Instructions:

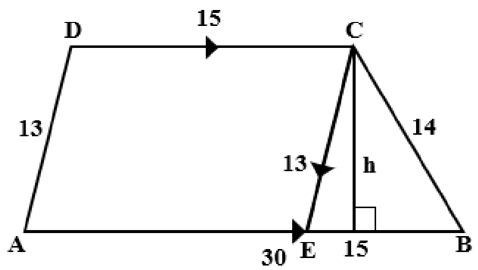
1. Fill in the Response Sheet with your Name, Class and the Institution through which you appear, in the specified places.
2. Diagrams given are only Visual aids; they are not drawn to scale.
3. You may use separate sheets to do rough work.
4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.
5. Duration of the Test: 2 pm to 4 pm (2 hours).

Question 1.

ABCD is a trapezium in which AB is parallel to CD. If AB = 30 cm, CD = 15 cm, AD = 13 cm and BC = 14 cm, then the area of the trapezium (in square cm) is

A) 263 B) 248 C) 252 D) 293

Solution: (C)



$CE \parallel DA$

$CEAD \rightarrow$ parallelogram

$DC = AE = 15\ cm$

$AD = CE = 13\ cm$

and $BE = 30 - 15 = 15\ cm$

In $\triangle BCE$

$s = \frac{13+14+15}{2} = 21$

$\Delta = s\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= 84\ cm^2$

Also, Area of $\triangle BCE = \frac{1}{2}\times 15\times h = 84$

$h = \frac{84\times 2}{15} = \frac{56}{5}\ cm$

Area of trapezium ABCD $= \frac{1}{2}(30 + 15)\times\frac{56}{5}$

$= \frac{1}{2}\times 45\times\frac{56}{5} = 252\ cm^2$

Question 2.

If $a + b = 2$, where a, b are real and $4^a + 4^b = 6$, then the numerical value of $2^{2(2a-1)} + 2^{2(2b-1)}$ is

A) 8 B) 12 C) 36 D) 1

Solution: (D)

$a + b = 2$

$4^a + 4^b = 6$

square on both side

$\left(4^a + 4^b\right)^2 = 6^2$

$4^{2a} + 4^{2b} + 2\times 4^a \times 4^b = 36$

$4^{2a} + 4^{2b} + 2\times 4^{a+b} = 36$

$4^{2a} + 4^{2b} + 2\times 4^2 = 36$

$4^{2q} + 4^{2b} + 32 = 36$

$4^{2a} + 4^{2b} = 4$

Now

$2^{2(2a-1)} + 2^{2(2b-1)}$

$\frac{2^{4a}}{2^2} + \frac{2^{4b}}{2^2}$

$= \frac{4^{2a}+4^{2b}}{4} = \frac{4}{4} = 1$

Question 3.

If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $x^{33} + x^{23} + x^{27} + x^{17} + 2$ is

A) 1 B) 2 C) 0 D) 4

Solution: (B)

$\left(x+\frac{1}{x}\right)^2=3$

$x^2+\frac{1}{x^2}+2=3$

$x^2+\frac{1}{x^2}=1$

$\Rightarrow x^4+\frac{1}{x^4}=-1$

$\Rightarrow x^8+\frac{1}{x^8}=-1$

Now

$x^{33}+x^{23}+x^{27}+x^{17}+2$

$x\left[x^{32}+x^{22}+x^{26}+x^{16}\right]+2$

$x.x^{24}\left[x^8+\frac{1}{x^2}+x^2+\frac{1}{x^8}\right]+2$

$x^{25}\left[\left(x^8+\frac{1}{x^8}\right)+\left(x^2+\frac{1}{x^2}\right)\right]+2$

$x^{25}[(-1)+(1)]+2=2$

Question 4.

The solution x of the equation $(5x)^x=5^{5^5}$ is of the form a^b, then $a+b$ is

A) 5 B) 10 C) 20 D) 9

Solution: (D)

$(5x)^x=5^{5^5}$

$log(5x)^x=log5^{5^5}$

$xlog(5x)=5^5log5$

$=5^4\times5^1\times log5$

$=5^4\times log5^5$

$xlog(5x)=5^4\times log\left(5\times5^4\right)$

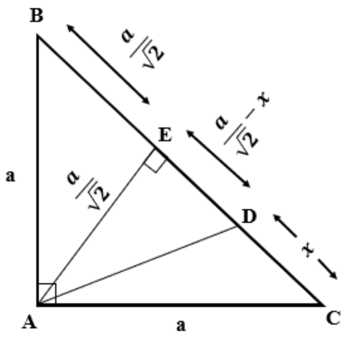
by comparing

$x=5^4=a^b,\ a+b=5+4=9$

Question 5. ABC is a right-angled isosceles triangle in which $\angle A = 90^{\circ}$. D is a point on BC. Then $\frac{BD^2+CD^2}{AD^2}$ is equal to

A) 1 B) 2 C) 3 D) 4

Solution: (B)



Draw $AE \perp AC$
$\triangle ABC$ is right isosceles $\triangle$.
So $BE = EC = \frac{AC}{2}$

$BC = \sqrt{2}a$

$BE = EC = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}} = AE$

$AD^2 = AE^2 + FD^2$

$= \left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{\sqrt{2}} - x\right)^2$

$= \frac{a^2}{2} + \frac{a^2}{2} + x^2 - \frac{2ax}{\sqrt{2}}$

$= a^2 + x^2 - \sqrt{2}ax$

NOW

$\frac{BD^2+CD^2}{AD^2}$

$\frac{\left(\frac{2a}{\sqrt{2}}-x\right)^2+x^2}{a^2+x^2-\sqrt{2}ax}$

$\frac{2a^2+x^2-2\sqrt{2}ax+x^2}{a^2+x^2-\sqrt{2}ax}$

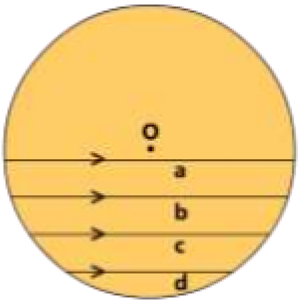
$\frac{2\left(a^2+x^2-\sqrt{2}ax\right)}{\left(a^2+x^2-\sqrt{2}ax\right)} = 2$

Question 6.

There are four equidistant parallel chords of a circle whose lengths are a,b,c,d as shown in the figure.

The numerical value of $\frac{a^2-d^2}{b^2-c^2}$ is

A) 1 B) 2 C) 0 D) 4



Solution: (C)

$R^2 = y^2 + \left(\frac{a}{2}\right)^2 - (1)$

$R^2 = (y + x)^2 + \left(\frac{b}{2}\right)^2 - (2)$

$R^2 = (y + 2x)^2 + \left(\frac{c}{2}\right)^2 - (3)$

$R^2 = (y + 3x)^2 + \left(\frac{d}{2}\right)^2 - (4)$

$eq\ \ (1) - (4)$

$0 = y^2 - (y + 3x)^2 + \frac{(a^2)}{4} - \frac{d^2}{4}$

$0 = y^2 - y^2 - 6xy - 9x^2 + \frac{(a^2-d^2)}{4}$

$a^2 - d^2 = 4\left(6xy + 9x^2\right)$

$a^2 - d^2 = 12x(2y + 3x) \ - (5)$

Now eq (2) − (3)

$0 = y^2 + x^2 + 2xy - y^2 - 4xy - 4x^2 + \frac{b^2-c^2}{4}$

$b^2 - c^2 = 4\left(3x^2 + 2xy\right)$

$b^2 - c^2 = 4x(3x + 2y)$ ____ (6)

So $\frac{a^2-d^2}{b^2-c^2} = \frac{12x(2y+3x)}{4x(3x+2y)} = 3$

Question 7.

In an arithmetic progression, the entries are integers. The sum of the first two terms is —2. The product of the second and third term is 5. The least number of the term which exceeds 2022 is

A) 508 B) 507 C) 510 D) 515

Solution: (B)

We are given that,

$a_2 \times a_3 = 5$

$\because a_2$ and a_3 can hold only integer values.

Case 1:

$a_2 =- 1$ and $a_3 =- 5$

$\Rightarrow d = a_3 - a_2 =- 5 - (- 1) =- 4$

Thus, we have $a_1 = 3$

We are also given that, $S_2 =- 2$

$\Rightarrow 2a_1 + d =- 2$

But this can not be true. Hence, it is discarded.

Case 2:

$a_2 = 1$ and $a_3 = 5$

$\Rightarrow d = a_3 - a_2 = 5 - (1) = 4$

Thus, we have $a_1 =- 3$

We are also given that, $S_2 =- 2$

$\Rightarrow 2a_1 + d =- 2$

$\Rightarrow 2(- 3) + 4 =- 6 + 4 =- 2$

Hence, $a_1 = 1$ and $d = 4$.

Now, we must find the least number of the term which exceeds 2022.

Let a_n be the required term of the AP.

So, we have $a_n > 2022$

$a_1 + (n - 1)d > 2022$

$1 + (n - 1)4 > 2022$

$(n - 1)4 > 2022 - 1$

$(n - 1)4 > 2021$

$(n - 1) > \frac{2021}{4}$

$(n - 1) > 505.25$

$n > 506.25$

$\because n$ can hold only integer value. Hence, $n = 507$.

Question 8.

The maximum ~~number~~ **length** of equal pieces that can be cut from the two lengths of wire of 74 cm and 92 cm, with a piece of 2 cm left out of each (in cm) is

A) 16 B) 18 C) 20 D) 14

(there is correction in question)

Solution: (B)

We are given that a piece of 2 cm length will be left out of each wire. Hence, the remaining length of the wire $=$ 72 cm and 90 cm.

Now, the maximum length of equal pieces that can be cut from the two lengths of wires of 72 cm and 90 cm length $= HCF(72, 90)$

Factors of 72 & 90 are:

$72 = 2\times2\times2\times3 \times 3$

$90 = 2\times3\times3\times5$

$\Rightarrow HCF(72, 90) = 2\times3\times3 = 18$

Question 9.

The value of $\sqrt{\frac{343^4+49^8}{343^6+49^7}}$ is

A) 7 B) $\frac{1}{7}$ C) 49 D) $\frac{1}{49}$

Solution: (B)

We have, $\sqrt{\frac{343^4+49^8}{343^6+49^7}}$

$= \sqrt{\frac{(7^3)^4+(7^2)^8}{(7^3)^6+(7^2)^7}}$

$= \sqrt{\frac{7^{12}+7^{16}}{7^{18}+7^{14}}}$

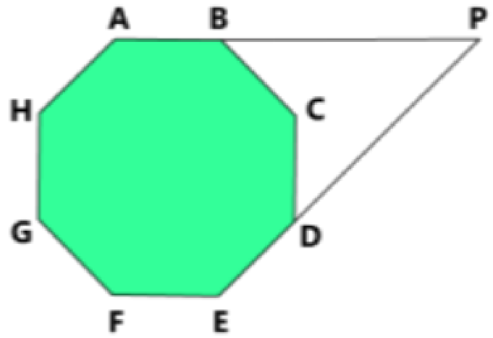
$= \sqrt{\frac{7^{12}}{7^{14}}\left(\frac{7+7^4}{7^4+7}\right)}$

$= \sqrt{\frac{7^{12}}{7^{14}}}$

$\Rightarrow \sqrt{\frac{1}{7^2}} = \frac{1}{7}$

Question 10.

In the adjoining figure, ABCDEFGH is a regular octagon. AB, ED are produced to meet at P. Then the measure of ∠BPD is equal to



A) $45°$ B) $40°$ C) $30°$ D) $35°$

Solution: (A)

We are given that ABCDEFGHA is a regular octagon.

We know that each interior angle of a regular polygon is given by $\frac{n-2}{n}\times 180°$.

Hence, Each interior angle of the given regular octagon ABCDEFGHA $= 135°$

Now, $\angle PBC = \angle PDC = 180° - 135° = 45°$ and reflex $\angle BCD = 360° - 135° = 225°$.

Finally, PBCD is a quadrilateral, hence the $\angle P + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle P + 45° + 45° + 225° = 360°$

$\Rightarrow \angle P = 360° - 315°$

$\Rightarrow \angle P = 45°$

Question 11.

If $4x^2 + \frac{1}{x^2} = 2$, then the value of $8x^3 + \frac{1}{x^3}$ is

A) 1 B) -1 C) 8 D) 0

Solution: (D)

We know that $a^3 + b^3 = (a + b)\left(a^2 + b^2 - ab\right)$

Thus, $8x^3 + \frac{1}{x^3} = \left(2x + \frac{1}{x}\right)\left(4x^2 + \frac{1}{x^2} - (2x)\left(\frac{1}{x}\right)\right)$

$= \left(2x + \frac{1}{x}\right)(2 - 2)\ \{\because 4x^2 + \frac{1}{x^2} = 2\ [Given]\}$

$= 0$

Question 12

For permissible real values of x, y, z, the value of the expression

$\frac{(2x+5y-3z)^3+(2x-5y+3z)^3+12x(2x+5y-3z)(2x-5y+3z)}{x^3}$ is

A) 16 B) 32 C) 64 D) 128

(their is correction in the question instead of 2x , there should be 12x written)

Solution: (C)

We know that,

$$a^3 + b^3 = (a + b)\left((a + b)^3 - 3ab\right)$$

Hence,

$$= \frac{(2x+5y-3z)^3+(2x-5y+3z)^3+12x(2x+5y-3z)(2x-5y+3z)}{x^3}$$

Substituting $b = 5y - 3z$ in above equation, we get

$$= \frac{(2x+b)^3+(2x-b)^3+12x(2x+b)(2x-b)}{x^3}$$

$$= \frac{8x^3+b^3+6bx(2x+b)+8x^3-b^3-6bx(2x-b)+12x(4x^2-b^2)}{x^3}$$

$$= \frac{16x^3+12bx^2+6b^2x-12bx^2+6b^2x+48x^3-12b^2x}{x^3}$$

$$= \frac{64x^3}{x^3}$$

$$= 64$$

Question 13 When $\theta \neq 0^\circ, 90^\circ$ the value of the expression

$\frac{(1+sec\theta+tan\theta)(1+cosec\theta+cot\theta)}{1+tan\theta+cot\theta+sec\theta+cosec\theta}$ is equal to

A) 1 B) 2 C) -1 D) $\frac{1}{2}$

Solution: (B)

$$\frac{\left(1+\frac{1}{cos\theta}+\frac{sin\theta}{cos\theta}\right)\left(1+\frac{1}{sin\theta}+\frac{cos\theta}{sin\theta}\right)}{1+\frac{sin\theta}{cos\theta}+\frac{cos\theta}{sin\theta}+\frac{1}{cos\theta}+\frac{1}{sin\theta}}$$

$$= \frac{\frac{(cos\theta+1+sin\theta)}{cos\theta}\times\frac{(sin\theta+1+cos\theta)}{sin\theta}}{\frac{cos\theta sin\theta+sin^2\theta+cos^2\theta+sin\theta+cos\theta}{sin\theta\cdot cos\theta}}$$

$$= \frac{(1+sin\theta+cos\theta)^2}{1+sin\theta+cos\theta+sin\theta cos\theta}$$

$= \frac{1+sin^2\theta+cos^2\theta+2sin\theta+2cos\theta+2\sin\theta\ cos\theta}{1+sin\theta+cos\theta+\sin\theta\ cos\theta}$

$= \frac{2(1+sin\theta+cos\theta+sin\theta cos\theta)}{(1+sin\theta+cos\theta+sin\theta cos\theta)}$

$= 2$

Question 14 The number of real ordered pairs (x, y) which satisfy

$4^{\frac{x}{y}+\frac{y}{x}} = 32,\ and\ log_3(x - y) = 1 - log_3(x + y)\ is$

A) 0 B) 1 C) 2 D) 3

Solution: (C)

$2^{2\left(\frac{x}{y}+\frac{y}{x}\right)} = 2^5$

$\frac{x^2+y^2}{xy} = \frac{5}{2}$ (Comparing the powers)

$x^2 + y^2 = \frac{5}{2}xy$ ...(1)

and

$log_3(x - y) = 1 - log_3(x + y)$

$log_3(x - y) + log_3(x + y) = 1$

$log_3\left(x^2 - y^2\right) = 1 = 0$

$\Rightarrow x^2 - y^2 = 3$ ... (2)

square equation (1)

$x^4 + y^4 + 2x^2y^2 = \frac{25}{4}x^2y^2$... (3)

square of eq (2)

$x^4 + y^4 - 2x^2y^2 = 9$

Subtracting Equation (4) from equation (3)

$4x^2y^2 = \frac{25}{4}x^2y^2 - 9$

$\frac{25}{4}x^2y^2 - 4x^2y^2 = 9$

$\frac{9x^2y^2}{4} = 9$

$x^2y^2 = 4$

$xy = \pm 2$

Putting above value in eq (1) & (2),

$x^2 + y^2 = \pm 5$

and $x^2 - y^2 = 3$

$x^2 + y^2 = -5$ is rejected because both x^2 and y^2 are positive.
$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

or $y^2 = 5 - x^2 = 1$
$y = \pm 1$
$(x, y) = (2, 1)(2, -1)(-2, 1)(-2, -1)$

but in $log_3(x - y)$ & $log_3(x + y)$

$x - y > 0$ & $x + y > 0$
$(x, y) = (2, 1), (2, -1)$ ($only$ 2 cases)

The number of real order pairs are 2.

Question 15

a, b are natural numbers such that $\frac{a}{b} + \frac{b}{a} = a + b$; then

A) a is odd and b is even.

B) $a,\ b$ are both even.

C) Such natural numbers a and b do not exist.

D) There is exactly one value of ‘a’ and ‘b’ which satisfy the equation.

Solution: (D)

$\frac{a}{b} + \frac{b}{a} = a + b$

$\Rightarrow\ a^2 + b^2 = ab(a + b)$

$\Rightarrow\ a^2 + b^2 - a^2b - ab^2 = 0$

$\Rightarrow\ a^2(1 - b) + b^2(1 - a) = 0$

$\Rightarrow\ a^2(b - 1) + b^2(a - 1) = 0$

$a, b \in N$

This is only possible when $a = b = 1$ otherwise $a^2(b - 1) + b^2(a - 1) > 0$.

Fill in the Blanks:

Question 16

The sum of all the roots of the equation $3^{\frac{x+2}{3x-4}} - 7 = 2\left(3^{\frac{5x-10}{3x-4}}\right)$ is____________.

Solution: 2

$$3^{\frac{x+2}{3x-4}} - 7 = 2\left(3^{\frac{5x-10}{3x-4}}\right)$$

$$3^{\frac{x+2}{3x-4}} - 7 = 2\left(3^{\frac{6x-8-x-2}{3x-4}}\right)$$

$$3^{\frac{x+2}{3x-4}} - 7 = 2\times3^{2} \times 3^{-\frac{(x+2)}{3x-4}}$$

Let us assume $3^{\frac{x+2}{3x-4}} = A$

$A - 7 = 18\times\frac{1}{A}$

$A^{2} - 7A = 18$

$A^{2} - 7A - 18 = 0$

$(A - 9)(A + 2) = 0$

$A = 9$ or $A = - 2$ (not possible)

$$3^{\frac{x+2}{3x-4}} = 9$$

$$\frac{x+2}{3x-4} = 2$$

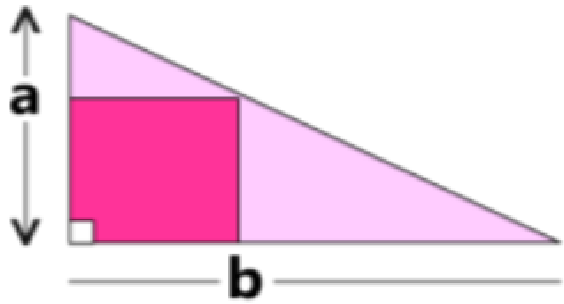
$x + 2 = 6x - 8$

$5x = 10$

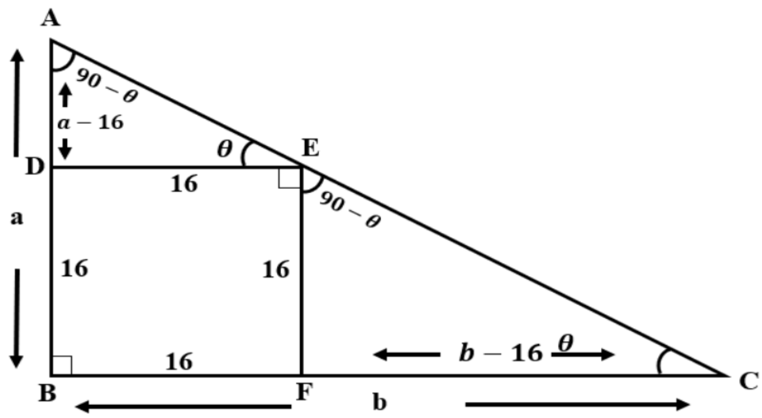
$x = 2$

Question 17

A square is inscribed in a right angled Triangle as shown in the figure. One leg of the triangle is twice the other. If the perimeter of the square is 64 cm, then the length of longer leg of the triangle (in cm) is__________.



Solution: 48 cm



Since, the perimeter of the square = 64 cm.

So, Sides of square = 16 cm.

$\triangle ADE \sim \triangle EFC$

$\frac{a-16}{16} = \frac{16}{2a-16}$ (Given that b = 2a)

$(a - 16)(a - 8) = 16\times 8$

$a^2 - 24a + 16\times 8 = 16\times 8$

$a^2 - 24a = 0$

$a(a - 24) = 0$

$a = 0\ (rejected)\ or\ 24$

$a = 24$

Since, b = 2a, the longer leg will be equal to 48 cm.

Question 18

If $cos\theta(tan\theta + 2)(2tan\theta + 1) = asec\theta + bsin\theta$, then $a + b$ is equal to______________.

Solution: 7

$cos\theta(tan\theta + 2)(2tan\theta + 1) = asec\theta + bsin\theta$

$cos\theta\left[2tan^2\theta + 5tan\theta + 2\right] = asec\theta + bsin\theta$

$2\frac{sin^2\theta}{cos\theta} + 5sin\theta + 2cos\theta = asec\theta + bsin\theta$

$\frac{2sin^2\theta+5\sin\theta\,cos\theta+2cos^2\theta}{cos\theta} = asec\theta + bsin\theta$

$\frac{2+5sin\theta\,cos\theta}{cos\theta} = asec\theta + bsin\theta$

$2sec\theta + 5sin\theta = asec\theta + bsin\theta$

$a = 2, b = 5$

$a + b = 7$

Question 19

The number of real roots of the equation $x^4 + x^3 + x^2 + 2 = 0$ is

Solution: 0

$x^4 + x^3 + x^2 + 2 = 0$

$x^4 + 2x^3 - x^3 + 2x^2 - 2x^2 + x^2 + 2x - 2x + 2 = 0$

$x^4 + 2x^3 + 2x^2 - x^3 - 2x^2 - 2x + x^2 + 2x + 2 =$

$x^2\left(x^2 + 2x + 2\right) - x\left(x^2 + 2x + 2\right) + 1\left(x^2 + 2x + 2\right) = 0$

$\left(x^2 + 2x + 2\right)\left(x^2 - x + 1\right) = 0$

For both the quadratic equation discriminant is negative hence

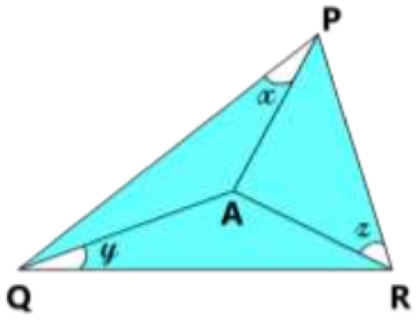
$x^2 + 2x + 2 > 0$ & $x^2 - x + 1 > 0$

$x^4 + x^3 + x^2 + 2 > 0$

Therefore, there is no real roots.

Question 20

In the adjoining figure, A is a point inside the triangle PQR, such that $AP = AQ = AR$. Given $x + 2y = 109^\circ$ and$3x - y = 54^\circ$. Then z (in degrees) is



Solution: 20º

In $\triangle PAQ$,

$AP \quad = AQ\ (given)$

$\Rightarrow \angle APQ\ =\ \angle AQP\ (angles\ opposite\ to\ equal\ side)$

$\Rightarrow \angle AQP\ =\ x$

Similarly,

$AQ \quad = AR\ (given)$

$\Rightarrow \angle AQR\ \ =\ \angle ARQ\ (angles\ opposite\ to\ equal\ side)$

$\Rightarrow \angle AQR\ \ =\ y$

Similarly,

$AR \quad = AP\ (given)$

$\Rightarrow \angle ARP\ =\ \angle APR\ (opposite\ angle\ of\ Equal\ ride)$

$\Rightarrow \angle APR\ =\ z$

$x\ +\ 2y\ =\ 109\ -\ \ (1)\ \ 3x\ -\ y\ =\ 54\ -\ \ (2)$

By multiplying 2 in eq^n (2) we get $6x\ -\ 2y\ =\ 108\ -$ (3)
solving 1 & (3) by elimination method,

$x\ +\ 2y\ =\ 109$

$6x\ -\ 2y\ =\ 108$

$7x\ =\ 217$

$x\ =\ 31$

Put this value in equation 1 we get $y\ =\ 39$

in $\triangle PQR$

$2x\ +\ 2y\ +\ 2z\ =\ 180$

$x\ +\ y\ +\ z\ = 90^{\circ}$

$z\ = 20^{\circ}$

Question 21

a, b, c are non-zero reals. Given $a\ +\ b\ +\ c\ =\ abc$ and $a^2\ =\ bc$ Then the minimum value of a^2 is

Solution: 3

$bc\ = a^2$

$\therefore a\ +\ b\ +\ c\ = abc$

$b\ +\ c\ = abc\ -\ a$

$b\ +\ c\ = a\left(a^2\right)-\ a$

$= a^3 - a$

Quadratic equation in (b, c)

$x^2 - (b + c)x + bc = 0$

$x^2 - (a^3 - a)x + a^2 = 0$

$D \geq 0$

$D = b^2 - 4ac$

$D = (a^3 - a)^2 - 4a^2 \geq 0$

$a^2(a^2 - 1)^2 - 4a^2 \geq 0$

$a^2(a^4 + 1 - 2a^2) - 4a^2 \geq 0$

$a^4a^2 - 2a^2 \cdot a^2 - 3a^2 \geq 0$

$a^2(a^4 - 2a^2 - 3) \geq 0$

$a^2(a^4 - 3a^2 + a^2 - 3) \geq 0$

$a^2[a^2(a^2 - 3) + (a^2 - 3)] \geq 0$

$a^2(a^2 - 3)(a^2 + 1) \geq 0$

$(a^2 - 3) \geq 0$ or $(a^2 + 1) \geq 0$

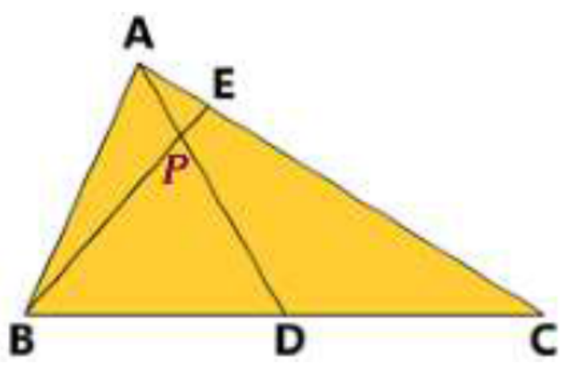
$a^2 \geq 3$ as $(a^2 + 1) = 0$ is not possible.

Question 22

In the given figure (not drawn to scale) $AC = 2AB$.

D, E are respectively points on BC and AC such that $\angle ABE = \angle CAD$.

If the triangle PBD is equilateral, and the measure of $\angle ABC$ is x°, then $x =$



Solution: 90°

$\Delta ABP \backsim \Delta CAD$ $[by\ AA\ similarity]$

$\frac{AB}{AC} = \frac{BP}{AD} = \frac{AP}{CD}$

$\frac{1}{2} = \frac{BP}{AD}$

$AD = 2BP$

So, $AP = AD - PD$
$\Rightarrow AP = BP$

$\angle ABP = \angle BAP = y$

$y + y + 120^{\circ} = 180^{\circ}$

$y = 30^{\circ}$

$\angle ABC = x^{\circ} = 30^{\circ} + 60^{\circ} = 90^{\circ}$

Question 23

The number of solutions x of the equation $(3|x| - 3)^2 = |x| + 7$ such that $\sqrt{x(x - 3)}$ exists is

Solution: 2

Let us find the domain of $\sqrt{x(x - 3)}$

$x(x - 3) \geq 0\ \ x > 3\ or\ x < 0$

Now, for the equation let us assume, $x > 0 \Rightarrow$

$|x| = x$

The equation will be

$(3x - 3)^2 = x + 7$

$9x^2 + 9 - 18x = x + 7$

$9x^2 - 19x + 2 = 0$

$9x^2 - 18x - x + 2 = 0$

$(9x - 1)(x - 2) = 0$

$x = \frac{1}{9}, x = 2$

Both the solutions will be discarded as they don't belong to the domain $x > 3$ or $x < 0$
So, let us now consider the equation for $x \leq 0$
The equation becomes $(-3x - 3)^2 =- x + 7$

$9x^2 + 9 + 18x =- x + 7$

$9x^2 + 19x + 2 = 0$

$(9x + 1)(x + 2) = 0$

$x = -\ 2, -\frac{1}{9}$

which lie in the domain $x < 0$.

Question 24

The difference between the fourth and first terms of a G.P. is 52. The sum of the first three terms is half of this difference. The n^{th} term of this G.P. just exceeds 2022. Then the value of n is

Solution: 8

$T_4 - T_1 = 52 \qquad ar^3 - a = 52 \qquad\qquad a\left(r^3 - 1\right) = 52$

$a + ar + ar^2 = \frac{1}{2}\times 52$

$a\left(1 + r + r^2\right) = 26 - (2)$

divide eq (1) by (2)

$r - 1 = 2$

$r = 3$

$Now\ a\left(3^3 - 1\right) = 52$

$a = 2$

$r^{th}\, tan\, m = T_n = ar^{n-1} > 2022$

$2(3)^{n-1} > 2022$

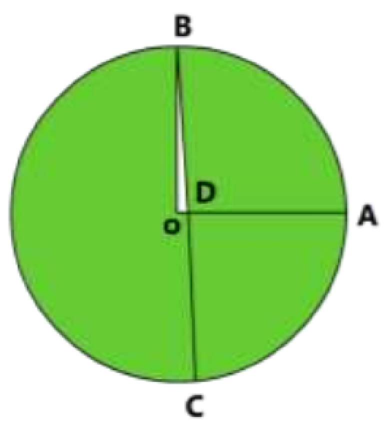
$(3)^{n-1} > 1011$

So least integer value of $n = 8$

Question 25.

In the adjoining figure, OA and OB are two perpendicular radii. With A as centre and AO as radius, an arc is drawn to cut the circle at C. BC cuts OA at D.

If $\angle ADC = x^{\circ}$, then $x =$



Solution: 75°

On joining A an C
we know that
$OA = AC$ ($As\ AC$ is cut from the centre O)
Also, we know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. Therefore,

$\angle BOA \ = 90^{\circ}\ \angle ACD \ = \frac{1}{2}\angle BOA \ = \frac{1}{2} \times 90^{\circ} \ = 45^{\circ}$

Now join O and C

In $\triangle AOC$

$AO = OC = AC$

$\therefore \angle AOC = \angle CAO = \angle ACO = 60^{\circ}$

Now, In $\triangle ADC$

$\angle ACD + \angle CAD + \angle ADC = 180^{\circ}$

$45^{\circ} + 60^{\circ} + x = 180^{\circ}$

$x + 105^{\circ} = 180^{\circ}$

$x = 180^{\circ} - 105^{\circ}$

$x = 75^{\circ}$

Question 26.

Three pipes p_1, p_2 and p_3 can fill a tank in 10 hours. After working at it together for 2 hours, p_1 is closed and p_2 and p_3 can fill it in 16 hours. The time required by p_1 to fill the tank alone is _____________hours.

Solution: 20 Hrs
Part time in 2 hours $= \frac{2}{10} = \frac{1}{5}$
Remaining part $= \left(1 - \frac{1}{5}\right) = \frac{4}{5}$
$\therefore (p_2 + p_3)$'s 16 hour's work $= \frac{4}{5}$
$(p_2 + p_3)$'s 1 hour's work $= \frac{4}{5\times16} = \frac{1}{20}$
$\therefore$ p_1's hour's work $= \{(p_2 + p_2 + p_3)'$'s 1 hour's work $\} - \{(p_2 + p_3)$'s 1 hour's work $\}$
$= \left(\frac{1}{10} - \frac{1}{20}\right) = \frac{1}{20}$
$\therefore$ p_1 alone can fill the tank in 20 hours.

Question 27.

The least number which when divided by 8, 9, 12 and 15 leaves 1 as remainder each time is __________.

Solution: 361

The least number which is completely divisible by 8, 9, 12 and 15 will be the LCM of these numbers.
⇒ The least number which is completely divisible by $8, 9, 12$ and $15 = LCM$ of $(8, 9, 12$ and $15)$
⇒ The least number which is completely divisible by $8, 9, 12$ and $15 = 360$
∴ The least number which when divided by $8, 9, 12$ and 15 , leaves the remainder 1 will be
$360 + 1 = 361$.

Question 28.

The sum of the digits of a two digit number is 15. If the digits are interchanged, the number of reverse digits is increased by 9. The original two digit number is ________.

Solution: 78

Let $x =$ units digit and $y =$ tens digit
So, the number can be expressed as $10y + x$
The number, if the digits are reversed, is $10x + y$
We have: $x + y = 15$ ....(i)

$10x + y = 10y + x + 9$

$9x - 9y = 9$

$x - y = 1$ ....(ii)

From the equation (i) and (ii)

$2x = 16$

$x = 8$

and $y = 7$
The number is 78.

Question 29.

The number of numbers divisible by 17 between 300 and 500 is _______.

Solution: 17

This question can be solved by the AP formula.
So, numbers divisible by 17 between 300 and 500 are $306, 323, 340, 357, ..., 493$
So,

$a = 306$

$d = 323 - 306 = 17$

$l = 493$

The AP formulae is $I = a + (n - 1)d$
Putting values

$493 = 306 + (n - 1)\times 17$

$493 = 306 + 17n - 17$

$17n = 204$

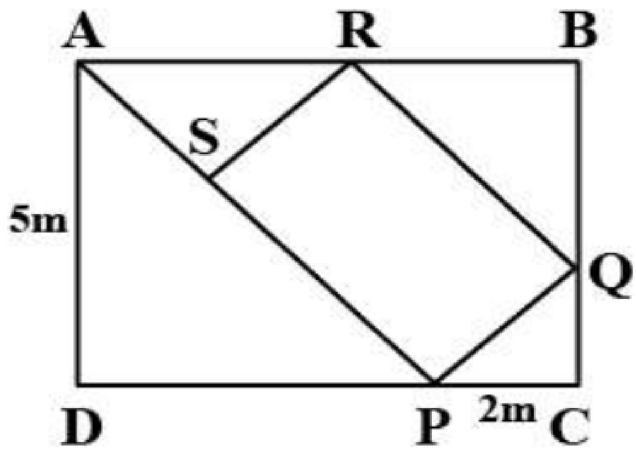
$n = 12$

Therefore, 12 numbers between 300 and 500 are divisible by 17.

Question 30.

ABCD is a non-standard billiards table.
$AD = 5\,m.$
A ball is projected from A along a line which makes 45° with AD.

It bounces at P on DC, again bounces respectively at Q and R as shown and reaches the line AP at S. The total distance covered by the ball is



Solution: $12\sqrt{2}m$

$In\ \triangle ADP$

$\angle DAP = \angle DPA = 45^{\circ}$

$\therefore AD = DP = 5\,m$

$So,\ AP = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = 5\sqrt{2}\,m$

$\because$ The ball bounces from P on DC

$\therefore \angle APD = \angle QPC = 45^{\circ}$

$\therefore$ In $\triangle PQC$

$PC = CQ = 2\,m\ PQ = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}\,m$

Similarly

In $\triangle QBR$

$BQ = BR$

$BQ = (BC - QC) = 5 - 2 = 3\ m$

$\therefore RQ = \sqrt{3^2 + 3^2} = \sqrt{9 + 9} = 3\sqrt{2}\ m$

$\because RS = PQ = 2\sqrt{2}\ m$

$\therefore$ Total distance travelled by ball $= AP + PQ + RQ + RS$

$= 5\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 2\sqrt{2}$

$= 12\sqrt{2}m$