

Question 1) When the temperature of a metal wire is increased from 0°C to 10°C , its length increases by 0.02%. The percentage change in its mass density will be closest to:

- (A) 0.06
- (B) 2.3
- (C) 0.008
- (D) 0.8

Answer: (A) 0.06

Solution:

Change in the length of the metal wire (Δl) when its temperature is changed by ΔT is given by

$$\Delta l = l\alpha\Delta T$$

Here, α = coefficient of linear expansion

$$\Delta l = 0.02\%, \Delta T = 10^{\circ}\text{C}$$

$$\text{Therefore, } \alpha = \Delta l / l\Delta T$$

$$= 0.02 / (100 \times 10)$$

$$\Rightarrow \alpha = 2 \times 10^{-5}$$

$$\text{Volume coefficient of expansion, } \gamma = 3\alpha = 6 \times 10^{-5}$$

$$\text{Since } \rho = M/V$$

$$(\Delta V/V) \times 100 = \gamma\Delta T = (6 \times 10^{-5} \times 10 \times 100) = 6 \times 10^{-2}$$

Volume increase by 0.06% therefore density decreases by 0.06%

Question 2) On a linear temperature scale Y, water freezes at -160°Y and boils at -50°Y . On this Y scale, a temperature of 340 K would be read as (water freezes at 273 K and boils at 373 K)

- (A) -73.70°Y
- (B) -233.70°Y
- (C) -86.3°Y
- (D) -106.3°Y

Answer: (C) -86.3°Y

Solution:

(Reading on any scale - LFP)/(UFP - LFP)

= constant for all scales

$$(340 - 273)/(373 - 273) = (Y - (-160))/(-50 - (-160))$$

$$\Rightarrow (67/100) = (Y + 160)/110$$

$$Y = -86.3^{\circ} \text{C}$$

Question 3) A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C . The coefficient of expansion of cubical metal is less than that of alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

(A) $W_1 > W_2$

(B) $W_1 = W_2$

(C) $W_1 < W_2$

(D) None of these

Answer: (C) $W_1 < W_2$

Solution:

$$W_1 = mg - Vd_a g$$

$$W_2 = mg - V'd'_a g = mg - V(1 + 50\gamma_b) \frac{d_a g}{(1 + 50\gamma_a)}$$

$$= mg - Vd_a g \left[\frac{1 + 50\gamma_b}{1 + 50\gamma_a} \right]$$

Given $\gamma_b < \gamma_a$

$$\therefore 1 + 50\gamma_b < 1 + 50\gamma_a \quad \text{or,} \quad \frac{1 + 50\gamma_b}{1 + 50\gamma_a} < 1$$

$$\therefore W_2 > W_1 \text{ or } W_1 < W_2$$

Question 4) A non-isotropic solid metal cube has a coefficient of linear expansion as $5 \times 10^{-5} / ^{\circ}\text{C}$ along the x-axis and $5 \times 10^{-6} / ^{\circ}\text{C}$ along the Y and the Z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6} / ^{\circ}\text{C}$ then the value of C is _____

Answer: 60

Solution:

Volume, $V = lbh$

$$\gamma = (\Delta V/V\Delta t) = (\Delta l/l\Delta t) + (\Delta b/b\Delta t) + (\Delta h/h\Delta t)$$

(γ = coefficient of volume expansion)

$$\Rightarrow \gamma = 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$$

$$= 60 \times 10^{-6} / ^\circ\text{C}$$

Therefore, value of $C = 60.00$

Question 5) A bakelite beaker has a volume capacity of 500 cc at 30°C . When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the volume of the beaker remains constant as the temperature is varied. If $\gamma_{\text{beaker}} = 6 \times 10^{-6} / ^\circ\text{C}$ and $\gamma_{\text{mercury}} = 1.5 \times 10^{-4} / ^\circ\text{C}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to

Answer: 20

Solution:

Volume capacity of the beaker, $V_0 = 500$ cc

$$V_b = V_0 + V_0\gamma_{\text{beaker}} \Delta T$$

When beaker is partially filled with V_m volume of mercury

$$V_b' = V_m + V_m\gamma_m \Delta T$$

$$\text{Unfilled volume } (V_0 - V_m) = (V_b - V_m')$$

$$\Rightarrow V_0 \gamma_{\text{beaker}} = V_m \gamma_m$$

$$\text{Therefore, } V_m = (V_0 \gamma_{\text{beaker}}) / \gamma_m$$

$$= (500 \times 6 \times 10^{-6}) / (1.5 \times 10^{-4})$$

$$= 20 \text{ cc}$$

Question 6) Two rods A and B of identical dimensions are at temperature 30°C . If A is heated upto 180°C and B upto $T^\circ\text{C}$, then the new lengths are the same. If the ratio of the coefficient of linear expansion of A and B is 4:3, then the value of T is

(A) 230°C

(B) 270°C

(C) 200°C

(D) 250°C

Answer: (A) 230°C

Solution:

Change in length in both rods are same

$$\text{i.e } \Delta l_1 = \Delta l_2$$

$$l\alpha_1\Delta\theta_1 = l\alpha_2\Delta\theta_2$$

$$(\alpha_1/\alpha_2) = (\Delta\theta_2/\Delta\theta_1)$$

$$(4/3) = (\theta - 30)/(180 - 30)$$

$$\theta = 230^\circ \text{ C}$$

Question 7) A rod, of length L at room temperature and uniform area of cross-section A , is made of a metal having a coefficient of linear expansion $\alpha/^\circ\text{C}$. It is observed that an external compressive force F , is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is

- (A) $F/A\alpha\Delta T$
- (B) $F/A\alpha(\Delta T - 273)$
- (C) $F/2A\alpha\Delta T$
- (D) $2F/A\alpha\Delta T$

Answer: (A) $F/A\alpha\Delta T$

Solution:

$$\text{Young's modulus } Y = \text{Stress/Strain} = (F/A)/((\Delta l/l))$$

Using coefficient of linear expansion

$$\alpha = \Delta l/l\Delta T$$

$$\Rightarrow \Delta l/l = \alpha \Delta T$$

$$\text{Therefore, } Y = F/A\alpha\Delta T$$

Question 8) At 40°C , a brass wire of 1mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to

(Coefficient of linear expansion and Young's modulus of brass are $10^{-5}/^\circ\text{C}$ and 10^{11} N/m^2 , respectively; $g = 10 \text{ m/s}^2$)

- (A) 9 Kg
- (B) 0.5 Kg

(C) 1.5 kg

(D) 0.9 kg

Answer: 6.28 Kg (Bonus)

Solution:

$$\Delta_{\text{temp}} = \Delta_{\text{load}}$$

$$\text{and } A = \pi r^2 = \pi(10^{-3})^2 = \pi \times 10^{-6}$$

$$L\alpha\Delta T = FL/AY$$

$$0.2 \times 10^{-5} \times 20 = (F \times 0.2) / (\pi \times 10^{-6}) \times 10^{11}$$

$$F = 20\pi \text{ N}$$

$$\text{Therefore, } m = F/g = 2\pi = 6.28 \text{ Kg}$$

Question 9) An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating it. The temperature should be raised by

(A) $3\alpha/PK$

(B) $3PK\alpha$

(C) $P/3\alpha K$

(D) $P/\alpha K$

Answer: (C) $P/3\alpha K$

Solution:

As we know, Bulk modulus

$$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$

$$V = V_0 (1 + \gamma\Delta t)$$

$$\frac{\Delta V}{V_0} = \gamma\Delta t \quad \therefore \quad \frac{P}{K} = \gamma\Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

Question 10) A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT . The net change in its length is zero. Let l be the length of the rod, A its area of cross-section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to:

(A) $l^2 Y \alpha \Delta T$

(B) $l A Y \alpha \Delta T$

(C) $A Y \alpha \Delta T$

(D) $A Y / \alpha \Delta T$

Answer: (C) $A Y \alpha \Delta T$

Solution:

Due to thermal expansion, change in length $(\Delta l) = l \alpha \Delta T$

Young's modulus (Y) = Normal Stress/Longitudinal Strain

$$Y = (F/A)/(\Delta l/l)$$

$$\Rightarrow \Delta l = F l / A Y$$

$$F l / A Y = l \alpha \Delta T$$

$$F = A Y \alpha \Delta T$$