omega^2$

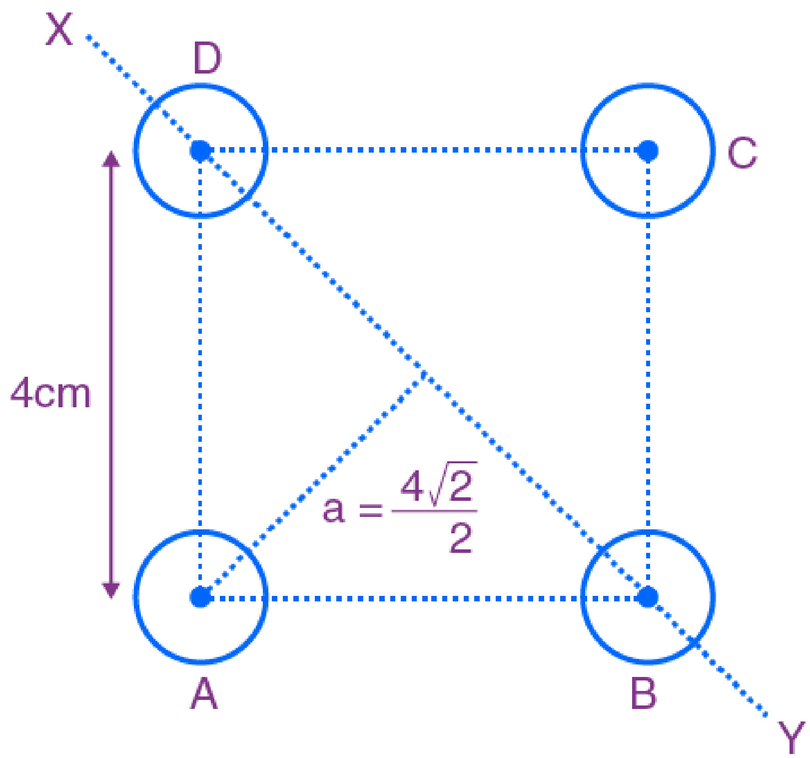
$I = (\frac{1}{2})\, mR^2$

Here, it is a solid cylinder

Question 6) Four solid spheres each of diameter √5 cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N x 10^{-4} kgm^2, then N is

Answer: 9

Solution:



$I_{xy} = I_A + I_B + I_C + I_D$

By symmetry

$I_A = I_C$

$I_B = I_D$

$I_{xy} = 2I_A + 2I_B$

$$I_{xy} = 2\left[\tfrac{2}{5}MR^2 + Ma^2\right] + 2\left[\tfrac{2}{5}MR^2\right]$$
$$I_{xy} = 4 \times \tfrac{2}{5}MR^2 + 2Ma^2$$
$$I_{xy} = \left[\tfrac{8}{5}MR^2 + 2Ma^2\right]$$
$$I_{xy} = \left[\tfrac{8}{5} \times 0.5 \times (\tfrac{\sqrt{5}}{2})^2 + 2 \times (0.5) \times (4 \times 2)^2\right] \times 10^{-4}$$

$= 9 \times 10^{-4}$

$= N \times 10^{-4}$

Therefore, N = 9

Question 7) A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is

(A) 1

(B) 2

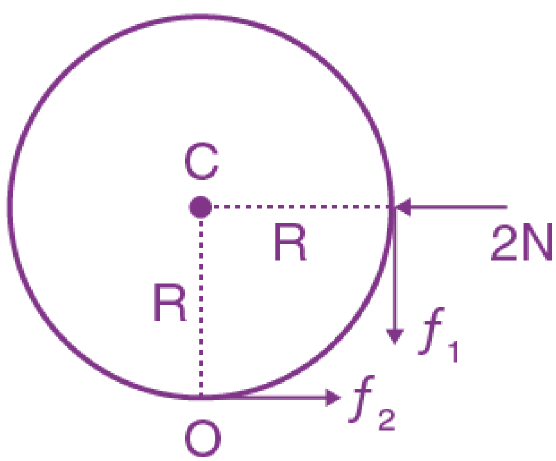
(C) 3

(4) 4

Answer: (4)

Solution:

The stick applies (2N) force so the point of contact O of the ring with ground tends to slide. But the frictional force f_2 does not allow this and creates a torque about 'c, which starts rolling the ring. Between the ring and the stick, a frictional force f_1 also acts.



$F - f_2 = ma$

$2 - f_2 = 2 \times 0.3$

Therefore, $f_2 = 1.4$ N

Applying $\tau = I\alpha$ about C

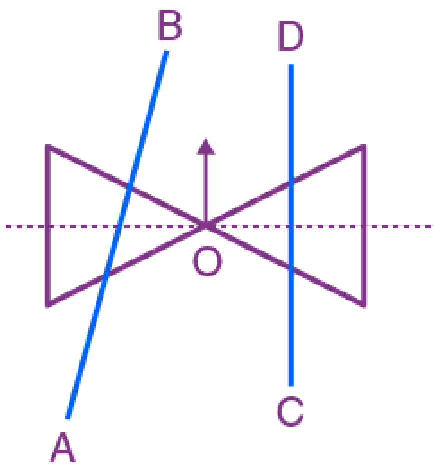
$(f_2 - f_1) R = I\alpha = I(a/R)$

$(1.4 - \mu \times 2) \times 0.5 = 2 \times (0.5)^2 \times (0.3/0.5)$ (since $I = MR^2$]

$\mu = 0.4 = 4/10 = P/10$

Therefore, $P = 4$

Question 8) A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD. It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



(A) go straight

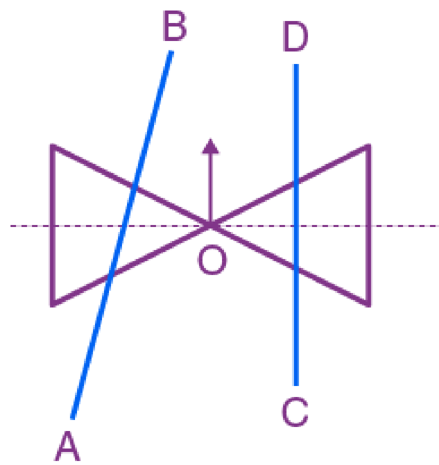
(B) Turn left and right alternately

(C) Turn left

(D) Turn right

Answer: (C) Turn left

Solution:



As shown in the diagram, the normal reaction of AB on the roller will shift towards O

This will lead to the tending of the system of cones to turn left.

Question 9) A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(A) $r\omega_0/4$

(B) $r\omega_0/3$

(C) $r\omega_0/2$

(D) $r\omega_0$

Answer: (C) $r\omega_0/2$

Solution:

From the conservation of angular momentum about any fix point on the surface,

$mr^2\omega_0 = 2mr^2\omega$

$\Rightarrow \omega = \omega_0/2$

$\Rightarrow v = (\omega_0/2)r$

Question 10) A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:

(A) 3F/2mR

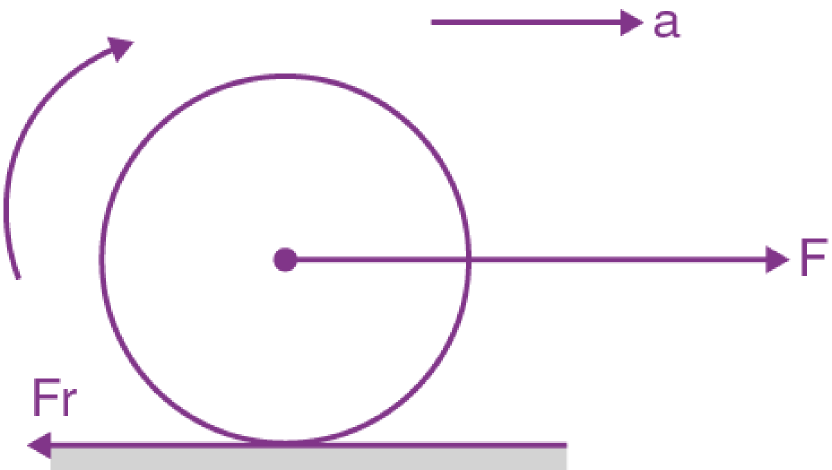
(B) F/3mR

(C) F/2mR

(D) 2F/3mR

Answer: (D) 2F/3mR

Solution:



$F - f_r = ma$ ——---(1)

$f_r R = I\alpha = (mR^2/2)\alpha$ ——---(2)

For pure rolling

$a = \alpha R$ ——------(3)

From (1), (2) and (3)

$F - (mR\alpha/2) = m\alpha R$

$\Rightarrow \alpha = 2F/3mR$