JEE Main Maths Complex Numbers Previous Year Questions With Solutions

Question 1:

Let a complex number z, $|z| \neq 1$, satisfy $\log_{1/\sqrt{2}} [(|z| + 11) / (|z| - 1)^2] \leq 2$. Then, the largest value of |z| is equal to

(a) 8

(b) 5

(c) 6

(d) 7

Solution:

Given that $|z| \neq 1$.

$\log_{1/\sqrt{2}} [(|z| + 11) / (|z| - 1)^2] \leq 2$

$\Rightarrow [(|z| + 11) / (|z| - 1)^2] \geq 1 / 2$

$\Rightarrow 2|z| + 22 \geq (|z| - 1)^2$

$\Rightarrow 2|z| + 22 \geq |z|^2 - 2|z| + 1$

$\Rightarrow |z|^2 - 4|z| - 21 \leq 0$

$(|z| - 7) (|z| + 3) \leq 0$

$|z| \leq 7$

The largest value of |z| = 7

Hence, option (d) is the answer.

Question 2:

The least value of |z| where z is a complex number which satisfies the inequality

$$exp\left(\frac{(|z|+3)(|z|-1)}{||z||+1}log_e 2\right) \geq log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$$

$i = \sqrt{-1}$, is equal to

(a) 2

(b) 3

(c) 8

(d) $\sqrt{5}$

Solution:

$2^{[\{(|z| + 3) (|z| - 1)\} / \{|z| + 1\}]} \geq \log_{\sqrt{2}}(16)$

$2^{[\{(|z| + 3) (|z| - 1)\} / \{|z| + 1\}]} \geq 2^3$

=> $[\{(|z| + 3) (|z| - 1)\} / \{|z| + 1\}] \geq 3$

=> $|z|^2 + 2 |z| - 3 \geq 3 |z| + 3$

=> $|z|^2 - |z| - 6 \geq 0$

=> $(|z| - 3) (|z| + 2) \geq 0$

=> $|z|_{min} = 3$

The least value of |z| = 3
Hence, option (b) is the answer.

Question 3:
If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then z_1/z_2 is
(a) purely imaginary
(b) purely real
(c) zero of purely imaginary
(d) Neither real nor imaginary
Solution:
Given that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$$\Rightarrow (z_1+z_2)+(\bar{z_1}+\bar{z_2}) = |z_1|^2+|z_2|^2 \Rightarrow |z_1|^2+|z_2|^2 = |z_1|^2+|z_2|^2+z_1\bar{z_2}+z_2\bar{z_1} \Rightarrow 0 = z_1\bar{z_2}+z_2\bar{z_1} \Rightarrow z_1\bar{z_2} = -z_2\bar{z_1} \Rightarrow \frac{\bar{z_1}}{\bar{z_2}} = -\frac{z_1}{z_2} \Rightarrow (\frac{\bar{z_1}}{z_2}) = -\frac{z_1}{z_2}$$

So z_1/z_2 is purely imaginary.
Hence, option (a) is the answer.

Question 4:
If z and w are two complex numbers such that |z| ≤ 1, |w|≤ 1 and

$$|z+iw| = |z-i\bar{w}| = 2$$

Then, z is equal to
(a) 1 or i
(b) i or -i
(c) i or -1
(d) 1 or -1
Solution:
Given |z| ≤ 1and |w| ≤ 1

$$|z+iw| = |z-i\bar{w}| = 2$$

Let $z = x_1 + iy_1$
$w = x_2 + iy_2$
So $x_1^2 + y_1^2 \leq 1$
$x_2^2 + y_2^2 \leq 1$
Therefore, $|z + iw| = |x_1 + y_1 + i(x_2+iy_2)| = 2$
$(x_1 - y_1)^2 + (y_2+x_2)^2 = 4$ ....(1)
Also $|z + iw| = |x_1 + y_1 + i(x_2+ iy_2)| = 2$
$(x_1 - y_2)^2+(y_1 - x_2)^2 = 4$ ....(2)
Subtract (2) - (1)

=> $(y_1 - x_2)^2 - (y_1 + x_2)^2 = 0$

$y_1^2 + x_2^2 - 2y_1x_2 - y_1^2 - x_2^2 - 2y_1x_2 = 0$

$y_1x_2 = 0$

$y_1 = 0$

=> $x_1^2 \leq 1$

=> $-1 \leq x \leq 1$

So z = 1+ i0 or -1 + i0

Hence, option (d) is the answer.

Question 5:

If 3/(2+ cos θ+i sin θ) = a+ib, then $[(a-2)^2+b^2]$ is

(a) 0

(b) -1

(c) 1

(d) 2

Solution:

Given that 3/(2+ cos θ+i sin θ) = a+ib

(3/(2+ cos θ+ i sin θ))((2 + cos θ)-i sin θ)/(2 + cos θ)- i sin θ)

= $((6 + 3 \cos \theta) - i\, 3 \sin \theta)/((2+\cos \theta)^2+\sin^2 \theta)$

= ((6 + 3 cos θ)- i(3 sin θ))/(5 + 4 cos θ)

Comparing with a+ib, we get;

a = (6+3 sin θ)/(5+4 cos θ)

b = -3 sin θ/(5+4 cos θ)

$(a - 2)^2 + b^2 = [(6 + 3 \sin \theta)/(5 + 4 \cos \theta)] - 2^2 + (-3 \sin \theta/(5+4 \cos \theta))^2$

$= ((-4 - 5 \cos \theta)^2 + 9 \sin^2\theta)/(5 + 4 \cos \theta)^2$

$= ((4 +5 \cos \theta)^2 + 9 \sin^2\theta)/(5 + 4 \cos \theta)^2$

$= (16 + 40 \cos \theta + 25 \cos^2\theta + 9 \sin^2\theta)/(5 + 4 \cos \theta)^2$

$= (16 + 40 \cos \theta + 16 \cos^2\theta + 9 (\sin^2\theta + \cos^2\theta))/(5 + 4 \cos \theta)^2$

$= (16 + 40 \cos \theta + 16 \cos^2\theta + 9)/(5 + 4 \cos \theta)^2$

$= (16 \cos^2\theta + 40 \cos \theta + 25)/(5 + 4 \cos \theta)^2$

$= (4 \cos \theta + 5)^2/(5+4 \cos \theta)^2$

= 1

Hence, option (c) is the answer.

Question 6:

If

$$\frac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$$

is a real number, then the argument of sin θ + i cos θ is

(a) $\pi - \tan^{-1}(4/3)$

(b) $-\tan^{-1}(3/4)$

(c) π- $\tan^{-1}$ (4/3)
(d) $\tan^{-1}$ (4/3)
Solution:
Let

$$z = \frac{3 + i\sin\theta}{4 - i\cos\theta} \times \frac{4 + i\cos\theta}{4 + i\cos\theta}$$
$$= \frac{12 - \sin\theta\cos\theta + i(4\sin\theta + 3\cos\theta)}{16 + \cos^2\theta}$$

Given that z is real.
4 sin θ + 3 cos θ = 0
⇒ tan θ = -3/4 [θ lies in 2nd quadrant]
arg (sin θ+ i cosθ) = π + $\tan^{-1}$ (cos θ /sin θ)
= π - $\tan^{-1}$(4/3)
Hence, option (a) is the answer.

Question 7:
If √(x+iy) = ±(a+ib), then √(-x-iy) is equal to
(a) ±(b+ia)
(b) ±(a-ib)
(c) (ai+b)
(d) None of these

Solution:
Given that √(x+iy) = ±(a+ib)
Squaring both sides, we get;
(x+iy) = $(a+ib)^2$
= $a^2 + 2aib - b^2$
= $a^2 - b^2 + 2aib$
Comparing real and imaginary part, we get;
x = a^2-b^2
y = 2ab
-x - iy = $-a^2 + b^2 - i(2ab)$
-x - iy = $(-b + ia)^2$
Taking square root, we get;
√(-x-iy) = ±(-b+ia)
Hence, option (d) is the answer.

Question 8:
The point in the set { z ∈ C: arg ((z-2)/(z-6i)) = π/2} (where C denotes the set of all complex number) lie on the curve which is

(a) hyperbola
(b) pair of lines
(c) parabola
(d) circle
Solution:
Given that arg ((z-2)/(z-6i) = π/2
Arg (z-2) - arg(z-6i) = π/2
z = x+iy
arg ((x-2)+iy) - arg(x+(y-6)i = π/2

$$\Rightarrow tan^{-1}\left(\frac{y}{(x-2)}\right) - tan^{-1}\left(\frac{(y-6)}{x}\right) = \frac{\pi}{2}$$

We know, $\tan^{-1}A - \tan^{-1}B = \tan^{-1}[(A-B)/(1+AB)]$

$$\Rightarrow \left(\frac{xy-(x-2)(y-6)}{x(x-2)+y(y-6)}\right) = tan\frac{\pi}{2}$$
$$\Rightarrow \left(\frac{xy-(x-2)(y-6)}{x(x-2)+y(y-6)}\right) = \frac{1}{0}$$

x(x-2) + y(y-6) = 0
$x^2 - 2x + 1 + y^2 - 6y + 9 - 10 = 0$
$(x - 1)^2 + (y - 3)^2 = \sqrt{(10)}^2$
The point lies on a circle.
Hence, option (d) is the answer.

Question 9:
The complex number (-√3+3i)(1-i)/(3+√3i)(i)(√3+√3i) when represented in the argand diagram lies in
(a) in the second quadrant
(b) in the first quadrant
(c) on the Y-axis (imaginary axis)
(d) on the X-axis (real axis)
Solution:
Given z = (-√3+3i)(1-i)/(3+√3i)(i)(√3+√3i)
Taking √3 outside
= $\sqrt{3}(-1+\sqrt{3}i)(1-i)/(\sqrt{3})^2(\sqrt{3}+i)(i)(1+i)$
= (1/√3)(-1+√3i)(1-i)/(-1+i√3)(1+i)
= (1-i)/√3(1+i)
Multiply numerator and denominator with (1-i)
= (1-i)(1-i)/√3(1+i)(1-i)
= $(1-i)^2/\sqrt{3}(1+1)$

= (1-2i-1)/2√3
= -2i/2√3
= -i/√3, purely imaginary
So, z lies on Y-axis.
Hence, option (c) is the answer.

Question 10:
The complex numbers sin x + i cos 2x and cos x - i sin 2x are conjugate to each other for
(a) x = nπ
(b) x = (n+½)π
(c) x = 0
(d) No values of x
Solution:
Given that sin x + i cos 2x and cos x - i sin 2x are conjugate to each other.

$$\overline{sinx + icos2x} = cosx - isin2x$$

sin x - i cos 2x = cos x - i sin 2x
Comparing real and imaginary parts
sin x = cos x ..(i)
cos 2x = sin 2x…(ii)
From equation (i) tan x = 1
From equation (ii) tan 2x = 1
=> 2tan x/(1-$\tan^2$x) = 1, not satisfied by tan x = 1.
Therefore no value of x is possible.
Hence option (d) is the answer.

Question 11:
The square roots of -7- 24√(-1) are
(a) ±(4+3√(-1))
(b) ±(3+4√(-1))
(c) ±(3-4√(-1))
(d) ±(4-3√(-1))
Solution:
-7- 24√(-1) = -7-24i (since √(-1) = i)
Let √(-7-24i) = x+iy
Squaring both sides
(-7-24i) = $(x+iy)^2$
= x^2-y^2+2xiy
Comparing real and imaginary part, we get
$x^2-y^2 = -7$
2xy = -24
solving above 2 equations, $x^2 = 9$ and $y^2 = 16$

$x = \pm 3$ and $y = \pm 4$
If $x = 3$, $y = -4$
If $x = -3$, $y = 4$
So the roots are $(-3+4i)$ and $(3-4i)$.
Hence, option (c) is the answer.

Question 12:
If 1, ω and ω^2 are the cube roots of unity, then $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)(1 - \omega^8 + \omega^{16})$ ... up to 2n factors is
(a) $2n$
(b) 2^{2n}
(c) 1
(d) -2^{2n}
Solution:
Use $1+\omega = -\omega^2$ and $1+\omega^2 = -\omega$
$(1 - \omega + \omega^2) = -2\omega$
$(1 - \omega^2 + \omega^4) = (1 - \omega^2 + \omega^3\,\omega) = (1 - \omega^2 + \omega) = -2\omega^2$
$(1 - \omega^4 + \omega^8) = 1-\omega+\omega^2 = -2\omega$
$(1 - \omega^8 + \omega^{16}) = (1 - \omega^2 + \omega) = -2\omega^2$
-2ω and $-2\omega^2$ are the successive factors.
So the product is $(-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)\ldots(-2\omega^2)$
$= (-2\omega)^n(-2\omega^2)^n$
$= 2^{2n}(\omega^3)^{2n}$
$= 2^{2n}$
Hence, option (b) is the answer.

Question 13:
Let z and w be two complex numbers such that

$$w = z\bar{z} - 2z + 2,$$

$|(z + i) / (z - 3i)| = 1$ and Re (w) has a minimum value. Then, the minimum value of $n \in N$, for which w^n is real, is equal to
Solution:
Consider $z = x + iy$
$|z + i| = |z-3i|$
$y = 1$
Now $w = x^2 + y^2 - 2x - 2iy + 2$
$w = x^2 + 1 - 2x - 2i + 2$
$Re(w) = x^2 - 2x + 3$
$Re(w) = (x-1)^2 + 2$
$Re(w)_{min}$ at $x = 1$
$z = 1 + i$

Now $w = 1 + 1 - 2 - 2i + 2$

$w = 2(1-i) = 2\sqrt{2}\, e^{i(-\pi/4)}$

$w^n = 2\sqrt{2}\, e^{i(-\pi/4)}$

If w^n is real, $n = 4$

Question 14:

Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \pi / 2$. Then the area of the triangle with vertices origin, z and w is equal to:

(a) 1/2

(b) 4

(c) 2

(d) 1 / 4

Solution:

$w = 1 - \sqrt{3}i$

$|w| = 2$

$|zw| = 1$

$\Rightarrow |z| = 1/|w|$

$= 1/2$

$\arg(z) - \arg(w) = \pi / 2$

Area of triangle = $(1/2) \times (1/2) \times 2$

$= 1/2$

Hence option a is the answer.

Question 15:

If $(z + i)/(z + 2i)$ is purely real, then the locus of z is

(a) x - axis

(b) y - axis

(c) y = x

(d) y = x/2

Solution:

Given that $(z + i)/(z + 2i)$ is purely real.

Let $z = x+iy$

Then $(z + i)/(z + 2i) = (x+iy + i)/(x+iy + 2i)$

Multiply and divide by the conjugate, and equate imaginary part to zero.

$[(-x(y+2)i) + ix(1+y)]/[(x^2 + (y+2)^2] = 0$

$\Rightarrow (-x(y+2)i) + ix(1+y) = 0$

$\Rightarrow x(y+2) = x(1+y)$

$\Rightarrow xy + 2x - x - xy = 0$

$\Rightarrow x = 0$

=> locus of z is y axis.

Hence, option b is the answer.