Disclaimer: Physics

Date: 20/09/2022

Subject: Physics — Class: Standard XI

Topic : circular motion — Time: 00:20 hrs

1. For a particle moving in a circle of radius $1 \mathrm{~m}$, speed increases uniformly from $5 \mathrm{~m/s}$ to $10 \mathrm{~m/s}$ in $10$ seconds. The value of angular acceleration of the particle is

A. $50 \mathrm{~rad/s^2}$

B. $5 \mathrm{~rad/s^2}$

C. $0.5 \mathrm{~rad/s^2}$ ✔

D. $0.05 \mathrm{~rad/s^2}$

Given, radius of the circle $r = 1 \mathrm{~m}$
initial speed of the particle $u = 5 \mathrm{~m/s}$
final speed of the particle $v = 10 \mathrm{~m/s}$

Since, angular speed is given as $\omega = \dfrac{\text{Linear Speed}}{\text{radius}}$

So, we have
initial angular speed $\omega_i = 5 \mathrm{~rad/s}$
final angular speed $\omega_f = 10 \mathrm{~rad/s}$
Time taken by the particle in such change is given as $t = 10 \mathrm{~sec}$
Thus, we have angular acceleration as
$\omega_f = \omega_i + \alpha t$

$$\Rightarrow \alpha = \frac{\omega_f - \omega_i}{t} = \frac{10-5}{10} = 0.5 \mathrm{~rad/s^2}$$

2. A particle is moving on a circular path of radius $3\ cm$ with a time varying speed of $v = 3t\ cm/s$, where $t$ is in seconds. The magnitude of tangential acceleration and the total acceleration of the particle at $t = 1\ s$ is

A. $3\ cm/s^2,\ 3\ cm/s^2$

B. $3\sqrt{2}\ cm/s^2,\ 3\sqrt{2}\ cm/s^2$

C. $\frac{3}{2} cm/s^2,\ 3\ cm/s^2$

D. $3\ cm/s^2,\ 3\sqrt{2}\ cm/s^2$ ✔

Given,
Radius of the circular path, $r = 3\ cm$
Speed, $v = 3t\ cm/s$
Total acceleration $\vec{a} = \vec{a_c} + \vec{a_t}$
$|\vec{a}| = \sqrt{a_c^2 + a_t^2}$

Centripetal acceleration, $(a_c) = \frac{v^2}{r} = \frac{9t^2}{3} = \frac{9t^2}{3} = 3t^2 = 3\ cm/s^2$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t) = 3\ cm/s^2$

Then, $|a| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}\ cm/s^2$

3. A ceiling fan is rotating with an angular velocity of $4\ rev/s$. It takes $40\ s$ to stop when it is switched off. The angular retardation during this interval is

A. $\frac{3\pi}{5} rad/s^2$

B. $-\frac{\pi}{5} rad/s^2$

C. $\frac{\pi}{5} rad/s^2$ ✔

D. $\frac{2\pi}{5} rad/s^2$

We know that $\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i}$

Given $\omega_i = 4\ \mathrm{rev/s} = 4 \times 2\pi\ \mathrm{rad/s}$
$= 8\pi\ \mathrm{rad/s}$

and $\omega_f = 0$

Retardation, $\therefore \alpha_{avg} = \left|\frac{0 - 8\pi}{40}\right| = \frac{\pi}{5} rad/s^2$

4. A car wheel rotates with an angular acceleration $\alpha = 4t^3 (\text{in rad}/s^2)$ where $t$ is the time taken (in seconds). If the wheel has initial angular velocity $\omega_0 = 20\ rad/s$, then the angular velocity $\omega(t)$ after $2\ s$ is

A. $20\ rad/s$

B. $36\ rad/s$ ✔

C. $4\ rad/s$

D. $16\ rad/s$

Initial angular velocity $w_0 = 20\ rad/s$
Given $\alpha = 4t^3$
$\because \alpha$ is a function of time
$\frac{d\omega}{dt} = 4t^3$
$d\omega = 4t^3 dt$
Integrating on both sides, $\int_{\omega_0}^{\omega} d\omega = \int_0^t 4t^3 dt$
$\omega - \omega_0 = \left[t^4\right]_0^2$
$\Rightarrow \omega = \omega_0 + \left[t^4\right]_0^2$
$= 20 + 16 = 36\ rad/s$

5. An exhaust fan is rotating with an angular velocity of $216 \text{ rad/min}$. When it is switched off, it is observed that the angular retardation of the fan is $\alpha = \frac{3}{2}\sqrt{t} \text{ rad/min}^2$. The time taken by the fan to stop completely is

- A. $36 \text{ min}$ ✔
- B. $6 \text{ min}$ ✘
- C. $17 \text{ min}$ ✘
- D. $22 \text{ min}$ ✘

Initial angular velocity $(\omega_0) = 216 \text{ rad/min}$
When switched off,
Final angular velocity $(\omega) = 0 \text{ rad/min}$
Given that $\alpha = \frac{3}{2}\sqrt{t}$
Since $\alpha$ is function of time and angular velocity of the fan is decreasing

$$\alpha = -\frac{d\omega}{dt}$$

$$\Rightarrow -\frac{d\omega}{dt} = \frac{3}{2}\sqrt{t}$$

$$\Rightarrow d\omega = -\frac{3}{2}\sqrt{t}dt$$

Integrating on both sides,

$$\int_{\omega_0}^{0} d\omega = -\int_0^t \frac{3}{2}\sqrt{t}dt$$

$$\Rightarrow \omega_0 = \frac{3}{2}\left(\frac{t^{3/2}}{3/2}\right)\Bigg|_0^t$$

$$\Rightarrow \omega_0 = t^{3/2}$$

$$\Rightarrow t = (\omega_0)^{2/3} = (216)^{2/3}$$

$$= 6^2 = 36 \text{ minutes}$$