Question 1: Let (x_0, y_0) be the solution of the following equations.

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$. Then x_0 is

(a) 1/6

(b) 1/3

(c) 1/2

(d) 6

Solution:

Given that $(2x)^{\ln 2} = (3y)^{\ln 3}$ ..(i)

Taking log on both sides

log 2 log (2x) = log 3 log (3y)

log 2 (log 2 + log x) = log 3 (log 3 + log y) ..(ii)

Also $3^{\ln x} = 2^{\ln y}$ ..(iii)

Taking log on both sides

log x log 3 = log y log 2

log y = log x log 3/log 2 …(iv)

Substitute (iv) in (ii)

log 2 (log 2 + log x) = log 3 (log 3 + log x log 3/log 2)

$(\log 2)^2 + \log 2 \log x = (\log 3)^2 + \log x (\log 3)^2/\log 2$

$[\log 2 - (\log 3)^2/\log 2] \log x = (\log 3)^2 - (\log 2)^2$

$\log x = [(\log 3)^2 - (\log 2)^2]/[((\log 2)^2 - (\log 3)^2)/\log 2]$

log x = - log 2

$\log x = \log 2^{-1}$

$\Rightarrow x = 2^{-1}$

= 1/2

Hence option c is the answer.

Question 2: If $3^x = 4^{x-1}$, then x =

Solution:

Given that $3^x = 4^{x-1}$

Take log on both sides

$\log 3^x = \log 4^{x-1}$

$x \log 3 = (x-1) \log 4$

$= x \log 4 - \log 4$

$\log 4 = x(\log 4 - \log 3)$

So $x = \log 4/(\log 4 - \log 3)$

$= 1/(1 - \log 3/\log 4)$

$= 1/(1 - \log_4 3)$

Hence the value of x is $1/(1 - \log_4 3)$.

Question 3: The number of positive integers satisfying the equation $x + \log_{10}(2^x + 1) = x \log_{10}5 + \log_{10} 6$ is

(a) 0

(b) 1

(c) 2

(d) infinite

Solution:

$x + \log_{10}(2^x + 1) = x \log_{10}5 + \log_{10} 6$

$x [1 - \log_{10}5] + \log_{10}(2^x + 1) = \log_{10} 6$

$x [\log_{10} 10 - \log_{10} 5] + \log_{10}(2^x + 1) = \log_{10} 6$

$x \log_{10}2 + \log_{10}(2^x + 1) = \log_{10} 6$

$\log_{10} 2^x (2^x + 1) = \log_{10} 6$

$(2^x)^2 + 2^x - 6 = 0$

$2^x = 2$ or $2^x = -3$ (rejected)

=> $x = 1$

So number of positive integers = 1

Hence option b is the answer.

Question 4: If $\log_7 2 = k$, then $\log_{49} 28$ is equal to

(a) $(1+2k)/4$

(b) $(1+2k)/2$

(c) $(1+2k)/3$

(d) none of the above

Solution:

Given $\log_7 2 = k$

$\log_{49} 28 = \log_{7^2} 28$

$= \frac{1}{2} \log_7 28$

$= \frac{1}{2} \log_7 (4 \times 7)$

$= \frac{1}{2} \log_7 4 + \frac{1}{2} \log_7 7$

$= \frac{1}{2} \log_7 2^2 + \frac{1}{2}$

$= \log_7 2 + \frac{1}{2}$

$= k + \frac{1}{2}$

$= (2k+1)/2$

Hence option b is the answer.

Question 5: If $\log_a x = b$ for permissible values of a and x then which is/are correct.

(a) If a and b are two irrational numbers, then x can be rational

(b) If a is rational and b is irrational, then x can be rational

(c) If a is irrational and b is rational, then x can be rational

(d) If a and b are rational, then x can be rational

Solution:

Given that $\log_a x = b$

=> $x = e^b/a$

Case 1: Let, b be any irrational number and a be equal to $a = e^b$ that means a is irrational because e is irrational. So $x = e^b/a$ becomes rational.

Hence option a is correct.

Case 2: Let a be rational and b is irrational.

If we take any rational number as a and b = ln a, then $x = e^b/a = e^{\ln a}/a = a/a = 1$.

So x is rational.

Hence option b is correct.

Case 3: Let a be irrational and b is rational,

If we consider any rational number as b and $a = e^b$

Then $x = e^b/e^b = 1$ is rational.

Hence option c is correct.

Case 4: let a and b be rational.

If we take a = any rational number and b = 0, x can be rational.

Hence option d is correct.

So option a, b, c and d are correct.

Question 6: If x = 9 is a solution of $\ln(x^2 + 15a^2) - \ln(a-2) = \ln(8ax/(a-2))$ then

(a) a = 3/5

(b) a = 3

(c) x = 15

(d) x = 2

Solution:

Given $\ln(x^2 + 15a^2) - \ln(a-2) = \ln(8ax/(a-2))$

$\ln[(x^2 + 15a^2)/(a-2)] = \ln(8ax/(a-2))$

=> $(x^2 + 15a^2)/(a-2) = (8ax/(a-2))$

=> $x^2 + 15a^2 = 8ax$

=> $x^2 + 15a^2 - 8ax = 0$ ..(i)

Given x = 9 is a root.

=> $81 + 15a^2 - 72a = 0$

=> $5a^2 - 24a + 27 = 0$

=> (5a-9) (a-3) = 0

=> a = 9/5 or a = 3

Put value of a in (i)

When a = 9/5, we get x = 9 or x = 27/5

When a = 3, we get x = 9 or x = 15

Hence option b and c are correct.

Question 7: If $\log_{10} 5 = a$ and $\log_{10} 3 = b$ then

(a) $\log_{30} 8 = 3(1-a)/(b+1)$

(b) $\log_{40} 15 = (a+b)/(3-2a)$

(c) $\log_{243} 32 = (1-a)/b$

(d) none of these

Solution:

Given $\log_{10} 5 = a$ and $\log_{10} 3 = b$

We check all the given options.

$\log_{30} 8 = \log 2^3/\log (3\times10)$

= 3 log 2/(log 3 + log 10)

= 3 log (10/5)/(1 + log 3)

= 3 (1 - log 5)/(1 + log 3)

= 3(1 - a)/(1+b)

Hence option a is correct.

$\log_{40} 15 = \log 15/\log 40$

$= \log(3\times5)/\log(10\times4)$

$= (\log 3 + \log 5)/(1 + \log 2^2)$

$= (\log 3 + \log 5)/(1 + 2 \log 2)$

$= (\log 3 + \log 5)/(1 + 2 \log (10/5))$

$= (\log 3 + \log 5)/(1 + 2 (1 - \log 5))$

$= (\log 3 + \log 5)/(1 + 2 - 2 \log 5)$

$= (b+a)/(3 - 2a)$

Hence option b is correct.

$\log_{243} 32 = \log 32/\log 243$

$= \log 2^5/\log 3^5$

$= 5 \log 2/5 \log 3$

$= \log 2/\log 3$

$= (1 - \log 5)/\log 3$ (since $\log 2 = \log (10/5) = \log 10 - \log 5 = 1 - \log 5$)

$= (1-a)/b$

So option c is correct.

Hence option a, b and c are correct.

Question 8: The value of

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$$

is

(a) 8

(b) 1

(c) 2

(d) none of these

Solution:

$$\log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$$

$$=(\log_2 9)^{2\log^{2}_{\log_2 9}} \times 7^{\frac{1}{2}\log_7 4}$$

$$= 4 \times 2 = 8$$

Hence option a is the answer.

Question 9: If $\log_{10} 2 = a$ and $\log_{10} 3 = b$ then log 60 can be expressed in terms of a and b as

(a) a+b+1

(b) a-b+1

(c) a-b-1

(d) a+b-1

Solution:

Given $\log_{10} 2 = a$ and $\log_{10} 3 = b$

log 60 = log (10×2×3)

= log 10 + log 2 + log 3

= 1 + a + b

Hence option a is the answer.

Question 10: If $A = \log_2 \log_2 \log_4 256 + \log_{\sqrt{2}} 2$, then A is equal to

(a) 2

(b) 3

(c) 5

(d) 7

Solution:

We use the property $\log_a (m)^n = n \log_a m$

$\log_2 \log_2 \log_4 256 + \log_{\sqrt{2}} 2 = \log_2 \log_2 \log_4 (4)^4 + \log_{\sqrt{2}} \sqrt{2}^2$

$= \log_2 \log_2 4 + 2(2)$

$= \log_2 \log_2 2^2 + 4$

$= \log_2 2 + 4$

$= 1+4$ ($\log_a a = 1$)

$= 5$

Hence option c is the answer.