

Marking Scheme

Class XII

Mathematics (Code – 041)

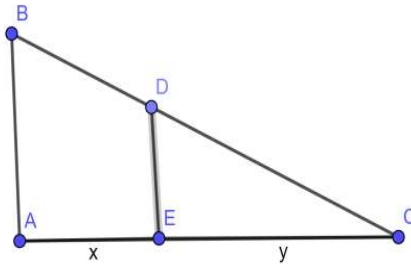
Section : A (Multiple Choice Questions- 1 Mark each)

Question No	Answer	Hints/Solution
1.	(c)	In a skew-symmetric matrix, the (i, j)th element is negative of the (j, i)th element. Hence, the (i, i)th element = 0
2.	(a)	$ AA' = A A' = (-3)(-3) = 9$
3.	(b)	The area of the parallelogram with adjacent sides AB and AC = $ \vec{AB} \times \vec{AC} $. Hence, the area of the triangle with vertices A, B, C = $\frac{1}{2} \vec{AB} \times \vec{AC} $
4.	(c)	The function f is continuous at $x = 0$ if $\lim_{x \rightarrow 0} f(x) = f(0)$ We have $f(0) = k$ and $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{8x^2} = \lim_{x \rightarrow 0} \frac{2\sin^2 \frac{x}{2}}{8x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{4x^2}$ $= \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{2x} \right)^2 = 1$ Hence, $k = 1$
5.	(b)	$\frac{x^2}{2} + \log x + C \left(\because f(x) = \int \left(x + \frac{1}{x} \right) dx \right)$
6.	(c)	The given differential equation is $4 \left(\frac{dy}{dx} \right)^3 \frac{d^2y}{dx^2} = 0$. Here, $m = 2$ and $n = 1$ Hence, $m + n = 3$
7.	(b)	The strict inequality represents an open half plane and it contains the origin as $(0, 0)$ satisfies it.
8.	(a)	Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k}$ $= \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{ \hat{i} + 2\hat{j} - 3\hat{k} } = \frac{7}{\sqrt{14}}$
9.	(c)	$\int_2^3 \frac{x}{x^2+1} = \frac{1}{2} [\log(x^2+1)]_2^3 = \frac{1}{2} (\log 10 - \log 5) = \frac{1}{2} \log \left(\frac{10}{5} \right)$ $= \frac{1}{2} \log 2$
10.	(c)	$(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$
11.	(d)	The minimum value of the objective function occurs at two adjacent corner points $(0.6, 1.6)$ and $(3, 0)$ and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region. So, the minimum value occurs at every point of the line-segment joining the two points.
12.	(d)	$2 - 20 = 2x^2 - 24 \Rightarrow 2x^2 = 6 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$
13.	(b)	$ adjA = A ^{n-1} \Rightarrow adjA = 25$
14.	(c)	$P(A' \cap B') = P(A') \times P(B')$ (As A and B are independent, A' and B' are also independent.) $= 0.7 \times 0.4 = 0.28$
15.	(c)	$ydx - xdy = 0 \Rightarrow ydx - xdy = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$ $\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x} + \log K, K > 0 \Rightarrow \log y = \log x + \log K$ $\Rightarrow \log y = \log x K \Rightarrow y = x K \Rightarrow y = \pm Kx \Rightarrow y = Cx$

16.	(a)	$y = \sin^{-1}x$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$ Again, differentiating both sides w. r. to x, we get $\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \left(\frac{-2x}{2\sqrt{1-x^2}} \right) = 0$ Simplifying, we get $(1-x^2)y_2 = xy_1$
17.	(b)	$ \vec{a} - 2\vec{b} ^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$ $ \vec{a} - 2\vec{b} ^2 = \vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 4\vec{b} \cdot \vec{b}$ $= \vec{a} ^2 - 4\vec{a} \cdot \vec{b} + 4 \vec{b} ^2$ $= 4 - 16 + 36 = 24$ $ \vec{a} - 2\vec{b} ^2 = 24 \Rightarrow \vec{a} - 2\vec{b} = 2\sqrt{6}$
18.	(b)	The line through the points (0, 5, -2) and (3, -1, 2) is $\frac{x}{3-0} = \frac{y-5}{-1-5} = \frac{z+2}{2+2}$ $or, \frac{x}{3} = \frac{y-5}{-6} = \frac{z+2}{4}$ Any point on the line is $(3k, -6k + 5, 4k - 2)$, where k is an arbitrary scalar. $3k = 6 \Rightarrow k = 2$ The z-coordinate of the point P will be $4 \times 2 - 2 = 6$
19.	(c)	$\sec^{-1}x$ is defined if $x \leq -1$ or $x \geq 1$. Hence, $\sec^{-1}2x$ will be defined if $x \leq -\frac{1}{2}$ or $x \geq \frac{1}{2}$. Hence, A is true. The range of the function $\sec^{-1}x$ is $[0, \pi] - \{\frac{\pi}{2}\}$ R is false.
20.	(a)	The equation of the x-axis may be written as $\vec{r} = t\hat{i}$. Hence, the acute angle θ between the given line and the x-axis is given by $\cos\theta = \frac{ 1 \times 1 + (-1) \times 0 + 0 \times 0 }{\sqrt{1^2 + (-1)^2 + 0^2} \times \sqrt{1^2 + 0^2 + 0^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

SECTION B (VSA questions of 2 marks each)

21.	$\sin^{-1}[\sin(\frac{13\pi}{7})] = \sin^{-1}[\sin(2\pi - \frac{\pi}{7})]$ $= \sin^{-1}[\sin(-\frac{\pi}{7})] = -\frac{\pi}{7}$ OR Let $y \in N(\text{codomain})$. Then $\exists 2y \in N(\text{domain})$ such that $f(2y) = \frac{2y}{2} = y$. Hence, f is surjective. $1, 2 \in N(\text{domain})$ such that $f(1) = 1 = f(2)$ Hence, f is not injective.	.1 1 1 1
22.	Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC be y m. Using similarity of triangles, we have $\frac{4}{1.6} = \frac{x+y}{y} \Rightarrow 3y = 2x$	$\frac{1}{2}$

	 <p>Differentiating both sides w.r.to t, we get $3 \frac{dy}{dt} = 2 \frac{dx}{dt}$</p> $\frac{dy}{dt} = \frac{2}{3} \times 0.3 \Rightarrow \frac{dy}{dt} = 0.2$ <p>At any time t seconds, the tip of his shadow is at a distance of $(x + y)$ m from AB.</p> <p>The rate at which the tip of his shadow moving</p> $= \left(\frac{dx}{dt} + \frac{dy}{dt} \right) \text{ m/s} = 0.5 \text{ m/s}$ <p>The rate at which his shadow is lengthening</p> $= \frac{dy}{dt} \text{ m/s} = 0.2 \text{ m/s}$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1/2</p>
23.	<p>$\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$</p> <p>Hence $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$ and $\vec{a} - \vec{b} = -4\hat{i} + (7 - \lambda)\hat{k}$</p> <p>$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ will be orthogonal if, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$</p> <p>i.e., if, $-24 + (49 - \lambda^2) = 0 \Rightarrow \lambda^2 = 25$</p> <p>i.e., if, $\lambda = \pm 5$</p> <p style="text-align: center;">OR</p> <p>The equations of the line are $6x - 12 = 3y + 9 = 2z - 2$, which, when written in standard symmetric form, will be</p> $\frac{x-2}{\frac{1}{6}} = \frac{y-(-3)}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$ <p>Since, lines are parallel, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$</p> <p>Hence, the required direction ratios are $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right)$ or $(1, 2, 3)$</p> <p>and the required direction cosines are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p>
24.	<p>$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$</p> <p>Let $\sin^{-1}x = A$ and $\sin^{-1}y = B$. Then $x = \sin A$ and $y = \sin B$</p> <p>$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \Rightarrow \sin B \cos A + \sin A \cos B = 1$</p> <p>$\Rightarrow \sin(A + B) = 1 \Rightarrow A + B = \sin^{-1}1 = \frac{\pi}{2}$</p> <p>$\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$</p> <p>Differentiating w.r.to x, we obtain $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
25.	<p>Since \vec{a} is a unit vector, $\therefore \vec{a} = 1$</p>	<p>$\frac{1}{2}$</p>

	$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12.$	
	$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$	$\frac{1}{2}$
	$\Rightarrow \vec{x} ^2 - \vec{a} ^2 = 12.$	$\frac{1}{2}$
	$\Rightarrow \vec{x} ^2 - 1 = 12$	
	$\Rightarrow \vec{x} ^2 = 13 \Rightarrow \vec{x} = \sqrt{13}$	$1/2$

SECTION C

(Short Answer Questions of 3 Marks each)

26.	$\int \frac{dx}{\sqrt{3-2x-x^2}}$ $= \int \frac{dx}{\sqrt{-(x^2+2x-3)}} = \int \frac{dx}{\sqrt{4-(x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{2}\right) + C \quad \left[\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \right]$	2 1												
27.	<p>P(not obtaining an odd person in a single round) = P(All three of them throw tails or All three of them throw heads)</p> $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{4}$ <p>P(obtaining an odd person in a single round)</p> $= 1 - \text{P(not obtaining an odd person in a single round)} = \frac{3}{4}$ <p>The required probability</p> $= \text{P('In first round there is no odd person' and 'In second round there is no odd person' and 'In third round there is an odd person')}$ $= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$ <p style="text-align: center;">OR</p> <p>Let X denote the Random Variable defined by the number of defective items.</p> $P(X=0) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$ $P(X=1) = 2 \times \left(\frac{2}{6} \times \frac{4}{5}\right) = \frac{8}{15}$ $P(X=2) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$ <table border="1"><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p_i</td><td>$\frac{2}{5}$</td><td>$\frac{8}{15}$</td><td>$\frac{1}{15}$</td></tr><tr><td>$p_i x_i$</td><td>0</td><td>$\frac{8}{15}$</td><td>$\frac{2}{15}$</td></tr></table> $\text{Mean} = \sum p_i x_i = \frac{10}{15} = \frac{2}{3}$	x_i	0	1	2	p_i	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$	$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$	1+1/2 1/2 1 2 1/2 1/2
x_i	0	1	2											
p_i	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$											
$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$											
28.	Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(i)$													

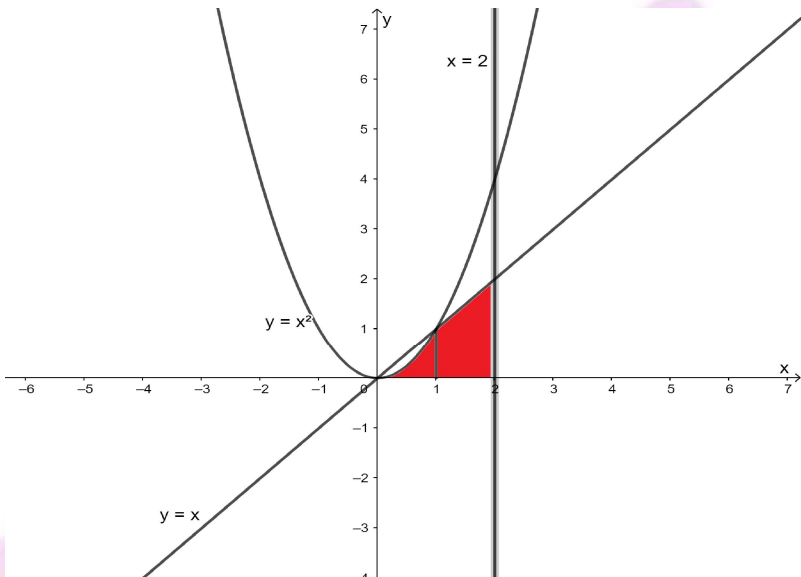
	<p>Using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$</p> $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}}{\sqrt{\sin(\frac{\pi}{6} + \frac{\pi}{3} - x)} + \sqrt{\cos(\frac{\pi}{6} + \frac{\pi}{3} - x)}} dx$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{..(ii).}$ <p>Adding (i) and (ii), we get</p> $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $2I = \int_{\pi/6}^{\pi/3} dx$ $= [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ <p>Hence, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \frac{\pi}{12}$</p> <p style="text-align: center;">OR</p> $\int_0^4 x-1 dx = \int_0^1 (1-x) dx + \int_1^4 (x-1) dx$ $= [x - \frac{x^2}{2}]_0^1 + [\frac{x^2}{2} - x]_1^4$ $= (1 - \frac{1}{2}) + (8 - 4) - (\frac{1}{2} - 1)$ $= 5$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
29.	<p>$ydx + (x - y^2)dy = 0$</p> <p>Reducing the given differential equation to the form $\frac{dx}{dy} + Px = Q$</p> <p>we get, $\frac{dx}{dy} + \frac{x}{y} = y$</p> <p>I.F = $e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$</p> <p>The general solution is given by</p> $x \cdot IF = \int Q \cdot IF dy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C, \text{ which is the required general solution}$ <p style="text-align: center;">OR</p> $xdy - ydx = \sqrt{x^2 + y^2} dx$ <p>It is a Homogeneous Equation as</p> $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} = \sqrt{1 + (\frac{y}{x})^2} + \frac{y}{x} = f\left(\frac{y}{x}\right).$ <p>Put $y = vx$</p> $\frac{dy}{dx} = v + x \frac{dv}{dx}$	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

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	<p>Hence, $\int \frac{(x^3+x+1)}{(x^2-1)} dx = \int \left(x + \frac{2x+1}{(x-1)(x+1)} \right) dx$</p> <p>$= \int \left(x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)} \right) dx$</p> <p>$= \frac{x^2}{2} + \frac{3}{2} \log x-1 + \frac{1}{2} \log x+1 + C$</p> <p>$= \frac{x^2}{2} + \frac{1}{2} (\log (x-1)^3(x+1)) + C$</p>	1
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SECTION D

(Long answer type questions (LA) of 5 marks each)

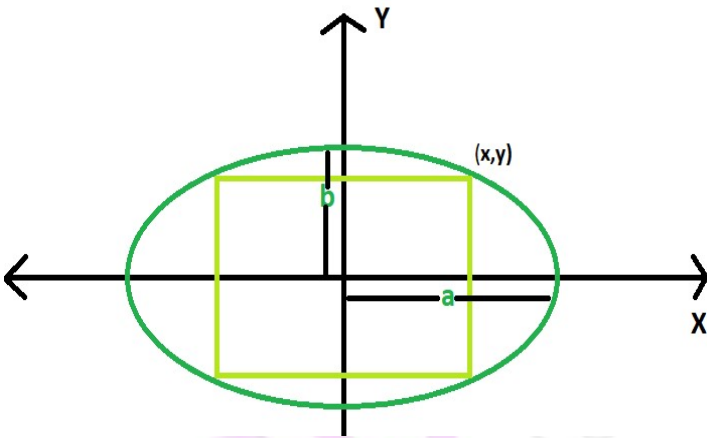
32.	 <p>The points of intersection of the parabola $y = x^2$ and the line $y = x$ are $(0, 0)$ and $(1, 1)$.</p> <p>Required Area $= \int_0^1 y_{\text{parabola}} dx + \int_1^2 y_{\text{line}} dx$</p> <p>Required Area $= \int_0^1 x^2 dx + \int_1^2 x dx$</p> <p>$= \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$</p>	<p>(Correct Fig: 1 Mark)</p> <p>$\frac{1}{2}$</p> <p>2</p> <p>1+1/2</p>
33.	<p>Let $(a, b) \in N \times N$. Then we have</p> <p>$ab = ba$ (by commutative property of multiplication of natural numbers)</p> <p>$\Rightarrow (a, b)R(a, b)$</p> <p>Hence, R is reflexive.</p> <p>Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$. Then</p> <p>$ad = bc$</p> <p>$\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers)</p> <p>$\Rightarrow (c, d)R(a, b)$</p> <p>Hence, R is symmetric.</p> <p>Let $(a, b), (c, d), (e, f) \in N \times N$ such that</p>	<p>1</p> <p>1+1/2</p>

	<p> $(a, b) R (c, d)$ and $(c, d) R (e, f)$. Then $ad = bc$, $cf = de$ $\Rightarrow adcf = bcde$ $\Rightarrow af = be$ $\Rightarrow (a, b) R (e, f)$ Hence, R is transitive. Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$. OR Let $A \in P(X)$. Then $A \subset A$ $\Rightarrow (A, A) \in R$ Hence, R is reflexive. Let $A, B, C \in P(X)$ such that $(A, B), (B, C) \in R$ $\Rightarrow A \subset B, B \subset C$ $\Rightarrow A \subset C$ $\Rightarrow (A, C) \in R$ Hence, R is transitive. $\emptyset, X \in P(X)$ such that $\emptyset \subset X$. Hence, $(\emptyset, X) \in R$. But, $X \not\subset \emptyset$, which implies that $(X, \emptyset) \notin R$. Thus, R is not symmetric. </p>	<p>2</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>2</p> <p>2</p>
34.	<p> The given lines are non-parallel lines. There is a unique line-segment PQ (P lying on one and Q on the other, which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the insects = PQ The position vector of P lying on the line $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ is $(6 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$ for some λ The position vector of Q lying on the line $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$ is $(-4 + 3\mu)\hat{i} + (-2\mu)\hat{j} + (-1 - 2\mu)\hat{k}$ for some μ $\overrightarrow{PQ} = (-10 + 3\mu - \lambda)\hat{i} + (-2\mu - 2 + 2\lambda)\hat{j} + (-3 - 2\mu - 2\lambda)\hat{k}$ Since, PQ is perpendicular to both the lines $(-10 + 3\mu - \lambda) + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)2 = 0$, i.e., $\mu - 3\lambda = 4$... (i) and $(-10 + 3\mu - \lambda)3 + (-2\mu - 2 + 2\lambda)(-2) + (-3 - 2\mu - 2\lambda)(-2) = 0$, i.e., $17\mu - 3\lambda = 20$... (ii) solving (i) and (ii) for λ and μ, we get $\mu = 1, \lambda = -1$. The position vector of the points, at which they should be so that the distance between them is the shortest, are $5\hat{i} + 4\hat{j}$ and $-\hat{i} - 2\hat{j} - 3\hat{k}$ $\overrightarrow{PQ} = -6\hat{i} - 6\hat{j} - 3\hat{k}$ The shortest distance = $\overrightarrow{PQ} = \sqrt{6^2 + 6^2 + 3^2} = 9$ OR </p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p>

	<p>Eliminating t between the equations, we obtain the equation of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which are the equations of the line passing through the origin having direction ratios $\langle 2, -4, 4 \rangle$. This line is the path of the rocket.</p> <p>When $t = 10$ seconds, the rocket will be at the point $(20, -40, 40)$. Hence, the required distance from the origin at 10 seconds =</p> $\sqrt{20^2 + 40^2 + 40^2} \text{ km} = 20 \times 3 \text{ km} = 60 \text{ km}$ <p>The distance of the point $(20, -40, 40)$ from the given line</p> $= \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} } = \frac{ -30\hat{j} \times (10\hat{i} - 20\hat{j} + 10\hat{k}) }{ 10\hat{i} - 20\hat{j} + 10\hat{k} } \text{ km} = \frac{ -300\hat{i} + 300\hat{k} }{ 10\hat{i} - 20\hat{j} + 10\hat{k} } \text{ km}$ $= \frac{300\sqrt{2}}{10\sqrt{6}} \text{ km} = 10\sqrt{3} \text{ km}$	<p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>2</p> <p>$\frac{1}{2}$</p>
35.	<p>$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$</p> <p>$A = 2(0) + 3(-2) + 5(1) = -1$</p> <p>$A^{-1} = \frac{\text{adj}A}{ A }$</p> <p>$\text{adj}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}, A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$</p> <p>$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$</p> <p>$= \frac{1}{(-1)} \begin{bmatrix} 0 + 5 - 6 \\ 22 + 45 - 69 \\ 11 + 25 - 39 \end{bmatrix}$</p> <p>$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)} \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3.$</p>	<p>$\frac{1}{2}$</p> <p>3</p> <p>$1 + \frac{1}{2}$</p>

SECTION E (Case Studies/Passage based questions of 4 Marks each)

36.	<p>(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in $(0, 12)$</p> <p>(ii) $f'(x) = -0.2x + m$ Since, 6 is the critical point, $f'(6) = 0 \Rightarrow m = 1.2$</p> <p>(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$</p> <p>$f'(x) = -0.2x + 1.2 = -0.2(x - 6)$</p> <table border="1"> <thead> <tr> <th>In the Interval</th><th>$f'(x)$</th><th>Conclusion</th></tr> </thead> <tbody> <tr> <td>$(0, 6)$</td><td>+ve</td><td>f is strictly increasing in $[0, 6]$</td></tr> <tr> <td>$(6, 12)$</td><td>-ve</td><td>f is strictly decreasing in $[6, 12]$</td></tr> </tbody> </table>	In the Interval	$f'(x)$	Conclusion	$(0, 6)$	+ve	f is strictly increasing in $[0, 6]$	$(6, 12)$	-ve	f is strictly decreasing in $[6, 12]$	<p>1</p> <p>1</p> <p>1+1</p>
In the Interval	$f'(x)$	Conclusion									
$(0, 6)$	+ve	f is strictly increasing in $[0, 6]$									
$(6, 12)$	-ve	f is strictly decreasing in $[6, 12]$									

	<p>OR</p> <p>(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$, $f'(x) = -0.2x + 1.2, f'(6) = 0$, $f''(x) = -0.2$ $f''(6) = -0.2 < 0$</p> <p>Hence, by second derivative test 6 is a point of local maximum. The local maximum value $= f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$</p> <p>We have $f(0) = 98.6, f(6) = 102.2, f(12) = 98.6$</p> <p>6 is the point of absolute maximum and the absolute maximum value of the function $= 102.2$.</p> <p>0 and 12 both are the points of absolute minimum and the absolute minimum value of the function $= 98.6$.</p>	<p>1</p> <p>1/2</p> <p>1/2</p>
37.	<p>(i)</p>  <p>Let $(x, y) = \left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)$ be the upper right vertex of the rectangle.</p> <p>The area function $A = 2x \times 2 \frac{b}{a}\sqrt{a^2 - x^2}$ $= \frac{4b}{a} x\sqrt{a^2 - x^2}, x \in (0, a)$.</p> <p>(ii) $\frac{dA}{dx} = \frac{4b}{a} \left[x \times \frac{-x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right]$ $= \frac{4b}{a} \times \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -\frac{4b}{a} \times \frac{2\left(x + \frac{a}{\sqrt{2}}\right)\left(x - \frac{a}{\sqrt{2}}\right)}{\sqrt{a^2 - x^2}}$ $\frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$ $x = \frac{a}{\sqrt{2}}$ is the critical point.</p> <p>(iii) For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} > 0$ and for the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} < 0$.</p> <p>Hence, by the first derivative test, there is a local maximum at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x = \frac{a}{\sqrt{2}}$</p> <p>Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$.</p> <p>OR</p>	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>

	<p>(iii) $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}, x \in (0, a)$.</p> <p>Squaring both sides, we get</p> $Z = A^2 = \frac{16b^2}{a^2}x^2(a^2 - x^2) = \frac{16b^2}{a^2}(x^2a^2 - x^4), x \in (0, a).$ <p>A is maximum when Z is maximum.</p> $\frac{dZ}{dx} = \frac{16b^2}{a^2}(2xa^2 - 4x^3) = \frac{32b^2}{a^2}x(a + \sqrt{2}x)(a - \sqrt{2}x)$ $\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}.$ $\frac{d^2Z}{dx^2} = \frac{32b^2}{a^2}(a^2 - 6x^2)$ $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}} = \frac{32b^2}{a^2}(a^2 - 3a^2) = -64b^2 < 0$ <p>Hence, by the second derivative test, there is a local maximum value of Z at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, Z is maximum at $x = \frac{a}{\sqrt{2}}$ hence, A is maximum at $x = \frac{a}{\sqrt{2}}$.</p> <p>Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$.</p>	1 1/2 1/2
38.	<p>(i) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:</p> $E_1 = PQ, E_2 = \bar{P}\bar{Q}, E_3 = \bar{P}Q, E_4 = P\bar{Q}$ <p>Let E = The shell fired from exactly one of them hits the plane.</p> $P(E_1) = 0.3 \times 0.2 = 0.06, P(E_2) = 0.7 \times 0.8 = 0.56, P(E_3) = 0.7 \times 0.2 = 0.14, P(E_4) = 0.3 \times 0.8 = 0.24$ $P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$ $P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$ $= 0.14 + 0.24 = 0.38$ <p>(ii) By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$</p> $= \frac{0.14}{0.38} = \frac{7}{19}$ <p>NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$</p> <p>Alternative way of writing the solution:</p> <p>(i) P(Shell fired from exactly one of them hits the plane)</p> $= P[(\text{Shell from A hits the plane and Shell from B does not hit the plane}) \text{ or } (\text{Shell from A does not hit the plane and Shell from B hits the plane})]$ $= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$ <p>(ii) P(Shell fired from B hit the plane/Exactly one of them hit the plane)</p> $= \frac{P(\text{Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane})}{P(\text{Exactly one of them hit the plane})}$	1 1 2 1 1

	$= \frac{P(\text{Shell from only } B \text{ hit the plane})}{P(\text{Exactly one of them hit the plane})}$ $= \frac{0.14}{0.38} = \frac{7}{19}$	1
		1

