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JEE Main Maths Functions Previous Year Questions With Solutions

Question 1:

The total number of functions, f : {1, 2, 3, 4} → {1, 2, 3, 4, 5, 6} such that f(1) + f(2) = f(3), is equal to

(A) 60

(B) 90

(C) 108

(D) 126

Solution:

Case 1:

If f(3) = 3 then f(1) and f(2) take 1 OR 2

Number of ways = 2×6

= 12

Case 2:

If f(3) = 5 then f(1) and f(2) take 2 OR 3

OR 1 and 4

Number of ways = 2×6×2

= 24

Case 3:

If f(3) = 2

then f(1) = f(2) = 1

Number of ways = 6

Case 4:

If f(3) = 4 then f(1) = f(2) = 2

Number of ways = 6

OR f(1) and f(2) take 1 and 3

Number of ways = 12

Case 5:

If f(3) = 6 then f(1) = f(2) = 3 => 6 ways

OR f(1) and f(2) take 1 and 5 => 12 ways

OR f(2) and f(1) take 2 and 4 => 12 ways

Total number of functions = 12 + 24 + 6 + 6 + 12 + 6 + 12 + 12

= 90

Hence, option (B) is the answer.

Question 2:

Let f be any function defined on R and let it satisfy the condition: $|f(x) - f(y)| \leq |(x - y)^2|$, $\forall$ x, y $\in$ R. If f (0) = 1, then :

(A) f (x) < 0, $\forall$ x $\in$ R

(B) f (x) can take any value in R

(C) f (x) = 0, ∀ x ∈ R
(D) f (x) > 0, ∀ x ∈ R

Solution:
Given that $|f(x) - f(y)| \le |(x - y)^2|$, ∀ x, y ∈ R
$|[f(x) - f(y)] / [(x - y)]| \le |x - y|$
$\lim_{x \to y} |[f(x) - f(y)] / [(x - y)]| \le 0$
$|f'(y)| \le 0$
$f'(y) = 0$
f (y) = c
As f(0) = 1, f (y) = 1, for all y belongs to R.
⇒ f(x) = 1
Hence, option (D) is the answer.

Question 3:
The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ in the interval [5/4, 2], where [t] is the greatest integer ≤ t, is

Solution:
Given that $f(x) = |5x - 7| + [x^2 + 2x]$
$= |5x - 7| + [(x + 1)^2] - 1$
Critical points of f(x) = 7/5, √5 - 1, √6 - 1, √7 - 1, √8 - 1, 2
The minimum or maximum value of f(x) occurs at critical points.
So f(5/4) = (¾) + 4
= 19/4
f(7/5) = 0 + 4 = 4
Since both |5x – 7| and $x^2 + 2x$ are increasing in nature after x = 7/5.
So, f(2) = 3 + 8
= 11
$f(7/5)_{min} = 4$
$f(2)_{max} = 11$
Therefore the sum of the maximum and minimum values of the function f(x) = 4 + 11
= 15

Question 4:
Let f : R → R be a continuous function such that f(3x) - f(x) = x. If f(8) = 7, then f(14) is equal to
(A) 4
(B) 10
(C) 11
(D) 16

Solution:
Given that f(3x) - f(x) = x ..(1)

$x \rightarrow x/3$

$\Rightarrow f(x) - f(x/3) = x/3$ ..(2)

Again $x \rightarrow x/3$

$\Rightarrow f(x/3) - f(x/9) = x/3^2$ ..(3)

Similarly,

$f(x/3^{n-2}) - f(x/3^{n-1}) = x/3^{n-1}$ ..(n)

Adding above equations and put $n \rightarrow \infty$

$\lim_{n\rightarrow\infty} [f(3x) - f(x/3^{n-1})] = x(1 + ⅓ + 1/3^2 + \ldots)$

$f(3x) - f(0) = 3x/2$

Put $x = 8/3$

$f(8) - f(0) = 4$

Given that $f(8) = 7$

So $f(0) = 3$

Put $x = 14/3$

$f(14) - 3 = 7$

$\Rightarrow f(14) = 10$

Hence, option (B) is the answer.

Question 5:

Let f :R→R be defined as $f(x) = 2x-1$ and g:R - {1} →R be defined as $g(x) = (x-½)/(x-1)$. Then the composition function f(g(x)) is:

(A) Both one-one and onto

(B) onto but not one-one

(C) Neither one-one nor onto

(D) one-one but not onto

Solution:

Given that $f(x) = 2x-1$

$g(x) = (x-½)/(x-1)$

$f(g(x)) = 2g(x) - 1$

$= 2[(x - 1/2)/(x - 1)] - 1$

$= x/(x - 1)$

$f(g(x)) = 1 + 1/(x - 1)$

So the function is one-one but not onto.

Hence, option (D) is the answer.

Question 6:

If f: R→ R is a function defined by $f(x) = [x - 1] \cos((2x - 1)/2)\pi$, where [.] denotes the greatest integer function, then f is:

(A) discontinuous only at $x = 1$

(B) discontinuous at all integral values of x except at $x = 1$

(C) continuous only at $x = 1$

(D) continuous for every real x

Solution:
Given that f(x) = [x - 1] cos ((2x - 1)/2)π
Doubtful points are x = n, n∈I
We find the LHL and RHL.
LHL = $\lim_{x \to n-}$[x - 1] cos ((2x - 1)/2)π
= (n-2) cos((2n - 1)/2)π
= 0
RHL = $\lim_{x \to n+}$[x - 1] cos ((2x - 1)/2)π
= (n-1) cos((2n - 1)/2)π
= 0
⇒ f(n) = 0
⇒ continuous for every real x.
Hence, option (D) is the answer.

Question 7:
The function f(x) = ((4x^3- 3x^2)/6) - 2sin x + (2x - 1)cos x
(A) increases in [1/2, ∞)
(B) decreases (-∞, 1/2]
(C) increases in (-∞, 1/2]
(D) decreases [1/2, ∞)

Solution:
Given that f(x) = ((4x^3- 3x^2)/6) - 2sin x + (2x - 1)cos x
f'(x) = (2x^2 - x) - 2 cos x + 2 cos x - sin x(2x-1)
= (2x - 1) (x - sin x)
When x>0, x - sin x > 0
When x < 0, x - sin x < 0
f'(x) ≥ 0 in x∈(-∞, 0] ∪ [1/2, ∞)
and f'(x) ≤ 0 in x∈ [0, ½]
⇒ f(x) increases in [1/2, ∞).
Hence, option (A) is the answer.

Question 8:
Let f: R → R be defined as

$$f(x) = \begin{cases} -55x & if x < -5 \\ 2x^3 - 3x^2 - 120x & if -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336 & if x > 4 \end{cases}$$

Let A = {x ∈ R : f is increasing}. Then A is equal to:
(A) (-5, -4) ∪ (4,∞)

(B) (-5, ∞)
(C) (-∞, -5) ∪ (4, ∞)
(D) (-∞, -5) ∪ (-4, ∞)

Solution:
Given

$$f(x)=\begin{cases} -55x & if x<-5 \\ 2x^3-3x^2-120x & if -5\le x\le 4 \\ 2x^3-3x^2-36x-336 & if x>4 \end{cases}$$

f'(x) = {-55 ; x< -5; $6(x^2 - x - 20)$; -5 < x < 4; $6(x^2 - x - 6)$; x > 4
= {-55 ; x< -5; 6(x - 5)(x + 4) ; –5 < x < 4; 6(x - 3)(x + 2) ; x > 4
Hence, f(x) is monotonically increasing in (-5, -4) ∪ (4, ∞)
Hence, option (A) is the answer.

Question 9:
Let f, g: N→N such that f(n + 1)= f(n)+ f(1) for all n ∈ N and g be any arbitrary function. Which of the following statements is NOT true?
(A) f is one-one
(B) If fog is one-one, then g is one-one
(C) If g is onto, then fog is one-one
(D) If f is onto, then f(n) = n for all n ∈ N

Solution:
Given that f(n + 1) = f(n) +f(1)
=> f(n + 1) - f(n) = f(1)
This implies an A.P. with common difference = f(1)
Here, the general term = T_n = f(1) + (n-1)f(1)
= nf(1)
f(n) = nf(1)
Therefore, f(n) is one-one.
For fog to be one-one, g should be one-one.
For f to be onto, f(n) must take all the values of natural numbers.
Since f(x) is increasing, f(1) =1
f(n) = n
If g is many one, then fog is many one.
So the statement “if g is onto, then fog is one-one” is not true.
Hence, option (C) is the answer.

Question 10:

Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then:
(A) $y = 273x$
(B) $2y = 91x$
(C) $y = 91x$
(D) $2y = 273x$

Solution:
Number of elements in A, $n(A) = 3$
Number of elements in B, $n(B) = 5$
Number of elements in A × B = 15
Number of one-one function is $x = 5 \times 4 \times 3$
$x = 60$
Number of one-one function is $y = 15 \times 14 \times 13$
$y = 15 \times 4 \times (14/4) \times 13$
$y = 60 \times 7/2 \times 13$
$2y = (13)(7x)$
$2y = 91x$
Hence, option (B) is the answer.

Question 11:
The number of bijective functions $f : \{1, 3, 5, 7, \ldots, 99\} \to \{2, 4, 6, 8, \ldots, 100\}$ such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq \ldots \geq f(99)$, is
(A) ${}^{50}P_{17}$
(B) ${}^{50}P_{33}$
(C) $33! \times 17!$
(D) $50!/2$

Solution:
Since the function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction
$f(3) > f(9) > f(15) \ldots > f(99)$
Therefore the number of ways = ${}^{50}C_{17} \times 1 \times 33!$
$= {}^{50}P_{33}$
Hence, option (B) is the answer.

Question 12:
If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f(5/4)$ is equal to
(A) -3/2
(B) -1/2
(C) 1/2
(D) 3/2

Solution:

Given that $g(x) = x^2 + x - 1$

$gof(x) = 4x^2 - 10x + 5$

$g(f(5/4)) = 4\times(5/4)^2 - 10\times(5/4) + 5$

$= (25/4) - (25/2) + 5$

$= (-25/4) + 5$

$= -5/4$

$g(f(x)) = (f(x))^2 + f(x) - 1$

$g(f(5/4)) = (f(5/4))^2 + f(5/4) - 1$

$-5/4 = (f(5/4))^2 + f(5/4) - 1$

$(f(5/4))^2 + f(5/4) + (1/4) = 0$

$\Rightarrow [f(5/4) + (1/2)]^2 = 0$

$\Rightarrow f(5/4) = -1/2$

Hence, option (B) is the answer.