JEE Main Maths Limits, Continuity and Differentiability Previous Year Questions With Solutions

Question 1: Solve

$$\lim_{x\to1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$$

Solution:

$$\lim_{x\to1} \frac{(2x-3)\ (\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)\ (2x+3)\times(\sqrt{x}+1)}$$

$$= \frac{-1}{5\ .\ 2}$$

$$= \frac{-1}{10}$$

Question 2: If f(9) = 9, f'(9) = 4, then

$$\lim_{x\to9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3} =$$

Solution:

Applying L - Hospitals rule,

$$\lim_{x\to9} \frac{\frac{1}{2\sqrt{f(x)}}\cdot f'(x)}{\frac{1}{2\sqrt{x}}}$$

$$= \frac{\frac{f'(9)}{\sqrt{f(9)}}}{\frac{1}{\sqrt{9}}}$$

$$= \frac{\frac{4}{3}}{\frac{1}{3}}$$

$$= 4$$

Question 3: Solve

$$\lim_{h\to 0}\ \frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

Solution:

Apply the L-Hospitals rule,

$$\lim_{h\to 0}\ \frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$
$$\lim_{h\to 0}\ \frac{2\,(a+h)\ \sin\,(a+h)+(a+h)^2\cos\,(a+h)}{1}$$
$$= 2a\ \ \sin a + a^2 \cos\ \ a$$

Question 4: Solve

$$\lim_{x\to\pi/4}\ \frac{\sqrt{2}\cos x-1}{\cot x-1}$$

Solution:

$$\lim_{x\to\pi/4}\ \frac{\sqrt{2}\cos x-1}{\cot x-1}$$

Apply the L-Hospitals rule,

$$= \lim_{x\to\pi/4}\ \frac{-\sqrt{2}\sin x}{-cosec^x}$$
$$\frac{-\sqrt{2}\times\frac{1}{\sqrt{2}}}{-(\sqrt{2})^2}$$
$$= \frac{1}{2}$$

Question 5: Solve

$$\lim_{x\to 0}\left[\frac{x}{\tan^{-1}2x}\right]$$

Solution:

Let

$$\tan^{-1}2x = \theta$$
$$\Rightarrow x = \frac{1}{2}\tan\theta \; and \; as \; x \to 0, \; \theta \to 0$$
$$\Rightarrow \lim_{x\to 0} \frac{x}{\tan^{-1}2x}$$
$$= \lim_{\theta\to 0} \frac{\frac{1}{2}\tan\theta}{\theta}$$
$$= \frac{1}{2}.(1)$$
$$= \frac{1}{2}$$

Question 6: Solve

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$$

Solution:

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

So,

$$\lim_{x\to 0+} \frac{|\sin x|}{x} = 1$$

and

$$\lim_{x\to 0-} \frac{|\sin x|}{x} = -1$$

Hence, the limit doesn't exist.

Question 7: Solve

$$\lim_{x\to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}$$

Solution:

Given,

$\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$

$= \lim_{x\to 0}\ \left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$

$= \lim_{x\to 0}\left(1+\left(\frac{1+tanx}{1-tanx}-1\right)\right)^{\frac{1}{x}}$

$= \lim_{x\to 0}\left(1+\left(\frac{2\,tanx}{1-tanx}\right)\right)^{\frac{1}{x}}$

$= e^{\lim_{x\to 0}\left(\frac{2\,tanx}{x(1-tanx)}\right)}$

$= e^2$

Question 8: Solve

$$\lim_{x\to 0}\ \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

Solution:

$$\lim_{x\to 0}\ \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$$

$$= \frac{\lim_{x\to 0}\ \left[\left(1+5x^2\right)^{1/5x^2}\right]^5}{\lim_{x\to 0}\ \left[\left(1+3x^2\right)^{1/3x^2}\right]^3}$$

$$= \frac{e^5}{e^3}$$

$$= e^2$$

$$[\because\ \ \lim_{x\to 0}\ (1+x)^{1/x} = e]$$

Question 9: Solve

$$\lim_{x\to 0}\ \frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$$

Solution:

$$\lim_{x\to 0} \frac{x\tan 2x-2x\tan x}{(1-\cos\ 2x)^2}$$
$$=\lim_{x\to 0} \frac{x(\tan\ 2x-2\tan x)}{(2\sin^2 x)^2}$$
$$=\lim_{x\to 0} \frac{1}{4}\frac{x\,(\tan 2x-2\tan x)}{\sin^4 x}$$
$$=\lim_{x\to 0} \frac{x}{4sin^4x}\left(\frac{2\,tanx}{1-tan^2x}-2\,tanx\right)$$
$$=\lim_{x\to 0} \frac{x}{4sin^4x}\left(\frac{2\,tanx-2\,tanx+2\,tan^3x}{1-tan^2x}\right)$$
$$=\lim_{x\to 0} \frac{x}{4sin^4x}\left(\frac{2\,tan^3x}{1-tan^2x}\right)$$
$$=\frac{1}{2}\lim_{x\to 0} \frac{x}{sinx}\cdot\frac{tan^3x}{sin^3x}\cdot\frac{1}{1-tan^2x}$$
$$=\frac{1}{2}\lim_{x\to 0} \frac{1}{\frac{sinx}{x}}\cdot\frac{1}{cos^3x}\cdot\frac{1}{1-tan^2x}$$
$$=\frac{1}{2}\times 1\times\frac{1}{1}\times\frac{1}{1}$$
$$=\frac{1}{2}$$

Question 10: The function

$$f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$$

is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

Solution:

Since the limit of a function is a + b as $x \to 0$, therefore to be continuous at a function, its value must be a + b at x = 0

$\Rightarrow$ f (0) = a + b

Question 11: Evaluate

$$f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2} & \text{if } x\neq 2\\ k & \text{if } x=2\end{cases}$$

Solution:

For continuous

$$\lim_{x\to 2}\ f(x) = f(2) = k$$
$$\Rightarrow\ k = \lim_{x\to 2}\ \frac{x^3+x^2-16x+20}{(x-2)^2}$$
$$= \lim_{x\to 2}\ \frac{(x^2-4x+4)\ (x+5)}{(x-2)^2}$$
$$= \lim_{x\to 2}\ \frac{(x-2)^2\ (x+5)}{(x-2)^2}$$
$$= 7.$$

Question 12: If

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x^2-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

, then find the condition for the function to be continuous or discontinuous.

Solution:

$$f(x) = \left\{\frac{x^2-4x+3}{x^2-1}\right\}$$

for x = 1

f (1) = 2,

$$f(1+) = \lim_{x\to 1+}\ \frac{x^2-4x+3}{x^2-1}$$
$$= \lim_{x\to 1+}\ \frac{(x-3)}{(x+1)}$$
$$= -1$$
$$f(1-) = \lim_{x\to 1-}\ \frac{x^2-4x+3}{x^2-1}$$
$$= -1$$
$$\Rightarrow\ f(1) \neq f(1-)$$

Hence, the function is discontinuous at x = 1.

Question 13: Which of the following functions have a finite number of points of discontinuity in R

([.] represents the greatest integer function)?

(A) tan x

(B) x[x]

(C) |x| / x

(D) sin[πx]

Solution:

f (x) = tanx is discontinuous when x = (2n + 1) π / 2, n ∈ Z

f (x) = x[x] is discontinuous when x = k, k ∈ Z

f (x) = sin [nπx] is discontinuous when nπx = k, k ∈ Z

Thus, all the above functions have an infinite number of points of discontinuity. But, if (x) = |x| / x is discontinuous when x = 0 only.

Question 14: The number of values of x ∈ [0, 2] at which f (x) = |x − (1/2)| + |x − 1|+ tan x is not differentiable is

(A) 0

(B) 1

(C) 3

(D) None of these

Solution:

|x − (1/2)| is continuous everywhere but not differentiable at x = 1 / 2, |x − 1| is continuous everywhere, but not differentiable at x = 1 and tan x is continuous in [0, 2] except at x = π/2.

Hence, f (x) is not differentiable at x = 1/2, 1, π/2.

Answer: (C) 3

Question 15:

$$\lim_{x\to\pi/2}(\sec\theta-\tan\theta)=$$

Solution:

$$\lim_{\theta\to\pi/2}\ \frac{1-\sin\theta}{\cos\theta}=\lim_{\theta\to\pi/2}\ \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}=0$$