JEE Main Maths Conic Section Previous Year Questions With Solutions

Question 1: The equation of $2x^2 + 3y^2 - 8x - 18y + 35 = k$ represents a ________.

Solution:

Given equation, $2x^2 + 3y^2 - 8x - 18y + 35 - k = 0$

Compare with $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$,we get

$a = 2, b = 3, h = 0, g = -4, f = -9, c = 35 - k$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 6(35 - k) + 0 - 162 - 48 - 0$

$\Delta = 210 - 6k - 210 = -6k$;

$\Delta = 0$, if $k = 0$

So, that given equation is a point if $k = 0$.

Question 2: The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix ___________.

Solution:

Let midpoint be (h, k) and moving point be $(at^2, 2at)$.

Let focus is $(a, 0)$.

So $h = [at^2 + a]/2$, $k = [2at + 0]/2$

=> $t = k/a$

So $2h = a(k/a)^2 + a$

$= (k^2/a) + a$

=> $k^2 = 2ah - a^2$

=> $k^2 = 2a(h - a/2)$

Replace (h, k) by (x, y)

$y^2 = 2a(x - a/2)$

Directrix is $(x - a/2) = -a/2$

=> $x = 0$

So $x = 0$ is the directrix.

Question 3: On the parabola $y = x^2$, the point least distance from the straight line $y = 2x - 4$ is ___________.

Solution:

Given, parabola $y = x^2$ .....(i)

Straight line $y = 2x - 4$ .....(ii)

From (i) and (ii),

$x^2 - 2x + 4 = 0$

Let $f(x) = x^2 - 2x + 4$,

$f'(x) = 2x - 2$.

For least distance, $f'(x) = 0$

$\Rightarrow 2x - 2 = 0$

$x = 1$

From $y = x^2$, $y = 1$

So, the point least distant from the line is (1, 1).

Question 4: The line $x - 1 = 0$ is the directrix of the parabola, $y^2 - kx + 8 = 0$. Then, one of the values of k is ________.

Solution:

The parabola is $y^2 = 4 * [k / 4] (x - [8 / k])$.

Putting $y = Y$, $x - (8/k) = X$, the equation is $Y^2 = 4 * [k/4] * X$

The directrix is $X + (k/4) = 0$,i.e. $x - (8/k) + (k/4) = 0$.

But $x - 1 = 0$ is the directrix.

So, $[8 / k] - (k / 4) = 1$

⇒ $k = -8, 4$

Question 5: The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is _____________.

Solution:

Tangent to the parabola $y = x^2$ at (2, 4) is $[1 / 2] (y + 4) = x * 2$ or

$4x - y - 4 = 0$

It is also a tangent to the circle so that the centre lies on the normal through (2, 4) whose equation is $x + 4y = \lambda$, where $2 + 16 = \lambda$.

Therefore, $x + 4y = 18$ is the normal on which lies (h, k).

$h + 4k = 18$ ..... (i)

Again, distance of centre (h, k) from (2, 4) and (0, 1) on the circle are equal.

Hence, $(h - 2)^2 + (k - 4)^2 = h^2 + (k - 1)^2$

So, $4h + 6k = 19$ .....(ii)

Solving (i) and (ii), we get the centre = $(-16 / 5, 53 / 10)$

Question 6: Find the equation of the axis of the given hyperbola $x^2/3 - y^2/2 = 1$ which is equally inclined to the axes.

Solution:

$x^2/3 - y^2/2 = 1$

Equation of tangent is equally inclined to the axis i.e., $\tan \theta = 1 = m$.

Equation of tangent $y = mx + \sqrt{(a^2m^2 - b^2)}$

Given equation is $[x^2/3] - [y^2/2] = 1$ is an equation of hyperbola which is of form $[x^2/a^2] - [y^2/b^2] = 1$.

Now, on comparing $a^2 = 3$, $b^2 = 2$

$y = 1 * x + \sqrt{(3-2)}$

$y = x + 1$

Question 7: If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then the value of p is ______.

Solution:

Slope of 1st curve $(dy / dx)_I = -4x / py$

Slope of 2nd curve $(dy / dx)_{II} = x / 4y$

For orthogonal intersection $(-4x / py)(x / 4y) = -1$

$x^2 = py^2$

On solving equations of given curves x = 3, y = 1

$p(1) = (3)^2 = 9$

p = 9

Question 8: If the foci of the ellipse $(x^2 / 16) + (y^2 / b^2) = 1$ and the hyperbola $(x^2 / 144) - (y^2 / 81) = 1 / 25$ coincide, then the value of b^2 is ________.

Solution:

Hyperbola is $(x^2 / 144) - (y^2 / 81) = 1 / 25$

$a = \sqrt{(144/25)}$; $b = \sqrt{(81/25)}$

$e_1 = \sqrt{(1 + b^2/a^2)}$

$= \sqrt{(1 + 81/144)}$

= 15/12

= 5/4

Therefore, foci = $(ae_1, 0) = ([12 / 5] * [5 / 4], 0) = (3, 0)$

Therefore, focus of ellipse = (4e, 0) i.e. (3, 0)

=> e = 3/4

use formula $e^2 = 1 - (b^2/a^2)$

Hence $b^2 = 16(1 - [9 / 16]) = 7$

Question 9: Let E be the ellipse $x^2 / 9 + y^2 / 4 = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1), respectively. Then

A) Q lies inside C but outside E

B) Q lies outside both C and E

C) P lies inside both C and E

D) P lies inside C but outside E

Solution:

The given ellipse is $[x^2 / 9] + [y^2 / 4] = 1$. The value of the expression $[x^2 / 9] + [y^2 / 4] - 1$ is positive for $x = 1$, $y = 2$ and negative for $x = 2$, $y = 1$. Therefore, P lies outside E and Q lies inside E. The value of the expression $x^2 + y^2 - 9$ is negative for both the points P and Q. Therefore, P and Q both lie inside C. Hence, P lies inside C but outside E.

Hence option d is the answer.

Question 10: The equation of the director circle of the hyperbola $[x^2 / 16] - [y^2 / 4] = 1$ is given by ______.

Solution:

Equation of the director circle of hyperbola is $x^2 + y^2 = a^2 - b^2$. Here $a^2 = 16$, $b^2 = 4$

Therefore, $x^2 + y^2 = 12$ is the required director circle.

Question 11: If m_1 and m_2 are the slopes of the tangents to the hyperbola $x^2 / 25 - y^2 / 16 = 1$ which pass through the point (6, 2), then find the relation between the sum and product of the slopes.

Solution:

The line through (6, 2) is $y - 2 = m (x - 6)$

$y = mx + 2 - 6m$

Now from condition of tangency, $(2 - 6m)^2 = 25m^2 - 16$

$36m^2 + 4 - 24m - 25m^2 + 16 = 0$

$11m^2 - 24m + 20 = 0$

Obviously its roots are m_1 and m_2, therefore $m_1 + m_2 = 24 / 11$ and $m_1m_2 = 20 / 11$.

Question 12: The eccentricity of the curve represented by the equation $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is __________.

Solution:

Equation $x^2 + 2y^2 - 2x + 3y + 2 = 0$ can be written as $(x - 1)^2 / 2 + (y + 3 / 4)^2 = 1 / 16$

$[(x - 1)^2] / (1/8) + [(y + 3 / 4)^2] / (1/16) = 1$, which is an ellipse with $a^2 = 1 / 8$ and $b^2 = 1 / 16$

$e = \sqrt{(1 - b^2/a^2)}$

$e^2 = 1 - 1 / 2$

$e = 1 / \sqrt{2}$

Question 13: The foci of the ellipse $25 (x + 1)^2 + 9 (y + 2)^2 = 225$ are at ___________.

Solution:

$[25(x + 1)^2/225] + [9(y + 2)^2/225] = 1$

Here, $a = \sqrt{[225 / 25]} = 15 / 5 = 3$

$b = \sqrt{[225/9]} = 15 / 3 = 5$

Major axis of the ellipse lies along the y-axis.

$b^2 = a^2(1 - e^2)$

$e = \sqrt{[1 - 9 / 25]} = 4 / 5$

Focus = $(-1, -2 \pm [15 / 3] * [4 / 5])$

$= (-1, -2 \pm 4)$

$= (-1, 2); (-1, -6)$

Question 14: The locus of a variable point whose distance from (2, 0) is 2/3 times its distance from the line $x = -9 / 2$, is __________.

Solution:

Let point P be (x_1, y_1) and Q(2, 0).

Given line => $x = -9/2$

=> $2x + 9 = 0$

Let PM be distance from P to line $2x + 9 = 0$.

So, PQ = (2/3)PM

$= (2/3)(x_1 + 9/2)$

$(x_1 + 2)^2 + y^2_1 = 4/9\ (x_1 + 9 / 2)^2$

$9\ [x_1^2 + y^2_1 + 4x_1 + 4] = 4(x_1^2 + (81 / 4) + 9x_1)$

$5x_1^2 + 9y^2_1 = 45$

$(x_1^2 / 9) + (y^2_1 / 5) = 1$,

Locus of (x_1, y_1) is $(x^2/9) + (y^2/5) = 1$, which is the equation of an ellipse.

Question 15: The equation of the ellipse whose latus rectum is 8 and whose eccentricity is $1 / \sqrt{2}$, referred to the principal axes of coordinates, is __________.

Solution:

Equation of ellipse = $(x^2/a^2) + (y^2/b^2) = 1$

Latus rectum = $2b^2/a = 8$

=> $b^2 = 4a$ ..(i)

Given $e = 1/\sqrt{2}$

We know $e^2 = 1 - (b^2/a^2)$

$1/2 = 1 - 4/a$

Solving we get a = 8.

So $b^2 = 4a = 32$

So $a^2 = 64$, $b^2 = 32$

Hence, the required equation of ellipse is $(x^2/64) + (y^2/32) = 1$.