Question 1: The positive integer value of n>3 satisfying the equation 1/sin(π/n) = 1/sin(2π/n) + 1/sin(3π/n)

(a) 8

(b) 6

(c) 5

(d) 7

Solution:

Given, 1/sin (π/n) = 1/sin (2π/n) + 1/sin (3π/n)

(1/sin (π/n) - 1/sin (3π/n)) = 1/sin (2π/n)

[sin 3π/n - sin π/n]/ (sin 3π/n sin π/n) = 1/sin 2π/n

Using the formula sin x - sin y = 2 cos(x + y) sin(x - y),

[(2 cos (2π/n) sin (π/n)/ sin (3π/n) sin (π/n)] sin (2π/n) = 1

[2 sin (2π/n) cos (2π/n)]/ sin (3π/n) = 1

Using the formula 2 sin A cos A = sin 2A,

sin (4π/n)/ sin (3π/n) = 1

sin (4π/n) = sin (3π/n)

sin(π - (4π/n)) = sin 3π/n

π - (4π/n) = 3π/n

π = (3π/n) + (4π/n)

π = 7π/n

So, n = 7

Hence option d is the answer.

Question 2: If A + B + C = 180^0 then the value of tan A + tan B + tan C is

(a) ≥3√3

(b) ≥2√3

(c) > 3√3

(d) > $2\sqrt{3}$

Solution:

Given A + B + C = 180^0

So A + B = 180 - C

tan (A + B) = tan (180 - C)

(tan A + tan B)/(1 - tan A tan B) = -tan C

=> (tan A + tan B) = -tan C (1 - tan A tan B)

=> tan A + tan B + tan C = tan A tan B tan C …(i)

Use A.M ≥ G.M

=> (tan A + tan B + tan C)/3 ≥ $(\tan A \tan B \tan C)^{1/3}$

=> tan A tan B tan C ≥ 3 $(\tan A \tan B \tan C)^{1/3}$ (using (i))

Cubing both sides

$\tan^2 A \tan^2 B \tan^2 C \geq 27$

tan A tan B tan C ≥ $3\sqrt{3}$

Hence option a is the answer.

Question 3: The number of values of θ in the interval (-π/2, π/2) such that θ ≠ nπ/5 for n = 0, ±1, ±2 and tan θ = cot 5θ as well as sin 2θ = cos 4θ

(a) 3

(b) 4

(c) 7

(d) 5

Solution:

Given tan θ = cot 5θ

= tan (π/2 - 5θ)

=> θ = nπ + π/2 - 5θ

=> 6θ = nπ + π/2

=> $\theta = n\pi/6 + \pi/12$ …(i)

$\sin 2\theta = \cos 4\theta$

=> $2 \sin^2 2\theta + \sin 2\theta - 1 = 0$

=> $2 \sin^2 2\theta + 2\sin 2\theta - \sin 2\theta - 1 = 0$

=> $\sin 2\theta = -1$ or $\sin 2\theta = ½$

=> $2\theta = \pi/6 , 5\pi/6$

=> $\theta = \pi/12 , 5\pi/12$

So $\theta = -\pi/4, \pi/12 , 5\pi/12$

θ takes 3 values.

Hence option a is the answer.

Question 4: For $x \in (0, \pi)$, the equation $\sin x + 2 \sin 2x - \sin 3x = 3$ has

(a) infinitely many solutions

(b) three solutions

(c) one solution

(d) no solution

Solution:

Given that $\sin x + 2 \sin 2x - \sin 3x = 3$ …(i)

Use $\sin 2x = 2 \sin x \cos x$

And $\sin 3x = 3 \sin x - 4 \sin^3 x$

Equation (i) becomes $\sin x + 4 \sin x \cos x - 3 \sin x + 4 \sin^3 x = 3$

$\sin x(1 + 4 \cos x - 3 + 4 \sin^2 x) = 3$

$\sin x(-2 + 4 \cos x + 4(1-\cos^2 x)) = 3$

$(-2 + 4 \cos x + 4(1-\cos^2 x)) = 3/\sin x$

$(-2 + 4 \cos x + 4(1-\cos^2 x)) = 3 \operatorname{cosec} x$

$-2 + 4 \cos x + 4 - 4\cos^2 x = 3 \operatorname{cosec} x$

$(2 - (4 \cos^2 x - 4 \cos x +1) + 1) = 3 \operatorname{cosec} x$

$3 - (2 \cos x - 1)^2 = 3 \operatorname{cosec} x$

When $x = \pi/2$, RHS = 3

LHS = 3, at $x = \pi/3$.

LHS and RHS are not equal at the same value of x. Hence, no solution.

Hence option d is the answer.

Question 5: If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is :

(a) 1/3

(b) 2/9

(c) -7/9

(d) -3/5

Solution:

Given that $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

(Use $\tan^2 x = \sec^2 x - 1$ and $\cos 2x = 2\cos^2 x - 1$)

$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$

$5\sec^2 x - 5 = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$(5 / t) = 9t + 12$

$9t^2 + 12t - 5 = 0$

$t = 1/3$ or $- 5/3$

$t = 1/3$ as $t \neq - 5/3$

$\cos^2 x = 1/3$, $\cos 2x = 2\cos^2 x - 1 = -1/3$

$\cos 4x = 2\cos^2 2x - 1$

$= (2/9) - 1$

$= -7/ 9$

Hence option c is the answer.

Question 6: If $\sqrt{2} \sin \alpha/\sqrt{(1 + \cos 2\alpha)} = 1/7$ and $\sqrt{((1 - \cos 2\beta)/2)} = 1/\sqrt{10}$, $\alpha, \beta \in (0, \pi/2)$, then tan (α + 2β) is

equal to

(a) 1

(b) -1

(c) 0

(d) 1/2

Solution:

We know $\cos 2x = 2\cos^2 x - 1$

Also $1+\cos 2x = 2\cos^2 x$

Given that $\sqrt{2}\sin\alpha/\sqrt{(1 + \cos 2\alpha)} = 1/7$

$\Rightarrow \sqrt{2}\sin\alpha/\sqrt{(2\cos^2\alpha)} = 1/7$

$\Rightarrow \tan\alpha = 1/7$

Given $\sqrt{((1 - \cos 2\beta)/2)} = 1/\sqrt{10}$

Use $\cos 2x = \cos^2 x - \sin^2 x$ in above equation

$\Rightarrow \sqrt{((1 - \cos^2\beta + \sin^2\beta)/2)} = 1/\sqrt{10}$

$\Rightarrow \sqrt{((\sin^2\beta + \sin^2\beta)/2)} = 1/\sqrt{10}$

$\Rightarrow \sqrt{((2\sin^2\beta)/2)} = 1/\sqrt{10}$

$\Rightarrow \sin\beta = 1/\sqrt{10}$

$\Rightarrow \tan\beta = 1/3$

We know $\tan 2\beta = (2\tan\beta)/(1 - \tan^2\beta)$

$= 2(1/3)/(1 - 1/9)$

$= 3/4$

We know $\tan(a+b) = (\tan a + \tan b)/(1 - \tan a \tan b)$

So $\tan(\alpha + 2\beta) = (\tan\alpha + \tan 2\beta)/(1 - \tan\alpha \tan 2\beta)$

$= (1/7 + 3/4)/(1 - (1/7) \times (3/4))$

$= 25/25$

$= 1$

Hence option a is the answer.

Question 7: Let the function f: (0, π) → R be defined by $f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$. Suppose, the function f has a local minimum at θ precisely when $\theta \in \{\lambda_1 \pi, \ldots, \lambda_r \pi\}$, where $0 < \lambda_1 < \cdots < \lambda_r < 1$.Then the value of $\lambda_1 + \cdots + \lambda_r$ is

(a) 1/4

(b) -2

(c) 1

(d) 1/2

Solution:

Given that $f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta + ((\sin\theta - \cos\theta)^2)^2$

$= 1 + \sin 2\theta + (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)^2$

$= 1 + \sin 2\theta + (1 - \sin 2\theta)^2$

$= 1 + \sin 2\theta + 1 + \sin^2 2\theta - 2\sin 2\theta$

$= 2 + \sin^2 2\theta - \sin 2\theta$

$= \sin^2 2\theta - \sin 2\theta + 2$

$= (\sin 2\theta - ½)^2 + 7/4$

Given that f has a local minimum at θ.

$\theta \in [0, \pi]$

$2\theta \in [0, 2\pi]$

f(θ) min. when $\sin 2\theta = ½$

So, $2\theta = \pi/6, 5\pi/6$

$\theta = \pi/12, 5\pi/12$

$\lambda_1 = 1/12$ and $\lambda_2 = 5/12$

$\lambda_1 + \lambda_2 = 6/12 = 1/2$

Hence option d is the answer.

Question 8: The number of distinct solutions of equation $(5/4)\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the

interval [0, 2π] is

(a) 2

(b) 8

(c) 4

(d) 1

Solution:

$(5/4)\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow (5/4)\cos^2 2x + (\cos^2 x)^2 + (\sin^2 x)^2 + (\cos^2 x)^3 + (\sin^2 x)^3 = 2$

$\Rightarrow (5/4)\cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2 \sin^2 x \cos^2 x + (\cos^2 x + \sin^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) = 2$

$\Rightarrow (5/4) \cos^2 2x + 1 - 2 \sin^2 x \cos^2 x + (\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) = 2$

$\Rightarrow (5/4) \cos^2 2x + 1 - 2 \sin^2 x \cos^2 x + (1 - 3 \sin^2 x \cos^2 x) = 2$

$\Rightarrow (5/4) \cos^2 2x - 5 \sin^2 x \cos^2 x = 0$

$(5/4) - (5/4)\sin^2 2x - 5\sin^2 x.\cos^2 x = 0$

$\Rightarrow (5/4) - (5/4)\sin^2 2x - (5/4)\sin^2 2x = 0$

$\Rightarrow \sin^2 2x = ½$

$\Rightarrow \sin 2x = \pm 1/\sqrt{2}$

So $x = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8$

Number of solutions in $[0, 2\pi]$ is 8.

Hence option b is the answer.

Question 9: For what and only what values of α lying between 0 and π is the inequality $\sin \alpha \cos^3\alpha > \sin^3\alpha \cos \alpha$ valid?

(a) $\alpha \in (0, \pi/4)$

(b) $\alpha \in (0, \pi/2)$

(c) $\alpha \in (\pi/4, \pi/2)$

(d) none of these

Solution:

$\sin \alpha \cos^3\alpha > \sin^3\alpha \cos \alpha$

=> $\sin \alpha \cos^3\alpha - \sin^3\alpha \cos \alpha > 0$

=> $\sin \alpha \cos \alpha(\cos^2\alpha - \sin^2\alpha) >0$ ..(i)

We know $\cos^2x - \sin^2x = \cos 2x$.

So (i) becomes

$(½)2\sin \alpha \cos \alpha \cos 2\alpha > 0$

=> $(½) \sin 2\alpha \cos 2\alpha > 0$

=> $(¼) \sin 4\alpha > 0$

=> $\sin 4\alpha > 0$

=> $4\alpha \in (0, \pi)$ (given that $0<\alpha<\pi$)

=> $\alpha \in (0, \pi/4)$

Hence option a is the answer.

Question 10: The solution of the equation tan θ. tan 2 θ = 1 is

(a) $n\pi + 5\pi/12$

(b) $n\pi - \pi/12$

(c) $2n\pi \pm \pi/4$

(d) $n\pi \pm \pi/6$

Solution:

We know $\tan 2\theta = 2 \tan \theta/(1 - \tan^2\theta)$

Given $\tan \theta. \tan 2\theta = 1$

=> $\tan \theta.2 \tan \theta/(1 - \tan^2\theta) = 1$

=> $2 \tan^2\theta = 1 - \tan^2\theta$

=> $3 \tan^2\theta = 1$

=> $\tan^2\theta = 1/3$

=> $\tan \theta = 1/\sqrt{3}$

=> $\theta = n\pi \pm \pi/6$

Hence option d is the answer.

Question 11: If sin θ = 3 sin (θ + 2α), then the value of tan (θ + α) + 2 tan α is

(a) 3

(b) 1

(c) 2

(d) 0

Solution:

Given that sin θ = 3 sin (θ + 2α)

=> sin (θ + α - α) = 3 sin (θ + α + α)

=> sin (θ + α) cos α - cos(θ + α) sin α = 3 sin (θ + α) cos α + 3 cos(θ + α) sin α

=> -2 sin (θ + α) cos α = 4 cos(θ + α) sin α

=>-sin (θ + α)/cos (θ + α) = 2 sin α/cos α

=> -tan (θ + α) = 2 tan α

=> tan (θ + α) + 2 tan α = 0

Hence option d is the answer.

Question 12: The value of tan 6^0 tan 42^0 tan 66^0 tan 78^0 is

(a) 1

(b) 4

(c) 2

(d) 0

Solution:

tan 6^0 tan 42^0 tan 66^0 tan 78^0 = (sin 6^0 sin 42^0 sin 66^0 sin 78^0)/(cos 6^0 cos 42^0 cos 66^0 cos 78^0)

= (cos 60 - cos 72)(cos 36 - cos 120)/(cos 60 + cos 72)(cos 36 + cos 120)

= (½ - sin 18)(cos 36 + ½)/ (½ + sin 18)(cos 36 - ½)

= [(½ - (√5 - 1)/4) ((√5+1)/4 + ½)]/[(½ + (√5 - 1)/4) ((√5+1)/4 - ½)]

= (3 - √5)(3 + √5)/(√5 +1)(√5 -1)

$= (9-5)/(5-1)$

$= 4/4$

$= 1$

Hence option a is the answer.

Question 13: The value of $\sqrt{3}$ cosec 20^0 - sec 20^0 is equal to

(a) -6

(b) 4

(c) 1

(d) 0

Solution:

$\sqrt{3}$ cosec 20^0 - sec 20^0 = $\sqrt{3}$/sin 20^0 - 1/cos 20^0

= ($\sqrt{3}$ cos 20^0 - sin 20^0)/sin 20^0 cos 20^0

= 4[($\sqrt{3}$/2) cos 20^0 - ½ sin 20^0)/2 sin 20^0 cos 20^0

= 4[(sin 60^0 cos 20^0 - cos 60^0 sin 20^0)/sin 20^0]

= 4[(sin (60^0-20^0))/sin 40^0]

= 4 sin 40^0/sin 40^0

= 4

Hence option b is the answer.

Question 14: The value of the expression (1 - 4 sin 10^0 sin 70^0)/2 sin 10^0 is

(a) -6

(b) 4

(c) 1

(d) 0

Solution:

(1 - 4 sin 10^0 sin 70^0)/2 sin 10^0 = (1 - 2(2 sin 10^0 sin 70^0))/2 sin 10^0

We know sin (90 - θ) = cos θ

And cos A - cos B = 2 sin (A+B)/2 sin (A-B)/2

$(1 - 2(2 \sin 10^0 \sin 70^0))/2 \sin 10^0 = [1 - 2(\cos 60^0 - \cos 80^0)]/2 \sin (90^0 - 80^0)$

$= (1- 2 \cos 60^0 + 2 \cos 80^0)/2 \cos 80^0$

$= (1 - 2(½) + 2 \cos 80^0)/2 \cos 80^0$

$= 2 \cos 80^0/2 \cos 80^0$

= 1

Hence option c is the answer.

Question 15: Let a, b, c be three non zero real numbers such that the equation √3a cos x + 2b sin x = c, x belongs to [-π/2, π/2] has two distinct roots α and β with α + β = π/3. Then the value of b/a is

(a) 1

(b) 4/3

(c) 1/2

(d) 0

Solution:

Given that √3a cos x + 2b sin x = c

Divide by a, we get

√3 cos x + (2b/a) sin x = c/a …(i)

Since α and β are the roots of (i)

√3 cos α + (2b/a) sin α = c/a …(ii)

√3 cos β + (2b/a) sin β = c/a …(iii)

Subtract (iii) from (ii)

=> √3 (cos α - cos β) + (2b/a) (sin α - sin β) = 0

We know cos A - cos B = -2 sin (A+B)/2 sin (A-B)/2

Also sin A - sin B = 2 cos (A+B)/2 sin (A-B)/2

=> √3 (-2 sin (α+β)/2 sin (α-β)/2) + (2b/a) (2 cos (α+β)/2 sin (α-β)/2) = 0

Given α + β = π/3

=> $\sqrt{3}$ (-2 sin π/6 sin (α-β)/2) + (2b/a) (2 cos π/6 sin (α-β)/2) = 0

=> $\sqrt{3}$ (-2 ×½ × sin (α-β)/2) + (2b/a) (2×$\sqrt{3}$/2 sin (α-β)/2) = 0

=> $\sqrt{3}$ (sin (α-β)/2) = (2b/a) ($\sqrt{3}$ sin (α-β)/2)

=> 1 = 2b/a

=> b/a = 1/2

Hence option c is the answer.