Question 1: If PQR be a triangle of area Δ with a = 2, b = 7/2 and c = 5/2, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R, respectively. Then (2 sin P - sin 2P)/(2 sin P + sin 2P) is equal to

(a) 3/4Δ

(b) 45/4Δ

(c) $(3/4\Delta)^2$

(d) $(45/4\Delta)^5$

Solution:

Given a = 2, b = 7/2 and c = 5/2

s = (a + b + c)/2

= 4

(2 sin P - sin 2P)/(2 sin P + sin 2P) = (2 sin P - 2 sin P cos P)/(2 sin P + 2 sin P cos P)

= 2 sin P(1 - cos P)/2 sin P(1 + cos P)

= (1 - cos P)/(1 + cos P)

= $2 \sin^2 (P/2)/2 \cos^2 (P/2)$

= $\tan^2 (P/2)$

= (s - b)(s - c)/s(s - a) (since tan (P/2) = √((s - b)(s - c)/s(s - a))

= $(s - b)^2(s - c)^2/s(s - a)(s - b)(s - c)$ (multiply numerator and denominator by (s - b)(s - c))

= $(4 - 7/2)^2(4 - 5/2)^2/\Delta^2$

= $(½ \times 3/2)^2/\Delta^2$

= $(3/4\Delta)^2$

Hence option (c) is the answer.

Question 2: Let ABC and ABC' be two non congruent triangles with sides AB = 4, AC = AC' = 2√2 and angle B = 30^0. The absolute value of the difference between the areas of these triangles is

Solution:

Given AB = 4, AC = AC' = 2√2

∠B = 30^0

Using cosine rule, cos B = $(a^2 + c^2 - b^2)/2ac$

$\sqrt{3}/2 = (a^2 + 16 - 8)/8a$

$\sqrt{3}/2 = (a^2 + 8)/8a$

$a^2 - 4\sqrt{3}a + 8 = 0$

Let a_1 and a_2 be the roots of this equation.

$a_1 + a_2 = 4\sqrt{3}$

$a_1a_2 = 8$

$|a_1 - a_2|^2 = (a_1 + a_2)^2 - 4a_1a_2$

$= 48 - 32$

$= 16$

$|a_1 - a_2| = 4$

$|\Delta_1 - \Delta_2| = ½ |a_1 - a_2|c \sin B$

$= ½\ 4 \sin 30^0 \times 4$

$= 4$

Question 3: If the angles of a triangle are in the ratio 1 : 3: 5 and θ denotes the smallest angle, then the ratio of the largest side to the smallest side of the triangle is

(a) $(\sqrt{3} \sin \theta + \cos \theta)/2 \sin \theta$

(b) $(\sqrt{3} \cos \theta - \sin \theta)/2 \sin \theta$

(c) $(\cos \theta + \sqrt{3} \sin \theta)/2 \sin \theta$

(d) $(\sqrt{3} \cos \theta + \sin \theta)/2 \sin \theta$

Solution:

Given angles are in the ratio 1 : 3 : 5.

So $\theta + 3\theta + 5\theta = 180$

$9\theta = 180$

So $\theta = 20^0$

Let $A = 20^0$, $B = 60^0$ and $C = 100^0$

$a/\sin A = b/\sin B = c/\sin C$

$c/a = \sin C/\sin A = \sin (120 - \theta)/\sin \theta$

$= (\sin 120 \cos \theta - \cos 120 \sin \theta)/\sin \theta$

$= ((\sqrt{3}/2) \cos \theta + ½ \sin \theta)/\sin \theta$

$= (\sqrt{3} \cos \theta + \sin \theta)/2 \sin \theta$

Hence option (d) is the answer.

Question 4: If R and r are the radii of the circumcircle and incircle of a regular polygon of n sides, each side being of length a, then a is equal to

(a) $2(R + r) \sin (\pi/2n)$

(b) $2(R + r) \tan (\pi/2n)$

(c) $2(R + r)$

(d) none of these

Solution:

We know $R = ½ a \operatorname{cosec} (\pi/n)$

$r = ½ a \cot (\pi/n)$

So $2(R + r) = a (1/\sin (\pi/n) + \cos (\pi/n)/\sin (\pi/n))$

$= a (2 \cos^2 (\pi/2n))/2 \sin (\pi/2n) \cos (\pi/2n)$

$= a \cot (\pi/2n)$

So $a = 2(R + r) /\cot (\pi/2n)$

$= 2(R + r) \tan (\pi/2n)$

Hence option (b) is the answer.

Question 5: In a triangle, the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is a third side of the triangle, then the ratio of the in-radius to the circumradius of the triangle is

(a) $3y/2x(x+c)$

(b) $3y/2c(x+c)$

(c) $3y/4x(x+c)$

(d) $3y/4c(x+c)$

Solution:

In-radius = area/semi perimeter

= Δ/s

Circumradius = abc/4Δ

In-radius/circumradius = $4\Delta^2$/abcs ..(i)

Let a and b be the two sides of the triangle.

a+b = x

ab = y

Given $x^2 - c^2 = y$

a+b+c = x+c

Semi perimeter, s = (a+b+c)/2 = (x+c)/2

So (i) becomes

In-radius/circumradius = $8\Delta^2$/yc(x+c)

Δ = ½ ab sin C

cos C = $(a^2+b^2-c^2)/2ab$

= $(x^2 - 2y - c^2)/2y$

= -1/2

=> C = 120^0

sin C = √3/2

Δ = ½ y. √3/2

= (√3/4)y

In-radius/circumradius = $8\Delta^2$/yc(x+c)

= $24y^2$/16yc(x+c)

= 3y/2c(x+c)

Hence option b is the answer.

Question 6: If d_1, d_2, d_3 are the diameters of the three escribed circles of a triangle, then $d_1d_2 + d_2d_3 + d_3d_1$ is

equal to

(a) $4s^2$

(b) Δ^2

(c) $4\Delta^2$

(d) $2\Delta^2$

Solution:

We know $d_1 = 2r_1$, $d_2 = 2r_2, d_3 = 2r_3$

We have $r_1 = \Delta/(s - a)$

$r_2 = \Delta/(s - b)$

$r_3 = \Delta/(s - c)$

$d_1d_2 + d_2d_3 + d_3d_1 = 4(r_1r_2 + r_2r_3 + r_3r_1)$

$= 4\Delta^2(1/(s - a)(s - b) + 1/(s - b)(s - c) + 1/(s - c)(s - a))$

$= 4\Delta^2(s - c + s - a + s - b)/(s - a)(s - b)(s - c)$

$= 4\Delta^2(3s - (a + b + c))/(s - a)(s - b)(s - c)$

$= 4\Delta^2(3s - 2s)/(s - a)(s - b)(s - c)$

$= 4\Delta^2 s/(s - a)(s - b)(s - c)$

Multiply numerator and denominator by s

$= 4\Delta^2 s^2/s(s - a)(s - b)(s - c)$

$= 4\Delta^2 s^2/\Delta^2$

$= 4s^2$

Hence option (a) is the answer.

Question 7: If in a triangle ABC, sin C + cos C + sin (2B + C) - cos (2B + C) = 2√2, the the triangle ABC is

(a) equilateral

(b) scalene

(c) isosceles right-angled

(d) obtuse-angled

Solution:

Given sin C + cos C + sin (2B + C) - cos (2B + C) = $2\sqrt{2}$

Rearranging the equation, we get

sin C + sin (2B + C) + cos C - cos (2B + C) = $2\sqrt{2}$ ..(i)

We know that sin A + sin B = 2sin (A + B)/2 cos (A - B)/2

So (i) => 2 sin (B + C) cos B + 2 sin (B + C) sin B = $2\sqrt{2}$

=> sin (π - A) cos B + sin (π - A) sin B = $\sqrt{2}$

=> sin A cos B + sin A sin B = $\sqrt{2}$

=> sin A(cos B + sin B) = $\sqrt{2}$

Divide both sides by $\sqrt{2}$

sin A((1/$\sqrt{2}$) cos B +(1/$\sqrt{2}$) sin B) = 1

We know sin π/4 = cos π/4 = 1/$\sqrt{2}$

=> sin A(sin π/4 cos B + sin π/4 sin B) = 1

=> sin A sin (B + π/4) = 1

=> sin A = 1 and sin (B + π/4) = 1

=> A = 90^0, and (B + π/4) = π/2

B = π/4 = 45^0

So C = 45^0

Thus the triangle is isosceles right angled.

Hence option (c) is the answer.

Question 8: In a triangle PQR, P is the largest angle and cos P = ⅓. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the length of PN, QL and RM are consecutive even integer. Then possible length(s) of the side(s) of the triangle is (are)

(a) 16

(b) 18

(c) 24

(d) 22

Solution:

Given that cos P = 1/3

cos P = $[(2x+2)^2 + (2x+4)^2 - (2x+6)^2]/2(2x+2)(2x+4)$

⅓ = $[4x-16]/2(4x^2+12x+8)$

=> $8x^2 + 24x + 16 = 12x^2 - 48$

=> $4x^2 - 24x - 64 = 0$

=> $(x-8)(x-2) = 0$

=> $x = 8$

So the sides of the triangle are 18, 20, 22.

Hence option b and option d are correct.

Question 9: In a triangle, ABC, AD, BE, and CF are the altitudes, and R is the circumradius then the radius of the circle DEF is

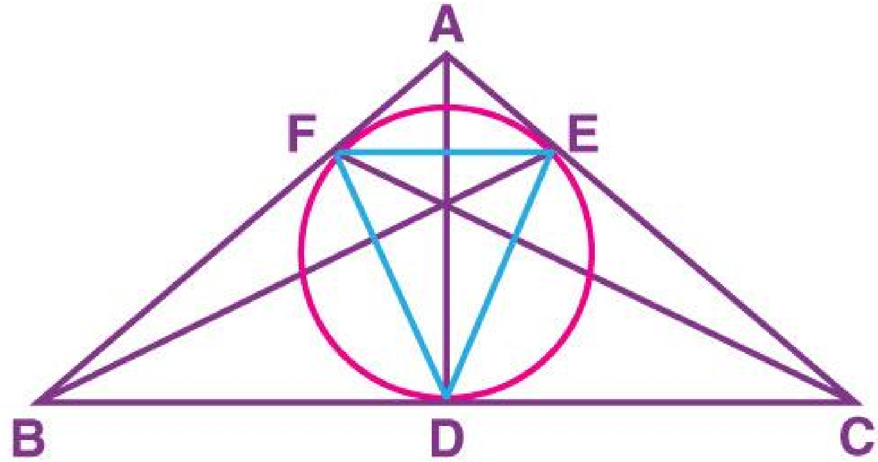
(a) 2R

(b) R

(c) R/2

(d) none of these

Solution:



Consider triangle DEF

EF = a cos A

DE = c cos C

DF = b cos B

Let R_1 is the circumradius of ΔDEF.

R_1 = (a cos A) (b cos B) (c cos C)/4× ½ DF×DE sin ∠EDF

∠EDF = 180 - 2A

So R_1 = (abc cos A cos B cos C)/2 b cos B c cos C sin (180 - 2A)

= a cos A/2 sin 2A

= a cos A/4 sin A cos A

Put a = 2R sin A

R_1= 2R sin A/4 sin A

= R/2

Hence option (c) is the answer.

Question 10: The sides of a triangle are in the ratio 5: 8: 11. If θ denotes the angle opposite to the largest side of the triangle, then the value of $\tan^2 \theta/2$ is equal to

(a) 1/21

(b) 7/48

(c) 7/3

(d) 7/12

Solution:

Given the sides are in the ratio 5: 8: 11

Let the sides be 5x, 8x, and 11x.

Then s = (5x + 8x + 11x)/2 = 12x

θ is opposite to the side 11x.

$\tan^2 \theta/2$ = (s - 5x)(s - 8x)/s(s - 11x)

$= (12x - 5x)(12x - 8x)/12x(12x - 11x)$

$= 7x \times 4x/12x \times x$

$= 28x^2/12x^2$

$= 7/3$

Hence option (c) is the answer.