BANKING OF ROADS

1. A car is moving on a levelled circular road of radius of curvature $300 \mathrm{~m}$. If the coefficient of static friction is 0.3 and acceleration due to gravity is $10 \mathrm{~ms}^{-2}$, then maximum allowable speed for the car will be (in kmh^{-1})

A. 30

B. 81

C. 108

D. 162

To avoid slipping, frictional force will act in a direction to provide the necessary centripetal force. i.e frictional force will act towards centre of circular curvature.

Here, f is magnitude of frictional force.

$$\therefore f=\frac{m v_{max}^2}{r}$$

$$\therefore \mu_s m g=\frac{m v_{max}^2}{r}$$

Here, it is given that

$r=300 \mathrm{~m}, \mu_s=0.3, g=10 \mathrm{~ms}^{-2}$

Applying the formula,

$v_{max}=\sqrt{\mu_s r g}=\sqrt{0.3 \times 300 \times 10}=30 \mathrm{~ms}^{-1}$

Or, $30 \times \frac{18}{5} \mathrm{kmh}^{-1}=108 \mathrm{~kmh}^{-1}$

2. What should be the value of coefficient of static friction between the tyre and the road, when a car travelling at speed of $60\ kmh^{-1}$ makes a level turn of radius $40\ \mathrm{m}$?

A. 0.5

B. 0.66

C. 0.71

D. 0.80

Applying equation for frictional force as centripetal force at the circular turn gives,

$$f = \frac{mv^2}{r}$$

Here, f is magnitude of frictional force.

$$\therefore \mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{rg} = \frac{\left(\frac{60 \times 5}{18}\right)^2}{40 \times 9.8} = 0.71$$

3. What is the smallest radius of a circular path on which a cyclist can travel with uniform speed of $36\ kmh^{-1}$, with angle of inclination (from vertical, for cyclist) 45° and $g = 10\ ms^{-2}$?

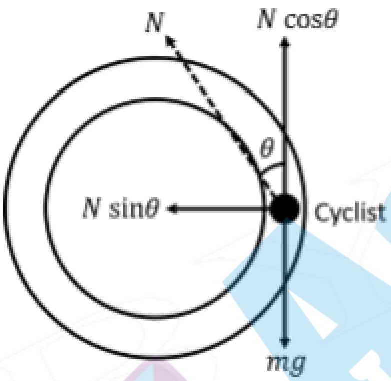
A. $20\ \mathrm{m}$

B. $10\ \mathrm{m}$

C. $30\ \mathrm{m}$

D. $40\ \mathrm{m}$

The components of Normal reaction for cyclist need to satisfy the equation of dynamics i.e. supporting the weight & centripetal force.



So, we have

$N \sin\theta = \frac{mv^2}{r}$.......(i)

$N \cos\theta = mg$..........(ii)

On dividing (i) by (ii), we get

$\tan\theta = \frac{v^2}{rg}$

$\Rightarrow r = \frac{v^2}{g\tan\theta} = \frac{10 \times 10}{10 \times \tan 45^\circ} = 10\ \mathrm{m}$

.

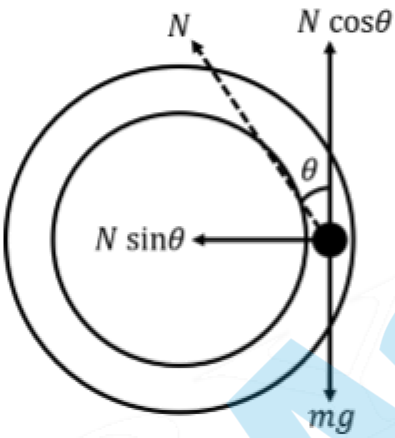
4. The angle which the bicycle and its rider must make with the vertical when going round a curve of $8.1 \mathrm{~m}$ radius at $9 \, ms^{-1}$ is (Take $g=10 \, m/s^2$)

A. 20°

B. 45°

C. 30°

D. 60°



The components of Normal reaction for cyclist need to satisfy the equation of dynamics i.e. supporting the weight & centripetal force.

So, we have

$N \sin \theta=\frac{m v^2}{r}$.......(i)

$N \cos \theta=m g$..........(ii)

On dividing (i) by (ii), we get

$\tan \theta=\frac{v^2}{r g}$

Here, $r=8.1 \mathrm{~m}, v=9 \, m s^{-1}$

Putting values,

$\tan \theta=\frac{v^2}{r g}=\frac{9 \times 9}{8.1 \times 10}=1$

$\theta=\tan ^{-1}(1)=45^{\circ}$

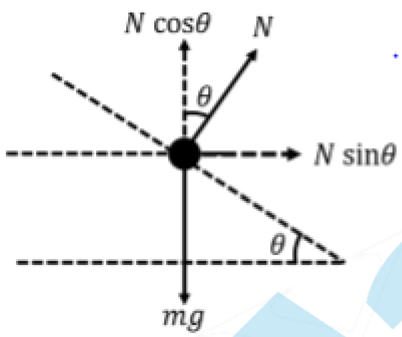
5. A car of mass $1000\ \mathrm{kg}$ negotiates a banked curve of radius $90\ \mathrm{m}$ on a frictionless road. If the banking angle is 45°, the speed of car is (Take $g = 10\ \mathrm{m/s^2}$)

A. $20\ \mathrm{ms^{-1}}$

B. $30\ \mathrm{ms^{-1}}$

C. $5\ \mathrm{ms^{-1}}$

D. $10\ \mathrm{ms^{-1}}$



By the free body diagram and writing the force equation we get,

$N\cos\theta = mg \ldots (i)$

$N\sin\theta = \dfrac{mv^2}{r} \ldots (ii)$

Now by dividing both the above equations, we get,

$\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$

where, v = speed and r = radius

For banking, $\tan\theta = \dfrac{v^2}{rg}$

$\Rightarrow\ \tan 45^{\circ} = \dfrac{v^2}{90 \times 10} = 1$

$\Rightarrow\ v^2 = 900$

$\Rightarrow\ v = \sqrt{900}\ \mathrm{m/s} = 30\ \mathrm{m/s}$