

JEE Arithmetic Progression Previous Year Questions With Solutions

Question 1: Let $a_1, a_2, a_3, \dots, a_{49}$ be in A.P. such that

$$\sum_{k=0}^{12} a_{4k+1} = 416 \text{ and } a_9 + a_{43} = 66. \text{ If } a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m, \text{ then } m \text{ is equal to}$$

(a) 68

(b) 34

(c) 33

(d) 66

Answer: (b)

Solution:

We know that the n th term of A.P. is $a_n = a + (n - 1)d$

$$a_9 + a_{43} = 66$$

$$\text{Therefore, } a + 8d + a + 42d = 66$$

$$\text{Or } a + 25d = 33 \dots\dots(1)$$

Now,

$$\sum_{k=0}^{12} a_{4k+1} = 416$$

$$\text{Therefore, } 13a + 312d = 416$$

$$\text{Or } a + 24d = 32 \dots(2)$$

Solving (1) and (2), we get

$$a = 8 \text{ and } d = 1$$

So,

$$\sum_{k=1}^{17} a_k^2 = 8^2 + 9^2 + \dots + 24^2$$

$$= (1^2 + 2^2 + \dots + 24^2) - (1^2 + 2^2 + \dots + 7^2)$$

Using the sum of squares of n natural numbers formula, we have

$$= [24 \times 25 \times 49]/6 - [7 \times 8 \times 15]/6$$

$$= 4760$$

$$= 140 \times 34$$

Thus, the answer is 34.

Question 2: Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-1/2$, then the greatest number amongst them is

(a) 16

(b) 27

(c) 7

(d) $21/2$

Answer: (a)

Solution:

Let 5 numbers be $a - 2d, a - d, a, a + d, a + 2d$

so, Sum of numbers $= 5a = 25$ or $a = 5$

Product of Numbers $= (a - 2d)(a - d)a(a + d)(a + 2d) = 2520$

$$\Rightarrow (25 - 4d^2)(25 - d^2) = 504$$

$$\Rightarrow 4d^4 - 125d^2 + 121 = 0$$

$$\Rightarrow 4d^4 - 4d^2 - 121d^2 + 121 = 0$$

$$\Rightarrow d^2 = 1 \text{ or } d^2 = 121/4$$

$$\Rightarrow d = \pm 1 \text{ or } d = \pm 11/2$$

For $d = \pm 1$, none of the terms is equal to $-1/2$. (Hence, rejected)

For $d = 11/2$, $a + 2d$ is the greatest term, $a + 2d = 5 + 11 = 16$

Question 3: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that for all $x \in \mathbb{R}$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is :

- (a) 0
- (b) 4
- (c) 3
- (d) 2

Answer: (c)

Solution:

Given: $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P.

Therefore,

$$f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying, A.M. \geq G.M. inequality,

$$\begin{aligned}\frac{(3^x + 3^{-x})}{2} &\geq \sqrt{3^x \cdot 3^{-x}} \\ \Rightarrow \frac{(3^x + 3^{-x})}{2} &\geq 1 \quad \dots (1)\end{aligned}$$

By A.M. \geq G.M. inequality,

$$\begin{aligned}\frac{2^{1+x} + 2^{1-x}}{2} &\geq \sqrt{2^{1+x} \cdot 2^{1-x}} \\ \Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} &\geq 2 \quad \dots (2)\end{aligned}$$

Adding (1) and (2), we get;

$$f(x) \geq 1 + 2 = 3$$

$$\text{i.e., } f(x) \geq 3$$

Thus, the minimum value of $f(x)$ is 3.

Question 4: If the 10th term of an A.P. is $1/20$ and its 20th term is $1/10$, then the sum of its first 200 terms is:

- (a) $201/4$
- (b) 100
- (c) 50
- (d) $201/2$

Answer: (d)

Solution:

10th term of an A.P. is $1/20$, and its 20th term is $1/10$.

So, $a_{10} = 1/20$ and $a_{20} = 1/10$

Now, $a_{20} - a_{10} = 10d$

$$(1/20) - (1/10) = 10d$$

$$\Rightarrow d = 1/200 \text{ and } a = 1/200$$

Now,

Sum of first 200 terms:

$$S_{200} = 200/2 [2(1/200) + 199(1/200)]$$

$$= (200/2) \times (1/200) \times [2 + 199]$$

$$= 201/2$$

Question 5: If three positive numbers a , b and c are in A.P. such that $abc = 8$, then the minimum possible value of b is :

- (a) 2
- (b) $4^{1/3}$
- (c) 8
- (d) 4

Answer: (a)

Solution:

For a set of positive numbers, the $AM \geq GM$.

Given: $abc = 8$

Since the numbers are in AP, their arithmetic mean is b .

The geometric mean of the three numbers:

$$(a+b+c)/3 = b$$

$$\Rightarrow b \geq (abc)^{1/3}$$

That means, $b \geq 2$.

Therefore, the minimum possible value of b is 2.

Question 6: Let $a_1, a_2, a_3, \dots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \dots, 11$

If $[a_1^2 + a_2^2 + \dots + a_{11}^2]/11 = 90$ then find the value of $[a_1 + a_2 + \dots + a_{11}]/11$.

Solution:

First term $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \dots, 11$

$$\text{And } [a_1^2 + a_2^2 + \dots + a_{11}^2]/11 = 90$$

$$a_{k-1} = [a_k + a_{k-2}]/2$$

Now,

Let the 11 terms be, $a - 5d, a - 4d, \dots, a, \dots, a + 4d, a + 5d$.

$$[(a + 5d)^2 + (a + 4d)^2 + \dots + a^2 + \dots + (a - 5d)^2]/11 = 90 \quad \{\text{since } [a_1^2 + a_2^2 + \dots + a_{11}^2]/11 = 90\}$$

$$11a^2 + 110d^2 = 990 \dots (i)$$

$$a^2 + 10d^2 = 90$$

$$a - 5d = 15 \quad \{\text{since the first term} = 15\}$$

$$a = 15 + 5d$$

Substituting $a = 15 + 5d$ in (i), we get;

$$\text{Now, } (15 + 5d)^2 + 10d^2 = 90$$

$$\Rightarrow d = -3, -9/7$$

$$\text{For } d = -3 \Rightarrow a_2 = 12$$

$$\text{For } d = -9/7 \Rightarrow a_2 = 13.7 \text{ (not possible, as } a_2 < 27/2)$$

$$\text{Thus, } [a_1 + a_2 + \dots + a_{11}]/11 = (11/2) \times [30 - 10(3)]/11 = 0$$

Question 7: A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months, his savings increased by Rs.40 more than the saving of immediately previous month. After how many months his total savings from the start of service will be Rs. 11040?

- (a) 18 months
- (b) 19 months
- (c) 20 months
- (d) 21 months

Answer: (d)

Solution:

From the question, savings are 200, 200, 200, 240, 280,..... up to n terms.

But 200, 240, 280,.....(n-2) terms are in AP.

Using the sum of n terms of an AP, we have

$$400 + (n - 2)/2 [2 \times 200 + (n - 2 - 1)40] = 11040$$

$$\Rightarrow (n - 2)[200 + 20n - 60] = 10640$$

$$\Rightarrow 20(n + 7)(n - 2) = 10640$$

$$\Rightarrow n^2 + 5n - 546 = 0$$

$$\Rightarrow (n + 26)(n - 21) = 0$$

-ve value is not possible.

$$n = 21$$

Therefore, total time = 21 months

Question 8: If 100 times the 100th term of an AP with a non-zero common difference is equal to the 50 times its 50th term, then the 150th term of this AP is

- (a) -150
- (b) 150 times its 50th term
- (c) 0
- (d) 150

Answer: (c)

Solution:

Given 100 times the 100th term of an AP = 50 times its 50th term.

$$100 \times T_{100} = 50 \times T_{50}$$

$$\Rightarrow 100(a + 99d) = 50(a + 49d)$$

$$\Rightarrow 2a + 198d = a + 49d$$

$$\Rightarrow a + 149d = 0$$

$$\Rightarrow T_{150} = 0$$

Question 9: If the n th term of an AP is $(2n - 1)$, then find the sum of its first n terms.

Solution:

$$\text{Let } a_n = (2n - 1)$$

$$a_1 = 2 \times 1 - 1 = 1$$

$$a_2 = 2 \times 2 - 1 = 4 - 1 = 3$$

$$\text{Now, } d = a_2 - a_1 = 3 - 1 = 2$$

$$\text{Sum of first } n \text{ terms} = (n/2) [2a + (n - 1)d]$$

$$= (n/2) [2 + 2n - 2]$$

$$= n^2$$

Question 10: The p th, q th and r th terms of an A.P are a , b , and c , respectively. Show that $(q - r)a + (r - p)b + (p - q)c = 0$.

Solution:

Let a , $a + d$, $a + 2d$, ..are in A.P.

$$p\text{th term} = a + (p - 1)d = a$$

$$q\text{th term} = a + (q - 1)d = b \text{ and}$$

$$r\text{th term} = a + (r - 1)d = c$$

$$\text{L.H.S.} = (q - r)a + (r - p)b + (p - q)c$$

$$= (q - r)(a + (p - 1)d) + (r - p)(a + (q - 1)d) + (p - q)(a + (r - 1)d)$$

Solving the above equation, we have;

$$a(q - r) + b(r - p) + c(p - q) = a[(q - r) + (r - p) + (p - q)] + d[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$$

$$= a(0) + d(0)$$

$$= 0$$

Therefore, $(q - r)a + (r - p)b + (p - q)c = 0$.