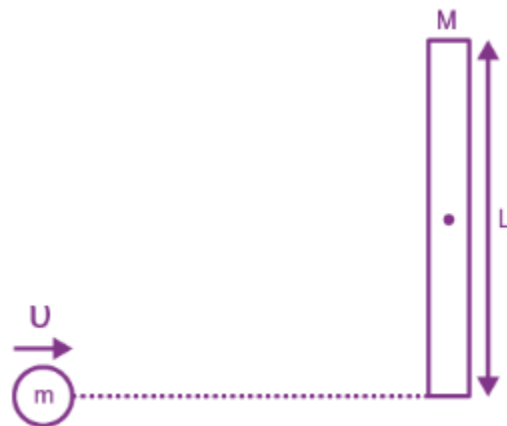


Question 1) A particle of mass m moving with a speed u collides elastically with the end of a uniform rod of mass M and length L perpendicularly as shown in the figure. If the particle comes to rest after collision, find the value of m/M .



(A) $1/3$

(B) $1/2$

(C) $1/4$

(D) $1/5$

Solution: (A) $1/3$

Applying conservation of angular momentum about the Centre of mass of rod

$$mv \left(\frac{L}{2} \right) = M \left(\frac{L^2}{12} \right) \omega$$

$$\omega = 6mu/ML$$

Conservation of Kinetic Energy

$$(1/2)mu^2 = (1/2)(ML^2/12)\omega^2$$

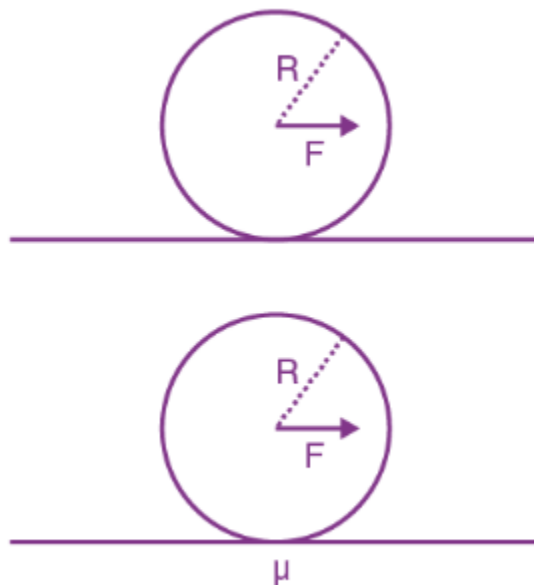
$$mu^2 = (ML^2/12)\omega^2$$

$$mu^2 = (ML^2/12)[36m^2u^2/M^2L^2]$$

$$\Rightarrow m/M = 1/3$$

Question 2) A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R , perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ . The center of mass of the object has an acceleration a . The acceleration due to gravity is g . Given that the object rolls without slipping, which of the following statement(s) is(are) correct?

- (A) For the same F , the value of a does not depend on whether the cylinder is solid or hollow
- (B) For a solid cylinder, the maximum possible value of a is $2\mu g$
- (C) The magnitude of the frictional force on the object due to the ground is always μmg
- (D) For a thin-walled hollow cylinder, $a = F/2m$



Answer (B, D)

Solution:

For solid cylinder,

$$F \times R = \frac{3}{2}mR^2 \times \frac{a}{R}$$

$$\Rightarrow a = 2F/3m$$

For hollow cylinder,

$$F \times R = 2mR^2 \times \frac{a}{R}$$

$$\Rightarrow a = F/3m$$

For solid cylinder

$$f = F - m \times (2F/3m) = (F/3) \leq \mu mg$$

$$\Rightarrow F \leq 3\mu mg$$

Therefore,

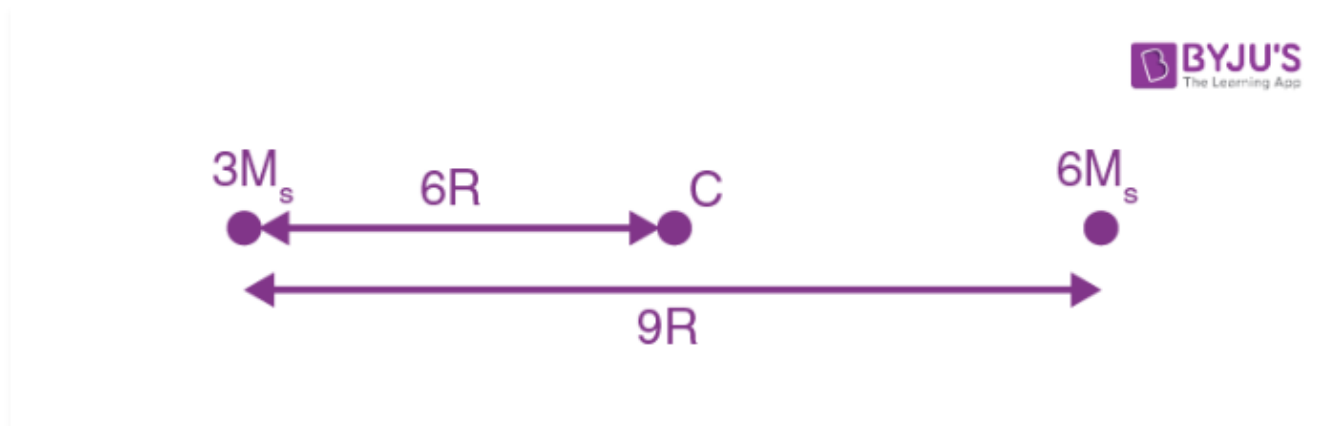
$$a \leq (2/3m) \times (3\mu mg)$$

$$\Rightarrow a_{\max} = 2\mu g$$

Question 3) The distance between two stars of masses $3M_s$ and $6M_s$ is $9R$. Here R is the mean distance between the centres of the Earth and the Sun, and M_s is the mass of the Sun. The two stars orbit around their common centre of mass in circular orbits with period nT , where T is the period of Earth's revolution around the Sun. The value of n is ____.

Answer: (9)

Solution:



Centre of mass of system lies at $6R$ from lighter mass

$$[3M_s \omega^2 \times 6R] = \frac{G(18M_s^2)}{81R^2}$$

$$\omega^2 = \frac{GM}{81R^3}$$

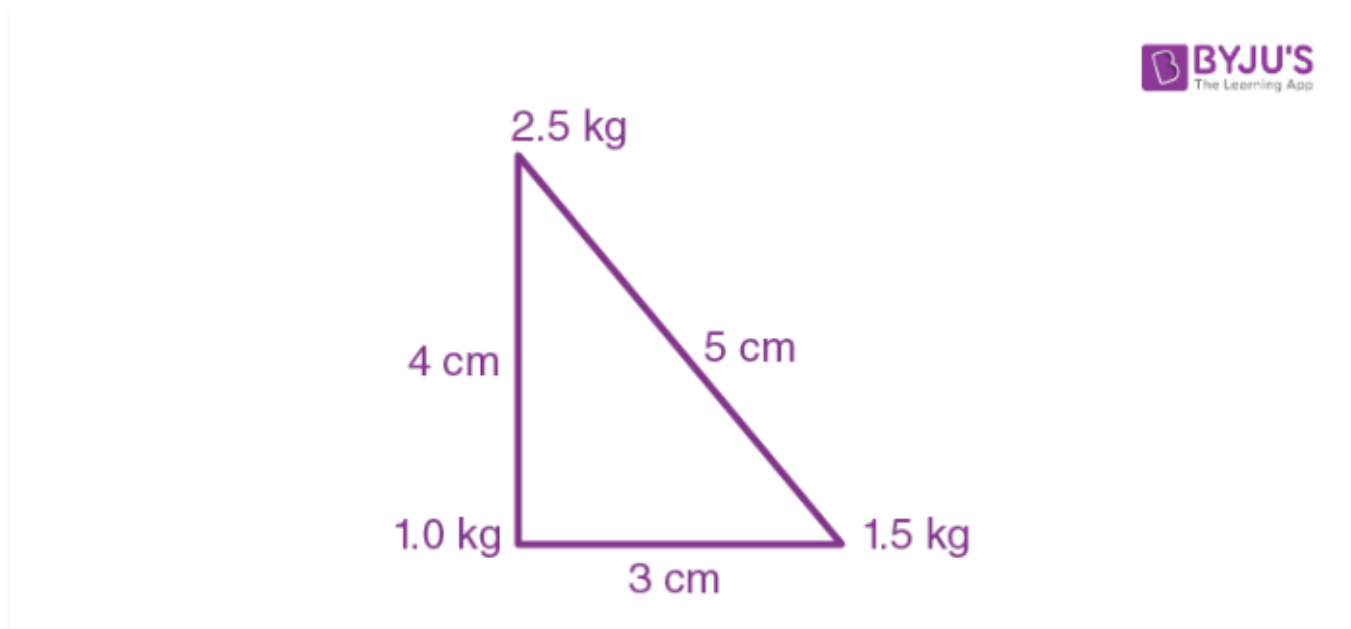
$$T' = 2\pi/\omega$$

$$T' = 2\pi \sqrt{\frac{81R^3}{GM_s}}$$

$$T' = 9T$$

$$n = 9$$

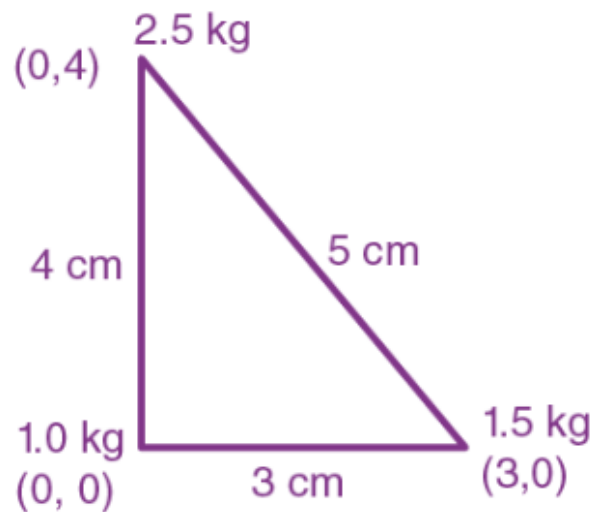
Question 4) Three-point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at a point:



- (A) 0.6 cm right and 2.0 cm above 1 kg mass
- (B) 1.5 cm right and 1.2 cm above 1 kg mass
- (C) 2.0 cm right and 0.9 cm above 1 kg mass
- (D) 0.9 cm right and 2.0 cm above 1 kg mass

Answer: (D) 0.9 cm right and 2.0 cm above 1 kg mass

Solution:



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

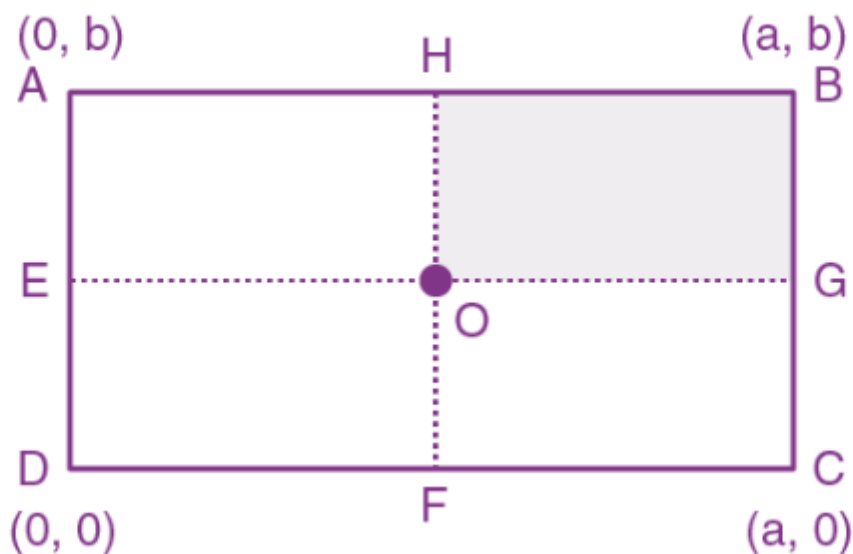
$$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = \frac{1.5 \times 3}{5} = 0.9 \text{ cm}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = \frac{2.5 \times 4}{5} = 2 \text{ cm}$$

Hence, centre of mass of system is at point (0.9, 2)

Question 5) A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be:



(A)

$$\left[\frac{3a}{4}, \frac{3b}{4} \right]$$

(B)

$$\left[\frac{5a}{3}, \frac{5b}{3} \right]$$

(C)

$$\left[\frac{2a}{3}, \frac{2b}{3} \right]$$

(D)

$$\left[\frac{5a}{12}, \frac{5b}{12} \right]$$

Answer: (D)

$$\left[\frac{5a}{12}, \frac{5b}{12} \right]$$

Solution:

From the figure

$$x_1 = a/4$$

$$x_2 = a/4$$

$$x_3 = 3a/4$$

$$x_{\text{com}} = (m_1x_1 + m_2x_2 + m_3x_3)/(m_1 + m_2 + m_3)$$

$$= (m(a/4) + m(a/4) + m(3a/4))/(m + m + m)$$

$$= 5a/12$$

$$y_1 = 3b/4$$

$$y_2 = b/4$$

$$y_3 = b/4$$

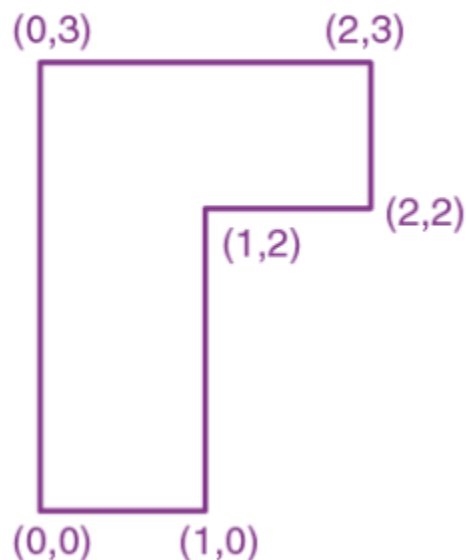
$$y_{\text{com}} = (m_1y_1 + m_2y_2 + m_3y_3)/(m_1 + m_2 + m_3)$$

$$= (m(3b/4) + m(b/4) + m(b/4))/(m + m + m)$$

$$= 5b/12$$

$$(x_{\text{com}}, y_{\text{com}}) = [5a/12, 5b/12]$$

Question 6) The coordinates of the centre of mass of a uniform flag-shaped lamina of mass 4 Kg. (The coordinates of the same are shown in figure) are



(A) (1.25 m, 1.50 m)

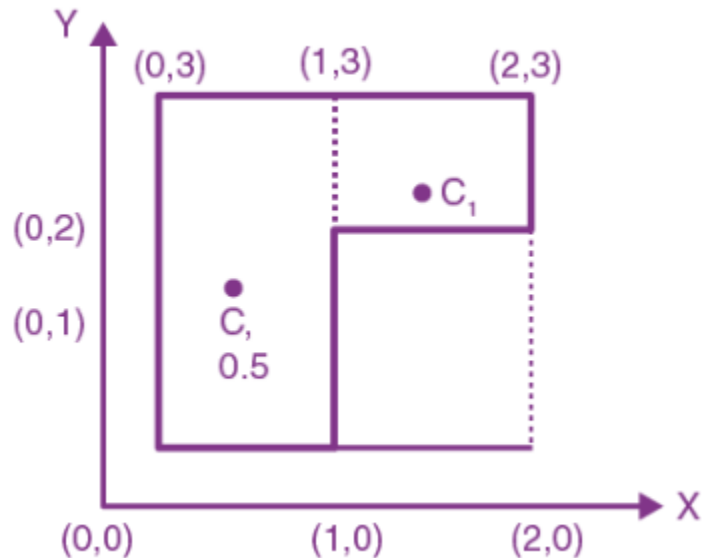
(B) (0.75 m, 1.75 m)

(C) (0.75 m, 0.75 m)

(D) (1 m, 1.75 m)

Answer: (B) (0.75 m, 1.75 m)

Solution:



For given lamina

$$m_1 = 1, C_1 = (1.5, 2.5)$$

$$m_2 = 3, C_2 = (0.5, 1.5)$$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

Therefore, the coordinates of the centre of mass of flag-shaped lamina is (0.75, 1.75)

Question 7) Two particles A and B initially at rest, move towards each other under the mutual force of attraction. At the instant when the speed of A is V and the speed of B is 2V, the speed of the centre of mass of the system is

- (A) 3 V
- (B) V
- (C) 1.5 V
- (D) zero

Answer: (D) zero

Solution:

Net external force on system is zero. Therefore centre of mass always remains at rest.

Question 8) A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is _____

(A) $W(d-x)/d$

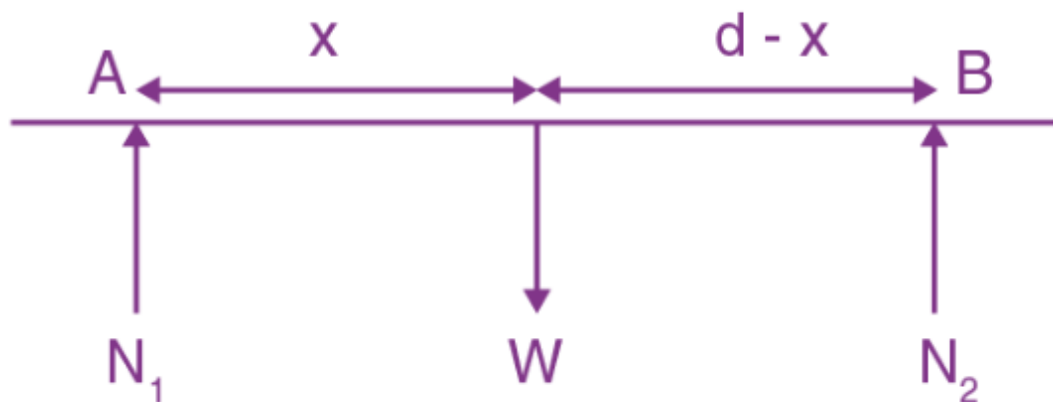
(B) Wx/d

(C) Wd/s

(D) $W(d-x)/x$

Answer: (A) $W(d-x)/d$

Solution:



N_1 = normal reaction on A

N_2 = Normal reaction on B

W = weight of the rod

In vertical equilibrium,

$$N_1 + N_2 = W \text{ ———(1)}$$

Torque balance about centre of mass of the rod

$$N_1 x = N_2 (d-x)$$

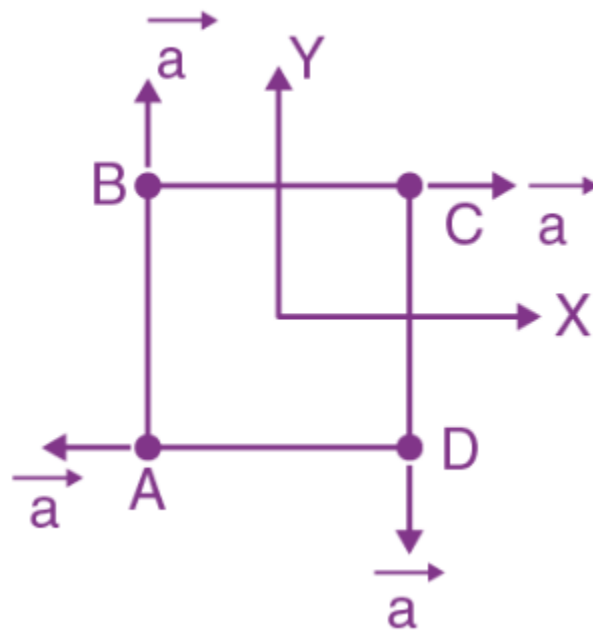
Putting the value of N_2 from equation (1)

$$N_1 x = (W - N_1)(d-x)$$

$$\Rightarrow N_1 x = Wd - Wx - N_1 d + N_1 x$$

$$N_1 = W(d-x)/d$$

Question 9) Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of the square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is



(A) $\frac{a}{5}(\hat{i} - \hat{j})$

(B) a

(C) Zero

(D) $\frac{a}{5}(\hat{i} + \hat{j})$

Answer: (A)

Solution:

Acceleration of centre of mass (a_{cm}) is given by

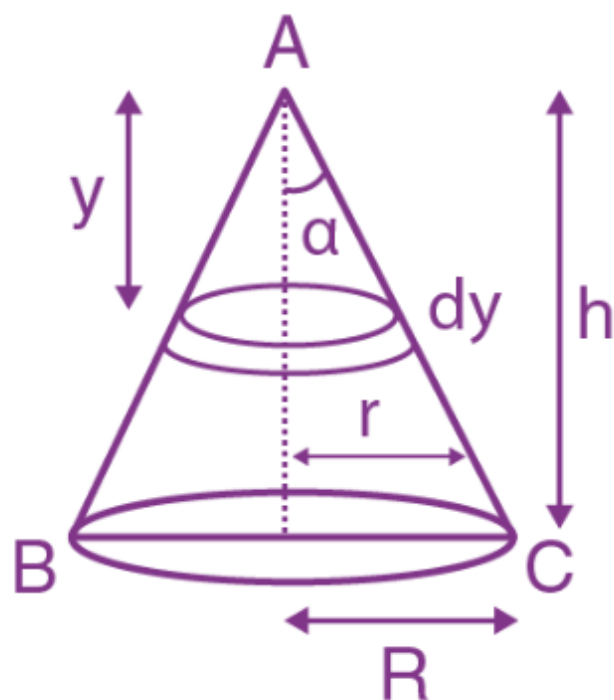
$$\begin{aligned}\therefore \vec{a}_{\text{cm}} &= \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots}{m_1 + m_2 + \dots} \\ &= \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m} \\ &= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})\end{aligned}$$

Question 10) Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h then z_0 is equal to:

- (A) $5h/8$
- (B) $3h^2/8R$
- (C) $h^2/4R$
- (D) $3h/4$

Answer: (D) $3h/4$

Solution:



Let density of cone = ρ .

$$\text{Centre of mass, } y_{cm} = \frac{\int y dm}{\int dm}$$

$$= \frac{\int_0^h y \pi r^2 dy \rho}{\frac{1}{3} \pi R^2 h \rho} = \frac{\int_0^h r^2 y dy}{\frac{1}{3} R^2 h} \quad \dots(i)$$

For a cone, we know that

$$\frac{r}{R} = \frac{y}{h} \quad \therefore r = \frac{y}{h} R$$

$$y_{cm} = \frac{\int_0^h 3y^3 dy}{h^3} = \frac{3 \left[\frac{y^4}{4} \right]_0^h}{h^3} = \frac{3}{4} h$$