JEE Main Maths Determinants Previous Year Questions With Solutions

Question 1:
Consider the following system of equations:
x + 2y - 3z = a
2x + 6y - 11 z = b
x - 2y + 7z = c,
Where a, b and c are real constants. Then the system of equations
(a) has a unique solution when 5a = 2b + c
(b) has an infinite number of solutions when 5a = 2b + c
(c) has no solution for all a, b and c
(d) has a unique solution for all a, b and c
Solution:
Given equations are x + 2y - 3z = a
2x + 6y - 11 z = b
x - 2y + 7z = c

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

= 20 - 50 + 30
= 0

$$\Delta_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

= 20a - 2(7b + 11c) - 3(- 2b - 6c)
= 20a - 14b - 22c + 6b + 18c
= 20a - 8b - 4c
= 4(5a - 2b - c)

$$\Delta_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

= 7b+ 11c - a(25) - 3(2c - b)

= 7b+ 11c - 25a - 6c + 3b
= - 25a + 10b+ 5c
= - 5(5a - 2b- c)

$$\Delta_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

= 6c + 2b- 2(2c - b) - 10a
= - 10a + 4b+ 2c
= - 2(5a - 2b- c)
For infinite solution,
$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$
⇒ 5a = 2b + c
Then the system of equations has an infinite number of solutions when 5a = 2b + c.
Hence, option (b) is the answer.

Question 2:
Let

$$P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$$

and

$$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

where ω = (- 1 + i √3) / 2, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP-I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to
Solution:
$|P^{-1}AP - I|^2$
$= |(P^{-1}AP - I)(P^{-1}AP - I)|$
$= |P^{-1}APP^{-1}AP - 2P^{-1}AP + I|$
$= |P^{-1}A^2P - 2P^{-1}AP + P^{-1}IP|$
$= |P^{-1}(A^2 - 2A + I)P|$
$= |P^{-1}(A - I)^2 P|$

$= |P^{-1}|\ |A - I|^2\ |P|$
$= |A - I|^2$

$$= \begin{bmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{bmatrix}^2$$

$= (1\ (\omega\ (\omega + 1) + \omega) - 7\omega + \omega^2 \,.\, \omega)^2$
$= (\omega^2 + 2\omega - 7\omega + 1)^2$
$= (\omega^2 - 5\omega + 1)^2$
$= (-6\omega)^2$
$= 36\omega^2$
$\Rightarrow \alpha = 36$
Hence, the value of α is equal to 36.

Question 3:
The maximum value of

$$f(x) = \begin{vmatrix} sin^2x & 1 + cos^2x & cos2x \\ 1 + sin^2x & cos^2x & cos2x \\ sin^2x & cos^2x & sin2x \end{vmatrix}$$

x ∈ R is:
(a) √7
(b) √5
(c) 5
(d) 3 / 4
Solution:
Given that

$$f(x) = \begin{vmatrix} sin^2x & 1 + cos^2x & cos2x \\ 1 + sin^2x & cos^2x & cos2x \\ sin^2x & cos^2x & sin2x \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 2 & 1 + cos^2x & cos2x \\ 2 & cos^2x & cos2x \\ 1 & cos^2x & sin2x \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & cos^2x & cos2x \\ 1 & cos^2x & sin2x \end{vmatrix}$$

= (–1)[2 sin2x – cos2x] = cos2x – 2sin2x
Maximum value = √5
Hence, option (b) is the answer.

Question 3:
The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to
(a) - 2
(b) - 1
(c) 1
(d) 0
Solution:
Given equations are kx + y + z = 1
x + ky + z = k
x + y + zk = k^2
These equations can be written as:

$$\Rightarrow A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}, B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$$

The condition for no solution to the given system of equations is |A| = 0.

$$D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$$

⇒ $k(k^2 - 1) - (k - 1) + (1 - k) = 0$
⇒ $(k - 1)(k^2 + k - 1 - 1) = 0$
⇒ $(k - 1)(k^2 + k - 2) = 0$
⇒ $(k - 1)(k - 1)(k + 2) = 0$
⇒ k = 1, k = -2
For k = 1, the given system of equations is identical. So, the given equations have no solution when k = -2.
Hence, option (a) is the answer.

Question 4:

If

$$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

then the value of det (A^4) + det (A^{10} - Adj (2A))^{10}) is equal to

Solution:

$$|A| = -2$$

$$det(A^4) = |A|^4 = 16$$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$adj(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$(adj(2A))^{10} = 2^{10} \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$$

$$= 2^{10} \begin{bmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10} \begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$|A^{10}$ - adj $(2A)^{10}| = 0$

$|A|^4 = 16$

Hence, the value of det (A^4) + det (A^{10} - Adj (2A))^{10}) = 16.

Question 5:

If x, y, z are in arithmetic progression with common difference d, x ≠ 3d, and the determinant of the matrix

$$\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix} = 0$$

is zero, then the value of k^2 is

(a) 6

(b) 36
(c) 72
(d) 12

Solution:

x, y, z are in AP, so x + z = 2y

And

$$\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix} = 0$$

$$R_1 \to R_1 + R_3 - 2R_2$$

$$\begin{bmatrix} 0 & 4\sqrt{2} + k - 10\sqrt{2} & 0 \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix} = 0$$

$$-(k - 6\sqrt{2})(4z - 5y) = 0$$

$$k = 6\sqrt{2}$$

$$4z = 5y(not\ possible)$$

$$\Rightarrow k^2 = 72$$

Hence, option (c) is the answer.

Question 6:

If 1, $\log_{10}(4^x - 2)$ and $\log_{10}[4^x + (18 / 5)]$ are in arithmetic progression for a real number x, then the value of the determinant

$$\begin{vmatrix} 2[x - (1/2)] & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$$

is equal to:

Solution:

1, $\log_{10}(4^x - 2)$ and $\log_{10}[4^x + (18 / 5)]$ are in arithmetic progression.

2. $\log_{10}(4^x - 2) = 1 + \log_{10}[4^x + (18 / 5)]$

$\log_{10}[4^x - 2]^2 = \log_{10}[10 . [4^x + (18 / 5)]$

$[4^x - 2]^2 = 10 . [4^x + (18 / 5)]$

$(4^x)^2 + 4 - 4 . 4^x = 10 . 4^x + 36$

$(4^x)^2 - 14 . 4^x - 32 = 0$

$(4^x)^2 + 2 . 4^x - 16 . 4^x - 32 = 0$

$4^x [4^x + 2] - 16 [4^x + 2] = 0$

$4^x = - 2$ or $4^x = 16$

Rejected the value of $4^x = -2$.
Therefore

$$\begin{vmatrix} 2[x-(1/2)] & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix}$$

= 3 (-2) - 1 (0 - 4) + 4 (1 - 0)
= - 6 + 4 + 4
= 2

Question 7:
Let α, β, γ be the roots of the equations, $x^3 + ax^2 + bx + c = 0$, (a, b, c ∈ R and a, b and a, b ≠ 0). The system of the equations (in u, v, w) given by αu + βv + γw = 0; βu + γv + αw = 0; γu + αv + βw = 0 has non-trivial solutions, then the value of a^2 / b is
(a) 5
(b) 1
(c) 0
(d) 3
Solution:
Let α, β, γ be the roots of the equations, $x^3 + ax^2 + bx + c = 0$
Given that αu + βv + γw = 0; βu + γv + αw = 0; γu + αv + βw = 0 has non-trivial solutions,
For non-trivial solutions,

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0$
$(\alpha + \beta + \gamma)((\alpha + \beta + \alpha)^2 - 3 \sum\alpha\beta) = 0$
$(-a)[a^2 - 3b] = 0$
$a^2 = 3b$ [because a ≠ 0]
$a^2/b = 3$
Hence, option (d) is the answer.

Question 8:
The solutions of the equation

$$\begin{vmatrix} 1+sin^2x & sin^2x & sin^2x \\ cos^2x & 1+cos^2x & cos^2x \\ 4sin2x & 4sin2x & 1+4sin2x \end{vmatrix} = 0$$

(0 < x < π), are:

(a) π/6, 5π/6

(b) 7π/12, 11π/12

(c) 5π/12, 7π/12

(d) π/12, π/6

Solution:

$R_1 \rightarrow R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ cos^2x & 1+cos^2x & cos^2x \\ 4sin2x & 4sin2x & 1+4sin2x \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+cos^2x & cos^2x \\ 0 & 4sin2x & 1+4sin2x \end{vmatrix} = 0$$

Therefore, 2 + 8 sin 2x – 4 sin 2x = 0

⇒ sin 2x = - 1 / 2

⇒ x = 7π/12, 11π/12

Hence, option (b) is the answer.

Question 9:

Let the system of linear equations

4x + λy + 2z = 0

2x - y + z = 0

μx + 2y + 3z = 0, λ, μ ∈ R

has a non-trivial solution. Then which of the following is true?

(a) μ = 6, λ ∈ R

(b) λ = 2, μ ∈ R

(c) λ = 3, μ ∈ R

(d) μ = - 6, λ ∈ R

Solution:

Given, 4x + λy + 2z = 0

2x - y + z = 0

μx + 2y + 3z = 0

For non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

4 (- 3 - 2) - λ (6 - μ) + 2 (4 + μ) = 0

- 20 - 6λ + λμ + 8 + 2μ = 0

- 12 - 6λ + λμ + 2μ = 0

- 6 (λ + 2) + μ (λ + 2) = 0

(λ + 2) (μ - 6) = 0

μ = 6, λ ∈ R [since for λ = -2 the first two equations will become identical]

Hence, option (a) is the answer.

Question 10:

Let A = [a_{ij}] and B = [b_{ij}] be two 3 × 3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is

(a) 1/9

(b) 1/81

(c) 1/3

(d) 3

Solution:

$b_i = (3)^{(i+j-2)}a_{ji}$

$$B = \begin{bmatrix} 3^0a_{11} & 3a_{21} & 3^2a_{31} \\ 3a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0a_{11} & 3a_{21} & 3^2a_{31} \\ 3a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{vmatrix}$$

Taking 3^2 common each from C_3 and R_3

$$|B| = 81\begin{vmatrix} a_{11} & 3a_{21} & a_{31} \\ 3a_{12} & 3^2a_{22} & 3a_{32} \\ a_{13} & 3a_{23} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 and R_2

$$|B| = 81 \times 9 \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

Given |B| = 81

81 = 81×9 |A| [since $|A| = |A|^T$]

|A| = 1/9

Hence option (a) is the answer.