Question 1: Perpendiculars are drawn from points on the line (x+2)/2 = (y+1)/-1 = z/3 to the plane x+y+z = 3. The feet of perpendiculars lie on the line is

(a) x/5 = (y-1)/8 = (z-2)/-13

(b) x/2 = (y-1)/3 = (z-2)/-5

(c) x/4 = (y-1)/3 = (z-2)/-7

(d) x/2 = (y-1)/-7 = (z-2)/5

Solution:

Given that the equation of the line is (x+2)/2 = (y+1)/-1 = z/3 = λ

So any point P on the line is x = 2λ-2, y = -λ-1, z = 3λ ..(i)

It lies on the plane x+y+z = 3

=> (2λ-2) + (-λ-1) + 3λ = 3

=> 4λ - 6 = 0

=> λ = 3/2

Substitute λ in (i) and get P

So P = (1, -5/2, 9/2) ..(ii)

We can observe that (-2, -1, 0) is a point on the line.

Let (x, y, z) be the foot of the perpendicular from point (-2, -1, 0) on the plane x+y+z = 3.

=> (x+2)/1 = (y+1)/1 = (z-0)/1

= $-(1(-2) + 1(-1) + 0(1) - 3)/(1^2+1^2+1^2)$

=> Q(x, y, z) = (0, 1, 2) ..(iii)

Direction ratios of PQ = (1, -7/2, 5/2) (from (ii) and (iii))

= (2, -7, 5) ..(iv)

From (iii) and (iv), equation of required line is x/2 = (y-1)/-7 = (z-2)/5

Hence option d is the answer.

Question 2: Two lines L_1: x = 5, y/(3-α) = z/-2 and L_2: x = α, y/-1 = z/(2-α) are coplanar. Then α can take values

(a) 1

(b) 2

(c) 3

(d) 4

Solution:

Given lines L_1: x = 5, y/(3-α) = z/-2 and

L_2: x = α, y/-1 = z/(2-α)

L_1 and L_2 are coplanar.

So

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

=> (5-α)(3-α)(2-α) - 2 = 0

=> (5-α)(6 - 3α - 2α + α^2 - 2) = 0

=> (5-α)(α-1)(α-4) = 0

=> α = 1, 4, 5

Hence option a and d is the answer.

Question 3: Let P be the image of the point (3, 1, 7) with respect to the plane x-y+z = 3. Then the equation of the plane passing through P and containing the straight line x/1 = y/2 = z/1 is

(a) x+y-3z = 0

(b) 3x+z = 0

(c) x-4y+7z = 0

(d) 2x-y = 0

Solution:

Equation of line passing through P is (x-3)/1 = (y-1)/-1 = (z-7)/1

Distance of point P from the given plane = -2(6)/3 = -4

(x-3)1 = (y-1)/-1 = (z-7)/1 = -4

=> x = -1, y = 5, z = 3

=> P= (-1, 5, 3)

Equation of plane passing through P is a(x+1) + b(y-5) + c(z-3) = 0

Normal of the plane is perpendicular to the line from which this plane passes through.

So a+2b+c = 0 ..(i)

The plane will also pass through the origin since the line passes through the origin.

So a-5b-3c = 0 ..(ii)

Solving (i) and (ii)

a/1 = b/-4 = c/7

Required equation of plane is (x+1) -4(y-5) +7(z-3) = 0

=> x-4y+7z = 0

Hence option c is the answer.

Question 4: The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y - 2z = 5 and 3x - 6y - 2z = 7 is

(a) 14x+2y+15z = 31

(b) 14x+2y-15z = 1

(c) -14x+2y+15z = 3

(d) 14x-2y+15z = 27

Solution:

Let plane P_1 => 2x + y - 2z = 5

P_2 => 3x - 6y - 2z = 7

Let P be the plane perpendicular to P_1 and P_2

Also P passes through (1, 1, 1).

Hence required equation =

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = 0$$

=> (x-1)(-2-12) - (y-1)(-4+6) + (z-1)(-12-3) = 0

=> 14x-14+2y-2+15z-15 = 0

=> 14x + 2y +15z = 31

Hence option a is the answer.

Question 5: If for some $\alpha \in R$, the lines L_1 : (x+1)/2 = (y-2)/-1 = (z-1)/1 and L_2 : (x+2)/α = (y+1)/(5-α) = (z+1)/1 are coplanar, then the line L_2 passes through the point:

(a) (2, -10, -2)

(b) (10, -2, -2)

(c) (10, 2, 2)

(d) (-2, 10, 2)

Solution:

A(-1,2,1), B(-2,-1,-1)

$$\left[\overrightarrow{AB}\overrightarrow{b_1}\overrightarrow{b_2}\right] = 0$$

$$\begin{vmatrix} -1 & -3 & -2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

-1(-1+α-5)+3(2-α)-2(10-2α+α) = 0

6-α+6-3α+2α-20 = 0

-8-2α = 0

$\alpha = -4$

L_2: (x+2)/-4 = (y+1)/9 = (z+1)/1

Check options. (2, -10, -2) satisfies above equation.

Hence option a is the answer.

Question 6: The distance of the point (1, -2, 3) from the plane x-y+z = 5 measured parallel to the line (x/2) = (y/3) = (z/-6) is:

(a) 1/7

(b) 7

(c) 7/5

(d) 1

Solution:

Equation of line through (1,-2,3) whose d.r.s. are (2, 3, -6)

(x-1)/2 = (y+2)/3 = (z-3)/-6 = λ

Any point on the line ($2\lambda+1$, $3\lambda-2$, $-6\lambda+3$)

Substitute in equation of plane

x-y+z = 5

$2\lambda+1- 3\lambda+2 -6\lambda+3 = 5$

$-7\lambda = -1$

$\lambda = 1/7$

Distance = $\sqrt{((2\lambda)^2+(3\lambda)^2+(6\lambda)^2)}$

= $\sqrt{(4\lambda^2+9\lambda^2+36\lambda^2)}$

= 7λ

= 1

Hence option d is the answer.

Question 7: Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x axis be 5. If R is the image of P in the xy-plane, then the length of PR is

Solution:

Let the coordinates of P are (a, b, c).

Coordinates of Q are (0, 0, c) and coordinates of R are (a, b, -c).

Given PQ is perpendicular to the plane x+y = 3

So PQ parallel to the normal of the given plane.

(ai+bj) is parallel to (i+j). (i, j are unit vectors)

Comparing we get a = b.

Since the midpoint of PQ lies on plane x+y = 3

So a/2 + b/2 = 3

=> a+b = 6

=> a = 3, b = 3

So distance of P from x axis = $\sqrt{(b^2+c^2)}$ = 5 (given)

=> $(b^2+c^2) = 25$

$c^2 = 25-9 = 16$

=> c = ±4

So PR = |2c|

= 8

Question 8: Let α, β, γ, δ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and α + γ = 1. Suppose the point (3, 2, -1) is the mirror image of the point (1, 0, -1) with respect to the plane αx + βy + γz = δ. Then which of the following statements is/are TRUE?

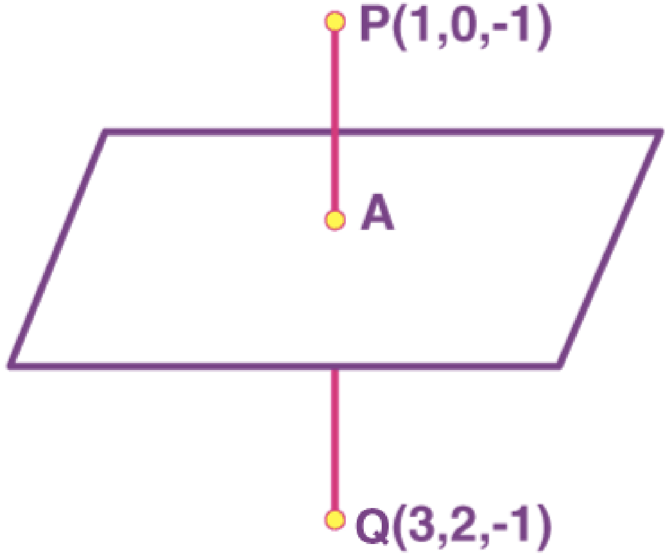
(a) α + β = 2

(b) δ - γ = 3

(c) δ + β = 4

(d) α + β + γ = δ

Solution:



Midpoint of PQ = A(2, 1, -1)

Direction ratios of PQ = 2, 2, 0

As PQ perpendicular to plane and midpoint lies on plane.

The equation of the plane is

$2(x-2) + 2(y-1) + 0(z+1) = 0$

$\Rightarrow x+y = 3$

Comparing with $\alpha x + \beta y + \gamma z = \delta$

We get $\alpha = 1$, $\beta = 1$, $\gamma = 0$ and $\delta = 3$

Hence options a, b, c are true.

Question 9: In R^3, consider the planes P_1: y = 0 and P_2: x+z = 1. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relation is (are) true?

(a) $2\alpha + \beta + 2\gamma + 2 = 0$

(b) $2\alpha - \beta + 2\gamma + 4 = 0$

(c) $2\alpha + \beta - 2\gamma - 10 = 0$

(d) $2\alpha - \beta + 2\gamma - 8 = 0$

Solution:

Let P_3 be $P_2 + \lambda P_1 = 0$

=> $(x+z-1) + \lambda y = 0$

=> $x+\lambda y+z-1 = 0$

Distance of the point (0, 1, 0) from P_3 is $|(\lambda-1)/\sqrt{(2+\lambda^2)}| = 1$

=> $(\lambda-1)^2 = (2+\lambda^2)$

=> $-2\lambda + 1 = 2$

=> $-2\lambda = 1$

=> $\lambda = -1/2$

Distance of point (α, β, γ) from P_3: $|(\alpha+\lambda\beta + \gamma-1) /\sqrt{(2+\lambda^2)}| = 2$

$|(\alpha-(\beta/2) + \gamma-1) /(3/2)| = \pm 2$

$\alpha- (\beta/2) + \gamma-1 =\pm 3$

=> $2\alpha- \beta + 2\gamma-2 =\pm 6$

=> $2\alpha- \beta + 2\gamma-8 = 0$ or $2\alpha- \beta + 2\gamma + 4 = 0$

Hence option b and d are the answers.

Question 10: If the straight lines (x-1)/2 = (y+1)/k = z/2 and (x+1)/5 = (y+1)/2 = z/k are coplanar, then the plane(s) containing these two lines is (are)

(a) $y + 2z = -1$

(b) $y + z = -1$

(c) $y - z = -1$

(d) $y - 2z = -1$

Solution:

Given that lines are coplanar.

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

=>

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$$

=> k = ±2

For k = 2, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0$$

=> y-z+1 = 0

=> y-z = -1

For k = -2, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0$$

=> y+z+1 = 0

=> y+z = -1

Hence option b and c are correct.