15th October 2022	Sub Junior Level	Screening test

Instructions:

1. Fill in the Response Sheet with your Name, Class and the Institution through which you appear, in the specified places.
2. Diagrams given are only Visual aids; they are not drawn to scale.
3. You may use separate sheets to do rough work.
4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.
5. Duration of the Test: 2 pm to 4 pm (2 hours).

Question 1

The value of $\sqrt{46.\,47.\,48.\,49\,+\,1}$ when simplified is

a) 2245   b) 2255   c) 2345   d) 2195

Solution: (b)

Given, $\sqrt{46.\,47.\,48.\,49\,+\,1}$

Let $a = 47,\ (a - 1) = 46,\ (a + 1) = 48\ and\ (a + 2) = 49$

$= \sqrt{(a - 1)a(a + 1)(a + 2) + 1}$

$= \sqrt{\left(a^2 + 2a\right)\left(a^2 - 1\right) + 1}$

$= \sqrt{\left(a^2 + a - 1\right)^2}$

$= a^2 + a - 1$

Substitute the value of a in the above expression, we get

$= 47^2 + 47 - 1$

$= 47(47 + 1) - 1$

$= 47 \times 48 - 1$

$= 2255$

Hence, the simplified value of $\sqrt{46.\,47.\,48.\,49\,+\,1}$ is 2255.

Question 2

Two regular polygons of same number of sides have side lengths $8\ cm$ and $15\ cm$. The length of the side of another regular polygon of the same number of sides whose area is equal to the sum of the areas of the given polygons is (in cm.)

a) 17   b) 23   c) 38   d) 120

Solution: (a)

Let the length of the side of the 3rd polygon be x cm.

Let n denote the no. of sides of all polygons

Area of any polygon is given by $= \frac{a \times p}{n}$

Where $a = \frac{s}{2 \tan\left(\frac{180°}{n}\right)}$; $s = $ side length and $p = s \times n$

Given, sum of areas of regular polygon with 8 cm and 15 cm = Area of 3rd polygon

$$\Rightarrow \frac{8n \times \frac{8}{2 \tan\left(\frac{180°}{n}\right)}}{2} + \frac{15n \times \frac{15}{2 \tan\left(\frac{180°}{n}\right)}}{2} = \frac{xn \times \frac{x}{2 \tan\left(\frac{180°}{n}\right)}}{2}$$

$$\Rightarrow \frac{64n}{2 \tan\left(\frac{180°}{n}\right)} + \frac{225n}{2 \tan\left(\frac{180°}{n}\right)} = \frac{x^2 n}{2 \tan\left(\frac{180°}{n}\right)}$$

$$\Rightarrow 289n = x^2 n$$

$$\Rightarrow x^2 = 289$$

$$\Rightarrow x = 17$$

Question 3

When $a = 2022, b = 2023$, the numerical value of $\left(\frac{a}{1+\frac{a}{b}} - \frac{b}{1-\frac{b}{a}} - \frac{2}{\frac{1}{a}-\frac{a}{b^2}} \right)$ is

a) 1   b) 2022×2023   c) $(2023)^2$ d) 0

Solution: (d)

Given, $a = 2022, b = 2023$

Substitute the values of a and b in $\left(\frac{a}{1+\frac{a}{b}} - \frac{b}{1-\frac{b}{a}} - \frac{2}{\frac{1}{a}-\frac{a}{b^2}} \right)$

$$= \left(\frac{2022}{1+\frac{2022}{2023}} - \frac{2023}{1-\frac{2023}{2022}} - \frac{2}{\frac{1}{2022}-\frac{2022}{(2023)^2}} \right)$$

$$= \left(\frac{2022 \times 2023}{2023+2022} - \frac{2023 \times 2022}{2022-2023} - \frac{2 \times 2022 \times (2023)^2}{(2023)^2-(2022)^2} \right)$$

$$= \left(\frac{2022 \times 2023}{4045} - \frac{2023 \times 2022}{-1} - \frac{2 \times 2022 \times (2023)^2}{(2023+2022)(2023-2022)} \right)$$

$$= \left(\frac{2022 \times 2023}{4045} + (2023 \times 2022) - \frac{2 \times 2022 \times (2023)^2}{4045} \right)$$

$$= \frac{2022 \times 2023}{4045}(1 - 2 \times 2023) + 2023 \times 2022$$

$$= \frac{2022 \times 2023}{4045}(-4045) + (2023 \times 2022)$$

$$= -2022 \times 2023 + (2023 \times 2022)$$

$$= 0$$

Hence, the numerical value of $\left(\frac{a}{1+\frac{a}{b}} - \frac{b}{1-\frac{b}{a}} - \frac{2}{\frac{1}{a}-\frac{a}{b^2}} \right)$ is 0.

Question 4

Two sides of a triangle are of lengths $5\ cm$ and $10\ cm$. The length of the altitude to the third side is equal to the average of the other two altitudes. The length of the third side (in cm) is

a) 12
b) 8
c) $\frac{20}{3}$
d) 9

Solution: (c)

Let the height to the side of length 5 cm be h_1, the height to the side of length 10 be h_2, the area be A, and the height to the unknown side be h_3.

Because the area of a triangle is $\frac{bh}{2}$, we get that $5(h_1) = 2A$ and $10(h_2) = 2A$, so, setting them equal,

$$h_2 = \frac{h_1}{2}.$$

From the problem, we know that $2h_3 = h_1 + h_2$.

On Substituting values, we get

Thus, the third side length is going to be $\frac{2A}{0.75h_1} = \frac{5}{0.75} = \frac{20}{3}$

Hence, the length of the third side (in cm) is $\frac{20}{3}$.

Question 5

a, b, c, d, e, f are natural numbers in some order among $4, 5, 6, 12, 20, 24$. The maximum value of $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$ is

a) 1
b) $5\frac{1}{2}$
c) $10\frac{1}{2}$
d) 12

Solution: (d)

Let $a = 12, b = 6, c = 20, d = 5, e = 24$ and $f = 4$

Then $\frac{a}{b} + \frac{c}{d} + \frac{e}{f} = \frac{12}{6} + \frac{20}{5} + \frac{24}{4} = 2 + 4 + 6 = 12$

Hence, the maximum value of $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$ is 12.

Question 6

Two consecutive natural numbers exist such that the square of their sum exceeds the sum of their squares by 112 ; then the difference of their squares is

a) 10   b) 12   c) 13   d) 15

Solution: (d)

Let the two natural numbers be n and $n + 1$

Square of the sum of those numbers $= (n + n + 1)^2 = (2n + 1)^2$

Sum of their squares $= n^2 + (n + 1)^2$

Therefore from the given data we get,

$(2n + 1)^2 = n^2 + (n + 1)^2 + 112$

$4n^2 + 4n + 1 = n^2 + n^2 + 2n + 1 + 112$

Rearrange the expression into a quadratic equation

$2n^2 + 2n - 112 = 0$

$2n^2 + 16n - 14n - 112 = 0$

$2n(n + 8) - 14(n + 8) = 0$

$(2n - 14)(n + 8) = 0$

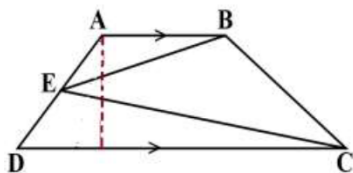From this we can say n is 7 or $- 8$, but since n is a natural number, it cannot be $- 8$.

Hence, n is 7, and $n + 1 = 8$.

The numbers are 7 and 8. Now, squares of 7 and 8 are $7^2 = 49 \ and \ 8^2 = 64$

Difference of their squares $= 64 - 49 = 15$.

Question 7

ABCD is a trapezoid with $AB \parallel CD$. Given $AB = 11 \ cm$ and $DC = 21 \ cm$ and the height of the trapezoid is $4 \ cm$. If E is the midpoint of AD, the area of triangle BEC (in cm^2) is

a) 32   b) 34   c) 28   d) 40

Solution: (a)

Area of trapezium $= \frac{1}{2} \times h \times (a + b) = \frac{1}{2} \times 4 \times (32) = 64\ cm^2$

We know that the area of a triangle formed by joining the midpoint of the non-parallel sides of a trapezium to the ends of the opposite sides is half of the area of a trapezium.

$\therefore Ar(\triangle BEC)\ = \frac{1}{2} \times Ar(Trap\ ABCD)\ = \frac{1}{2} \times 64\ = 32\ cm^2$

Hence, the area of triangle BEC (in cm^2) is 32.

Question 8

One-sixth of one-fourth of three-fourths of a number is 15, the number is

a) 1020   b) 320   c) 520   d) 480

Solution: (d)
Let the number be x.
According to the question,

$$\frac{1}{6} \times \frac{1}{4} \times \frac{3}{4} \times x = 15$$

$$\Rightarrow \frac{3x}{96} = 15$$

$$\Rightarrow 3x = 15 \times 96$$

$$\Rightarrow x = \frac{1440}{3}$$

$$\Rightarrow x = 480$$

Question 9

Two places A and B are connected by a straight road. Samrud and Saket start by motorbikes respectively from A and B at the same time; after meeting each other, they complete their journey in 90 minutes and 40 minutes respectively. If the speed of Samrud's bike is 16 km/hr., then the speed of Saket's bike (in km/hr.) is …

a) 20   b) 18   c) 24   d) 22

Solution: (c)

Let Samrud and Sanket meet at point M and take t hrs.
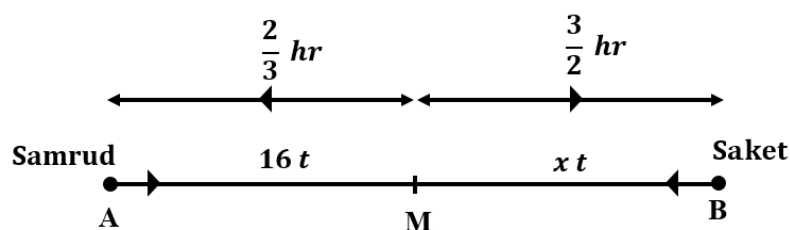
Given, Time taken by Samrud from M to B = 90 mins = $\frac{3}{2}$ hr

Time taken by Saket from M to A = 40 mins = $\frac{2}{3}$ hr

Let speed of Sanket is x km/hr

As we know, Speed = $\dfrac{Distance}{time}$

Distance travelled by Samrud from A to M = 16 km/hr $\times t$ …(i)

Distance travelled by Sanket from B to M = x km/hr $\times t$ …(ii)

Distance travelled by Samrud from M to B = 16 km/hr $\times \frac{3}{2}$ hr = 24 km …(iii)

Distance travelled by Sanket from M to A = x km/hr $\times \frac{2}{3}$ hr …(iv)

Now, Distance travelled by Sanket from B to M = Distance travelled by Samrud from M to B

$\Rightarrow xt = 24$ 　　　　　(From (ii) and (iii))

$\Rightarrow t = \dfrac{24}{x}$ hr … (v)

Now, Distance travelled by Samrud from A to M = Distance travelled by Sanket from M to A

$\Rightarrow 16t = \frac{2}{3}x$

$\Rightarrow t = \dfrac{2}{48}x = \dfrac{1}{24}x$ …(vi)

From (v) and (vi),

$\Rightarrow \dfrac{24}{x} = \dfrac{x}{24}$

$\Rightarrow x^2 = 24^2$

$\Rightarrow x = 24$ km/hr

Question 10

The length of a rectangle is increased by 60%. By what percent should the breadth be decreased to have the same area?

a) 35.5　　　　b) 37.5　　　　c) 38.25　　　　d) 36.5

Solution: (b)

Let length of rectangle =100 m
And the breadth of rectangle =100 m

As we know, area of the rectangle is (Length $\times$ Breadth)

Therefore, the original area = $100\times 100 = 10000^2$

Given that, the length of the rectangle is increased by 60%.
First we find 60% of the length then add it to the original length to find out the new length
i.e. $100 + 60\%$ of $100 = 100 + \frac{60}{100}\times 100 = 160 m$

And we assume that the length is decreasing at the x%

First we find x% of the breadth then subtract it to the original breadth to find out the new breadth i.e. $100 + x\%$ of $100 = 100 - \frac{x}{100} \times 100 = (100 - x)m$

Therefore, the new area $= 160 \times (100 - x)$

According to the question the area should be same i.e.

$\Rightarrow 160 \times (100 - x) = 10000$
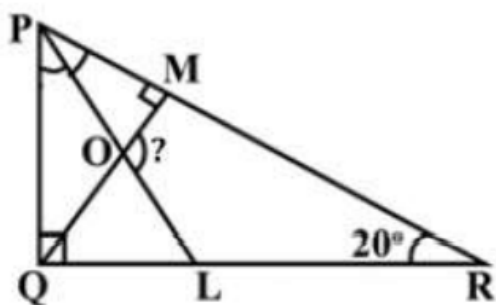
$\Rightarrow (100 - x) = \frac{10000}{160}$

$\Rightarrow x = 100 - \frac{125}{2}$

$\therefore x = 37.5\%$
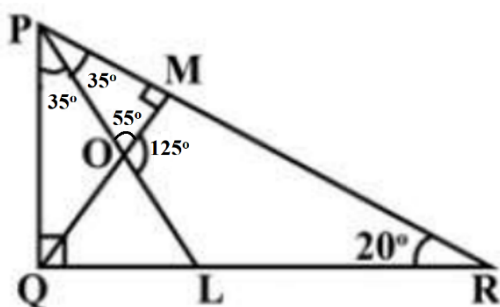
Therefore, the percent decrease in breadth is 37.5%.

Question 11

In the adjoining figure, PL is the bisector of ∠QPR. The measure of the angle MOL is …

a) 115°   b) 120°   c) 125°   d) 135°

Solution: (c)

Given, PL is angle bisector of $\angle QPR$.

$\angle PQR = 90^{\circ}$

from angle sum property of triangle, In $\triangle PQR$,

$\angle PQR + \angle PRQ + \angle QPR = 180^{\circ}$

$$90° + 20° + \angle QPR = 180°$$

$$\angle QPR = 180° - 110°$$

$$\angle QPR = 70°$$

Now, $\angle QPL = \angle RPL = \frac{\angle QPR}{2} = \frac{70°}{2} = 35°$ (PL is angle Bisector of ∠QPR)

In $\triangle POM,$

$$\angle MPO + \angle POM + \angle OMP = 180°$$

$$35° + \angle POM + 90° = 180°$$

$$\angle POM = 55°$$

Now, $\angle POM + \angle MOL = 180°$ (linear pair)

$$55° + \angle MOL = 180°$$

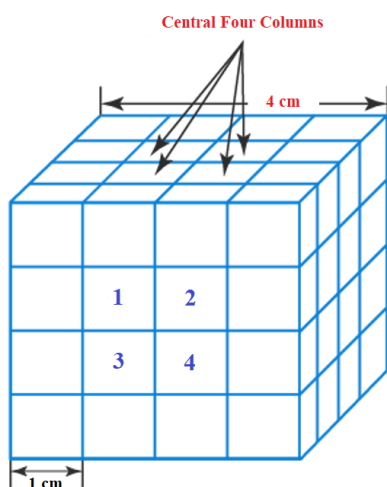$$\angle MOL = 180° - 55°$$

$$\angle MOL = 125°$$

Question 12

A four centimetre cube is painted blue on all its faces. It is then cut into Identical one centimetre cubes. Among them, the number of cubes with only one face painted is …

a) 12   b) 16   c) 18   d) 24

Solution: (d)

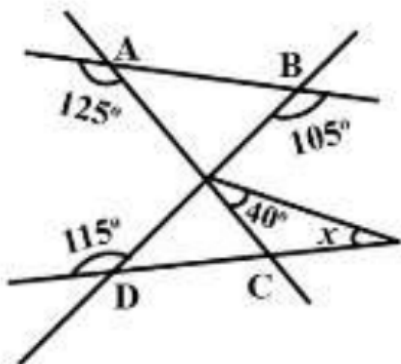A cube of 4 cm is shown below which is broken into sixty-four 1 cm cubes.

From the above diagram it is evident that the four cubes in the centre of a face of 4 cm cube do not have any of the faces painted.

The number of cubes with only one face painted
= No. of faces × Cubes painted only one face of 4 cm cube face

= 6 × 4 = 24

Question 13
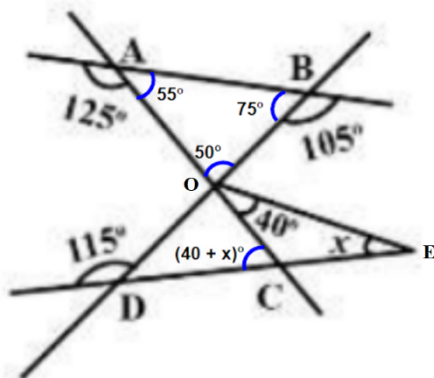
In the adjoining figure, the value of x (in degrees) is

a) 20°   b) 25°   c) 30°   d) 35°

Solution: (b)

$\angle OAB = 180^{\circ} - 125^{\circ} = 55^{\circ}$

in $\triangle OAB$

$\angle AOB + \angle OAB = 105^{\circ}$

$\angle AOB + 55^{\circ} = 105^{\circ}$

$\angle AOB = 50^{\circ}$

Now

$\angle AOB = \angle COD = 50^{\circ}$ [Vertically opposite]
So In $\triangle COD$

$\angle COD + \angle OCD = 115^{\circ}$

$50^{\circ} + \angle OCD = 115^{\circ}$

$\angle OCD = 65^{\circ}$

Now in $\triangle OCE$
$\angle COE + \angle CEO = \angle OCD$ [External Angle]

$40^{\circ} + x = 65^{\circ}$

$x = 25^{\circ}$

Question 14

Given here is a magic square. The numerical value of $a^2 + b^2 + c^2 + d^2 + e^2$ is

a	14	b	0
c	5	6	11
4	d	10	7
15	2	e	12

a) 324   b) 144   c) 274   d) 316

Solution: (a)

In the magic square, the sum of rows and columns are the same.
$\Rightarrow 0 + 11 + 7 + 12 = 30$

$\Rightarrow 15 + 2 + e + 12 = 30$

$\Rightarrow e = 14 + d + 10 + 7 = 30$

$\Rightarrow 21 + d = 30$

$\Rightarrow d = 9c + 5 + 6 + 11 = 30$

$\Rightarrow c + 22 = 30 \Rightarrow c = 8$

Now,

$a + b + 14 = 30 \Rightarrow a + b = 16$

Now, $a + c + 4 + 15 = 30$
$\Rightarrow a + c + 19 = 30 \Rightarrow a + c = 11$

Since, $c = 8$, So,
$a + 8 = 11a = 3$

Now, $a + b = 16$
$3 + b = 16b = 13$

Now, $a^2 + b^2 + c^2 + c^2 + e^2$

$= (3)^2 + (13)^2 + (8)^2 + (9)^2 + (1)^2$

$= 9 + 169 + 64 + 81 + 1$

$= 324$

Question 15

x % of 400 added to y % of 200 gives 100. If y % of 800 is 80, what percent of x is y ?

a) 60   b) 40   c) 50   d) 20

Solution: (c)

$\frac{x}{100} \times 400 + \frac{y}{100} \times 200 = 100$

$4x + 2y = 100 \ldots\ldots\ldots(1)$

if $\frac{y}{100} \times 800 = 80$

$y = 10$

From eq. (1)

$4x + 20 = 100$

$4x = 80$

$x = 20$
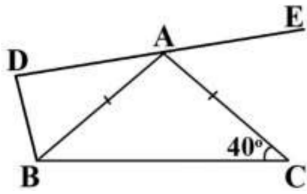
According to question

$\frac{?}{100} \times x = y$

$\frac{?}{100} \times 20 = 10$

$? = 50$

FILL IN THE BLANKS:

Question 16

In the adjoining figure, $AB = AC$ and $C = 40°$.

If $\angle ABD = (3x - 3)^\circ, \angle BDA = (2x + 8)^\circ$ and $\angle CAE = (x - 11)^\circ$ then $x =$_____

Solution: (21°)

$AB = AC$

So, $\angle B = \angle C = 40^\circ$

$In \triangle ABC,$

$\angle B + \angle C + \angle BAC = 180^\circ$

$40^\circ + 40^\circ + \angle BAC = 180^\circ$

$\angle BAC = 100^\circ$

In line DE, $\angle DAB + \angle BAC + \angle CAE = 180^\circ$,

$\angle DAB + 100^\circ + x - 11^\circ = 180^\circ$

$180^\circ - 5x - 5 + 100^\circ + x - 11^\circ = 180$

$- 4x + 84^\circ = 0$

$4x = 84$

$x = 21^\circ$

Question 17

If $a = 2022, b =- 2, c = 4044$ then the numerical value of $\dfrac{a(b^2-c^2)}{bc} + \dfrac{2b(c^2-a^2)}{ca} - \dfrac{c(2b^2-a^2)}{ab}$ is____

Solution: (1)

Given, $a = 2022,\ b =- 2,\ c = 4044$

$\dfrac{a(b^2-c^2)}{bc} + \dfrac{2b(c^2-a^2)}{ca} - \dfrac{c(2b^2-a^2)}{ab}$

$= \dfrac{a^2b^2-a^2c^2+2b^2c^2-2a^2b^2-2b^2c^2+a^2c^c}{abc}$

$= \dfrac{-a^2b^2}{abc}$

$=- \dfrac{ab}{c}$

$$=-\frac{(2022)(-2)}{4044}=1$$

Hence, the numerical value of $\frac{a(b^2-c^2)}{bc}+\frac{2b(c^2-a^2)}{ca}-\frac{c(2b^2-a^2)}{ab}$ is 1.

Question 18

If $a=\sqrt[3]{2}-\frac{1}{\sqrt[3]{2}}$, then the numerical value of $2a^3+6a$ is_____

Solution: (3)

$$a=(2)^{\frac{1}{3}}-\frac{1}{(2)^{\frac{1}{3}}}$$

By Cubing on both sides, we get, $a^3=\left[(2)^{\frac{1}{3}}-\left(2^{-\frac{1}{3}}\right)\right]^3$

Use $(a-b)^3=a^3-b^3-3ab(a-b)$

$$a^3=\left(2^{\frac{1}{3}}\right)^3-\left(2^{-\frac{1}{3}}\right)^3-3\left(2^{\frac{1}{3}}\right)\left(2^{-\frac{1}{3}}\right)[2^{\frac{1}{3}}-2^{-\frac{1}{3}})$$

$a^3=2-\frac{1}{2}-3\times a$ $\qquad$ $[a=(2)^{\frac{1}{3}}-\frac{1}{(2)^{\frac{1}{3}}}\ Given]$

$a^3=2-\frac{1}{2}-3a$

$a^3+3a=2-\frac{1}{2}$
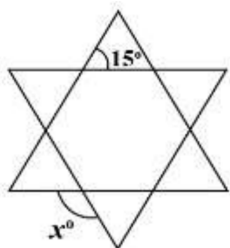
$a^3+3a=\frac{3}{2}$

$2a^3+6a=3$

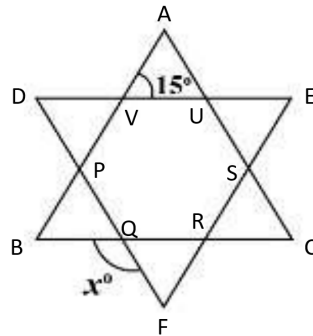Hence, the numerical value of $2a^3+6a$ is 3.

Question 19

In the adjoining figure, two equilateral triangles cut each other. The measure of the angle x° is_____degrees.

Solution: (165°)

Given, $\angle AVU = 15°$

$\angle AVU = DVP = 15°$ (*Vertically opposite angles*)

And also given ΔABC is equilateral triangle $\therefore \angle D = 60°$

Now, $\angle DPV = 180° - (60° + 15°)$

$\angle DPV = 105°$

$\therefore \angle BPQ = \angle DPV = 105^\circ$ (*Vertically opposite angles*)

And $\angle B = 60^\circ$ (given ΔABC is isosceles triangle)
$\angle x$ is a exterior angle of ΔPBQ

$\therefore \angle x = \angle BPQ + \angle PBQ \Rightarrow 105^\circ + 60^\circ \Rightarrow 165^\circ , \angle x \Rightarrow 165^\circ$

Hence, the measure of the angle x° is 165 degrees

Question 20

A vendor has four regular customers. He sells to the first customer half his stock of cakes and half a cake. He sells to the second customer half of the remaining stock and half a cake. He repeats this procedure for the third and the fourth customer also. Now, finally he is left with 15 cakes. The number of cakes he had in the beginning is

Solution: (289)

Let the stocks of the cake is x

$\frac{A}{\theta}$, to the first customer $\Rightarrow \frac{x}{2} + \frac{1}{2} = \frac{x+1}{2}$

to the second customer $\Rightarrow \frac{-1}{4} + \frac{1}{2} = \frac{x+1}{4}$

to the third customer $\Rightarrow \frac{x-3}{8} + \frac{1}{2} \Rightarrow \frac{x+1}{8}$

to the fourth customer $\Rightarrow \frac{x-7}{16} + \frac{1}{2} \Rightarrow \frac{x+1}{16}$

$x - \frac{x+1}{2} - \frac{x+1}{4} - \frac{x+1}{8} - \frac{x+1}{16} = 15$

$$\frac{16x - 8(x+1) - 4(x+1) - 2(x+1) - (x+1)}{16} = 15$$

$$\frac{16x - 8x - 8 - 4x - 4 - 2x - 2 - x - 1}{16} = 15$$

$$\frac{16x - 15x - 15}{16} = 15$$

$$x - 15 = 15 \times 16$$

$$x - 15 = 240$$

$$x = 240 + 15 = 255$$

Hence, the number of cakes he had in the beginning is 289.

Question 21

In the sequence 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, ..., the 2022nd term is_____
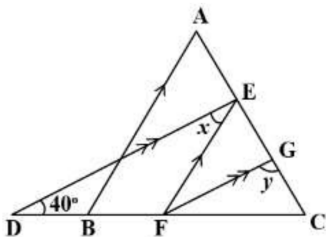
Solution: (1011)

Before every natural number '1' is added in this sequence.

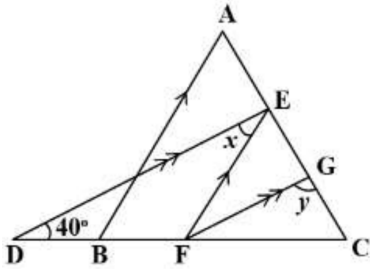So, in the 10th term we are getting '5'

Similarly, For the 2022th term we will be getting **1011.**

Question 22

In the adjoining figure, ABC is an equilateral triangle. AB and EF are parallel. DE and FG are parallel. $\angle BDE = 40°$. Then $x + y$ (in degrees) is_____

Solution: (100°)

Given, $\triangle ABC$ is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 180°$

Let, $\angle DEF = \angle EFG = x$ (alternate interior angle)

$\angle EDF = \angle GFC = 40°$ (Corresponding angle)

In, $\triangle CFG$,

$y + 40° + 60° = 180°$

$y = 80°$

According to the exterior angle property,

$y = \angle EFG + \angle GEF$

$y = x + \angle GEF$

$\angle GEF = y - x$

$60° = 80° - x$

$x = 20°$

Then $x + y = 20° + 80° = 100°$

Question 23

A gardener has to plant a number of rose plants in straight rows. First he tried 5 in each row; then he successively tried 6, 8, 9 and 12 in each row but always had 1 plant left Then he tried 13 in a row and to his pleasant surprise, no plant was left out. The smallest number of plants he could have had is

Solution: (3601)

Number of plant $= LCM(5, 6, 8, 9, 12) \times K + 1$
$N = 360K + 1$

Also it is divisible by 13

$k = 1 \to N = 361$ *It is not divisible by* 13

$k = 2 \quad \to N = 721$ *It is not divisible by* 13

$k = 3 \to N = 1081$ *It is not divisible by* 13

$k = 10 \qquad N = 3601 \Rightarrow$ *It is divisible by* 13

The smallest number of plants he could have had is 3601.

Question 24

A, B run a race $1\ km$ long straight path. If A gives B $40\ m$ start then, A wins by 19 seconds. If A gives B 30 seconds start, then B wins by $40\ m$. If B normally would take t_1 seconds to run the total $1\ km$ length and A normally would take t_2 seconds to run the total $1\ km$ length, then $t_1 - t_2$ (in seconds) is

Solution: (25)

According to the given situation,

Let a and b be the speeds of the A and B respectively,

$19 = (1000 - 40)/(b - t_2)$

$19 = \dfrac{960}{1000}t_1 - t_2$

$or\ t_2 = \dfrac{960}{1000}t_1 - 19 \text{.......} (i)$

$and,\ 30 = t_1 - (1000 - 40)/a$

$30 = t_1 - \dfrac{960}{1000}t_2\ or\ t_1 = 30 + \dfrac{960}{1000}t_2 \text{.........} (ii)$

Subtract (i) from (ii),

$t_1 - t_2 = -\dfrac{96}{100}t_1 + \dfrac{96}{100}t_2 + 19 + 30$

$t_1 - t_2 = \dfrac{96}{100}\left[t_2 - t_1\right] + 49$

$t_1 - t_2 - \dfrac{96}{100}\left[t_2 - t_1\right] = 49$

$t_1 - t_2 + \dfrac{96}{100}\left(t_1 - t_2\right) = 49$

$\left[t_1 - t_2\right]\left[1 + \dfrac{96}{100}\right] = 49$

$t_1 - t_2 = 25$

Question 25

David computed the value of 3^{19} as $11a2261467$. He found all the digits correctly except 'a'. The value of 'a' is

Solution: (6)

3^{19} as $11a2261467$

$1 + 1 + a + 2 + 2 + 6 + 1 + 4 + 6 + 7 = 30 + a$

Value of $a = 6$

Question 26

The sum of eight consecutive natural numbers is 124 . The sum of the next 5 natural numbers will be

Solution: (110)
Sum of eight consecutive natural numbers $= 124$

$x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 + x + 6 + x + 7 = 124$

$8x + 28 = 124$

$8x = 124 - 28$

$8x = 96$

$x = \dfrac{96}{8}$

$x = 12$

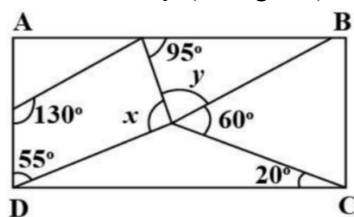Sum of next five natural numbers $= x + 8 + x + 9 + x + 10 + x + 11 + x + 12$
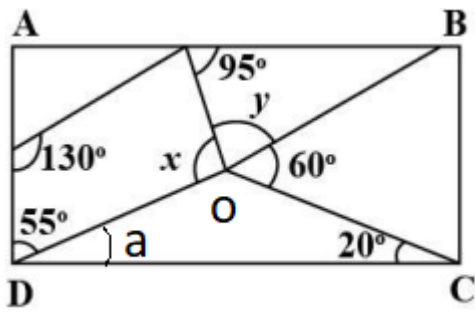
$= 12 + 8 + 12 + 9 + 12 + 10 + 12 + 11 + 12 + 12$

$= 20 + 21 + 22 + 23 + 24$

$= 110$

Question 27

In the adjoining figure, $ABCD$ is a rectangle. The value of $x + y$ (in degrees) is

Solution: (175°)

ABCD is a rectangle.

$\angle D = 90^{\circ}$

$55^{\circ} + a = 90^{\circ}$

$a = 90^{\circ} - 55^{\circ}$

$a = 35^{\circ}$

In triangle DOC,.

$\angle ODC + \angle DOC + \angle OCD = 180^{\circ}$

$35^{\circ} + \angle O + 20 = 180$

$\angle O = 180 - 55 = 125^{\circ}$

We know the angle of a circle is 360° So,

$x + y + 60^{\circ} + 125^{\circ} = 360^{\circ}$

$x + y = 360^{\circ} - 185^{\circ}$

$x + y = 175^{\circ}$

Question 28

If $A = (625)^{-3/4}$ and $B = (78125)^{3/7}$, then the value of $A \times B$ is

Solution: (1)

$A = (625)^{\frac{-3}{4}}, B = (78125)^{\frac{3}{7}}$

$A = \left(5^4\right)^{\frac{-3}{4}} = (5)^{-3} = \frac{1}{125}$

$B = \left(5^7\right)^{\frac{3}{7}} = 125$

$$A \times B = \frac{1}{125} \times 125 = 1$$

Question 29

A room is $5\ m\ 44\ cm$ long and $3\ m\ 74\ cm$ broad. The side of the largest square-slabs which can be paved of this room (in $cm.$) is

Solution: $34\ cm$

The side of the square slab is the HCF of $544\ \&\ 374\ cm$ is 34

$544 = 2\times2\times2\times2\times2\times17$

$374 = 2\times11\times17$

In both Common factor is 2×17

The side of the largest square-slabs $= 34\ cm$

Question 30

A company sells umbrellas in two different sizes, large and small. This year it sold 200 umbrellas, of which one-fourth were large. The sale of large umbrellas produced one-third of the company's income. If $a:b$ is the ratio of the price of a larger umbrella to the price of a smaller umbrella, then ab^2 is

Solution: (12)

Total umbrellas = 200

Large Umbrellas = 50

Small Umbrellas = 150

Let x = total income

Income for large and small umbrellas = $\dfrac{x}{3} : \dfrac{2x}{3}$

for 1 large umbrella and 1 small umbrella $= \dfrac{x}{3\times50} : \dfrac{2x}{3\times150}$

$= \dfrac{x}{3} : \dfrac{2x}{3\times3}$

$= 1 : \dfrac{2}{3}$

$a:b = 3:2$

$ab^2 = 3\times2^2 = 12$