CRACK JEE 2024

MOTION IN TWO DIMENSION - L1

INTRODUCTION

GRADE 11 | PHYSICS

ANTHE

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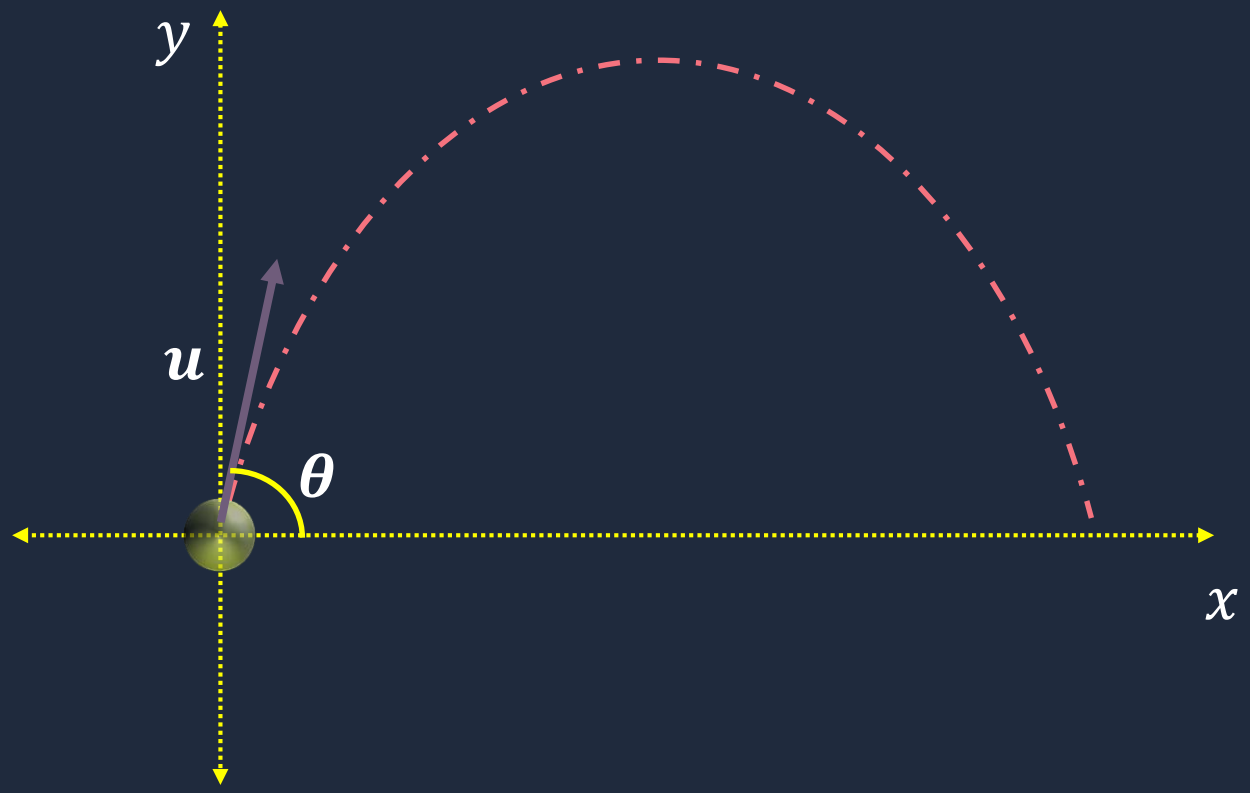
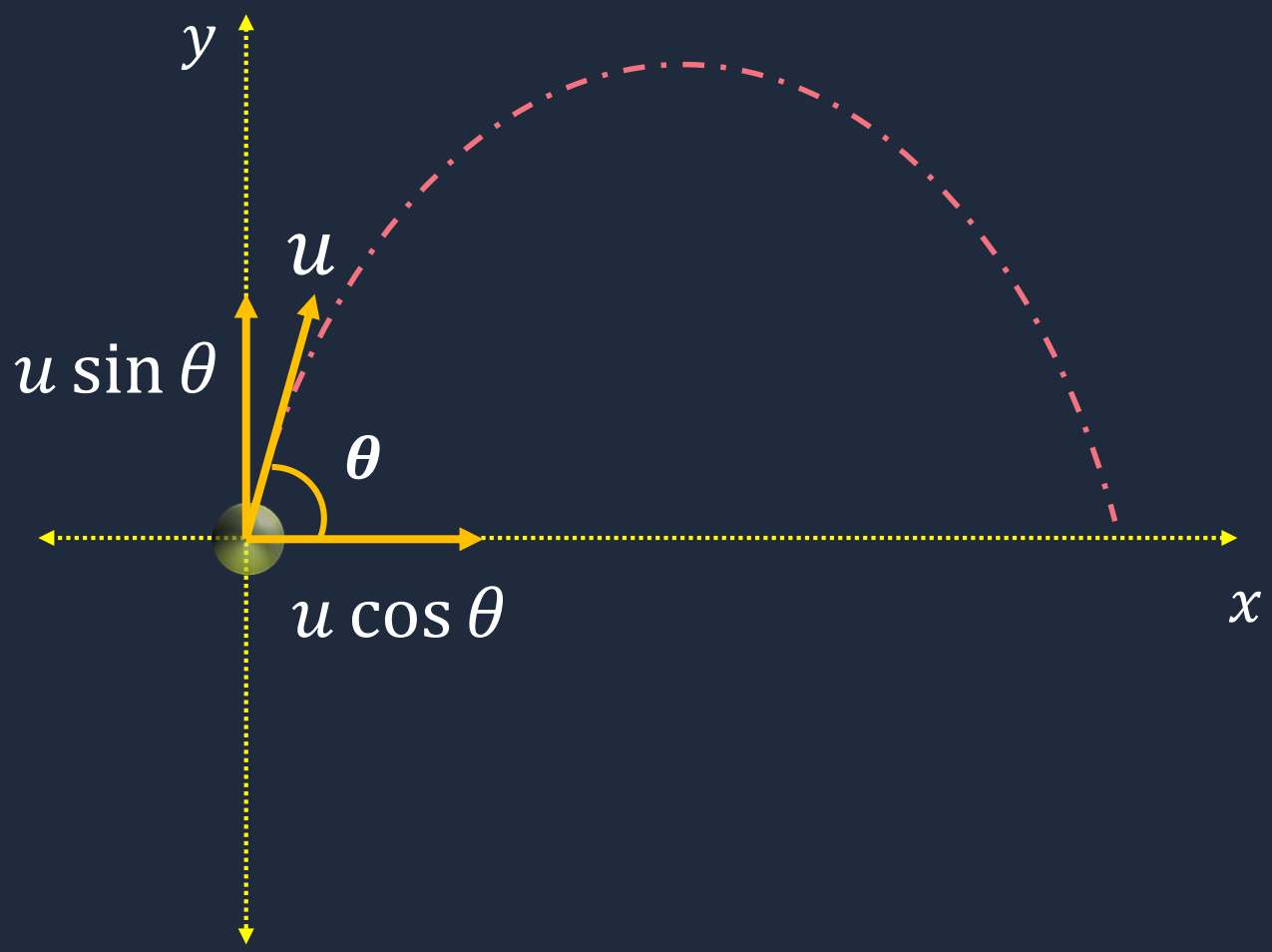
Projectile

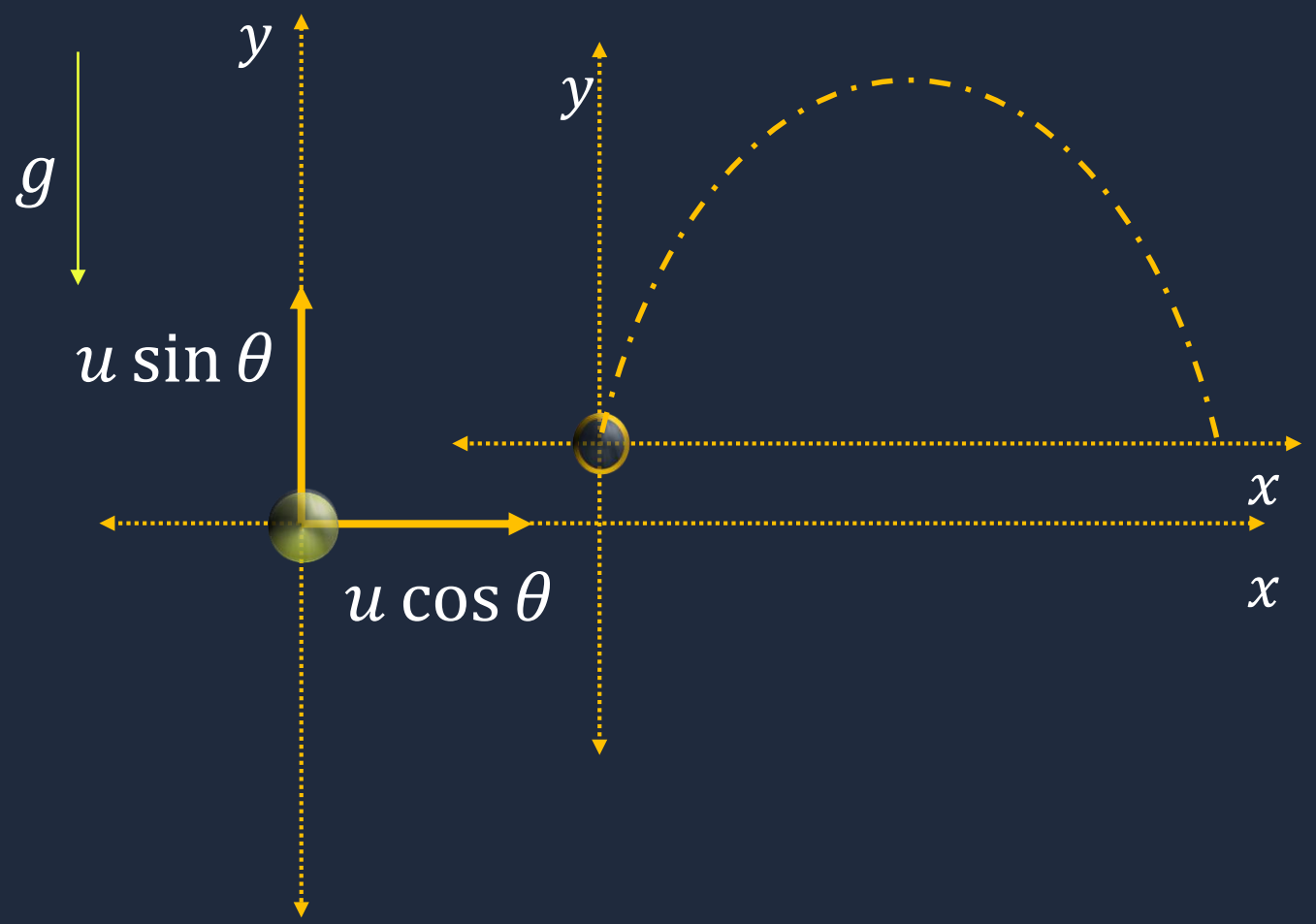


Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration due to gravity. The object is called projectile and its motion is called the trajectory.

Projectile Motion



Projectile Motion



Terminologies

➢ Velocity of Projection

The initial velocity with which a body is projected is called velocity of projection.

➢ Launch Angle

The angle of initial velocity with the horizontal with which the body is projected is called the launch angle.

➢ Horizontal Range

The horizontal distance travelled by the body during the entire motion is called the horizontal range.

Terminologies

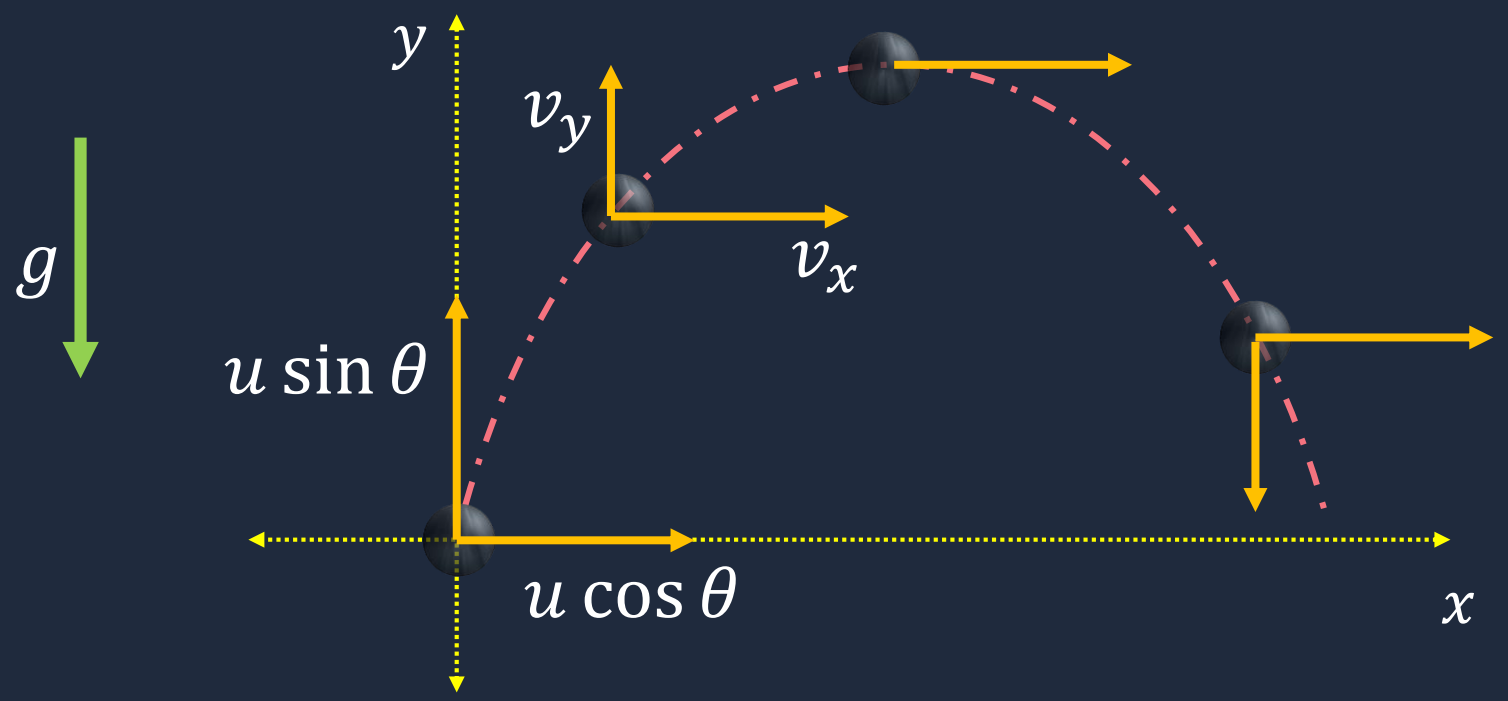
➢ Time of flight

The amount of time spent by a projectile in air is called time of flight.

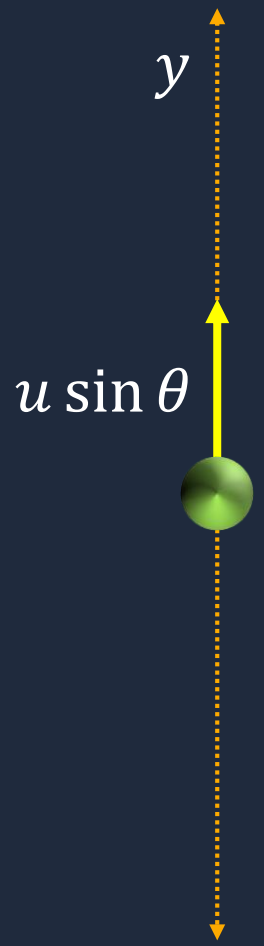
➢ Maximum Height (h_{max} or H)

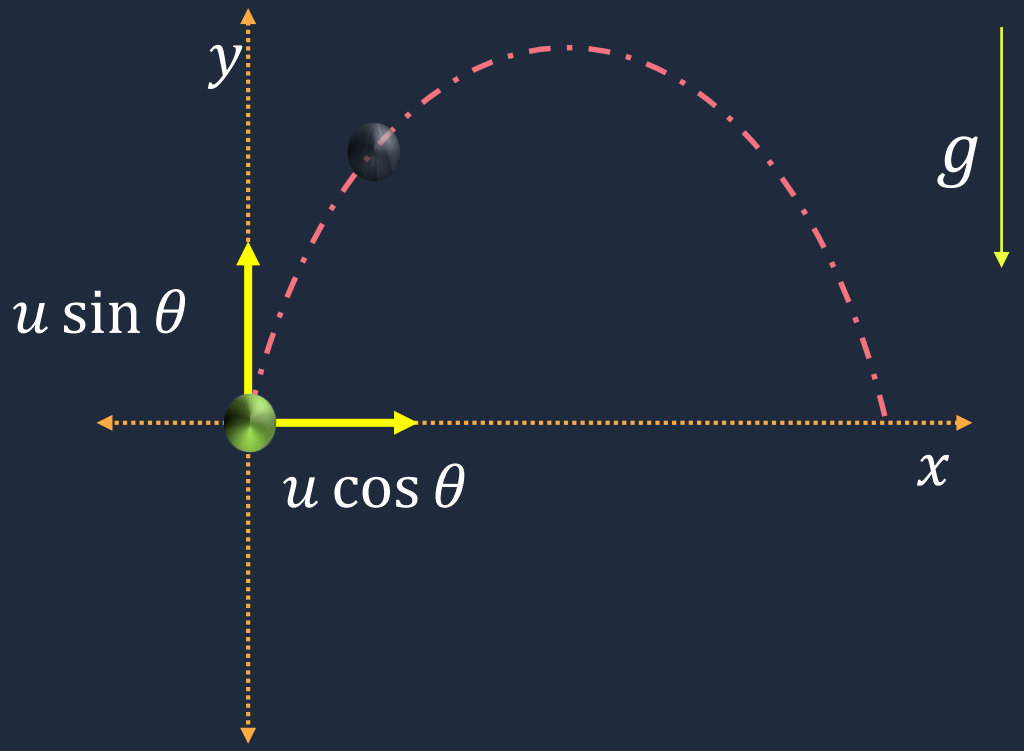
The height at which the vertical component of the velocity becomes zero is called maximum height.

Projectile Motion



Projectile Motion

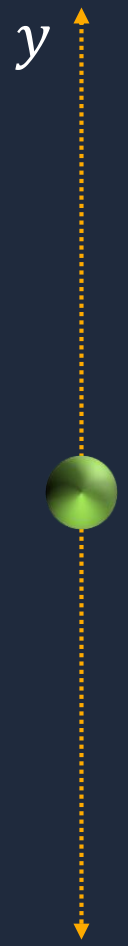




y-axis

Projectile Motion

y-axis



Time of Flight

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = u\sin\theta\, T - \frac{1}{2} g T^2$$

$$\boldsymbol{T = 0}$$

$$\boldsymbol{T = \frac{2u\sin\theta}{g}}$$

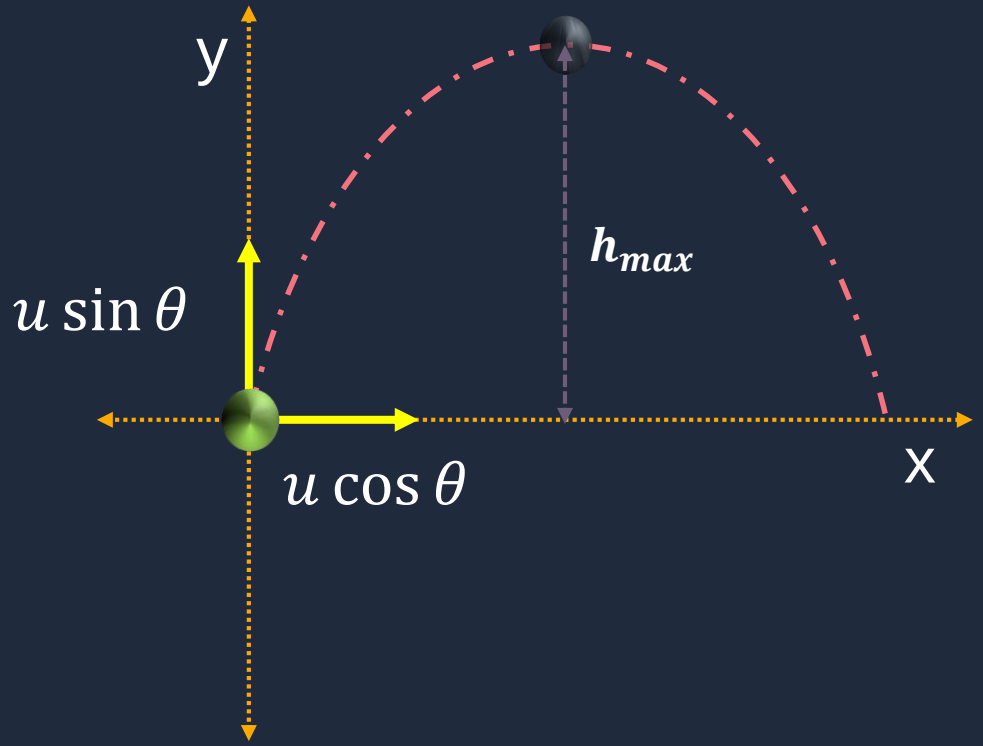
Maximum Height

$$v_y^2 - u_y^2 = 2a_y s_y$$

$$0 - u^2 \sin^2\theta = -2gh_{max}$$

$$\boldsymbol{h_{max} = \frac{u^2 \sin^2\theta}{2g}}$$

Projectile Motion



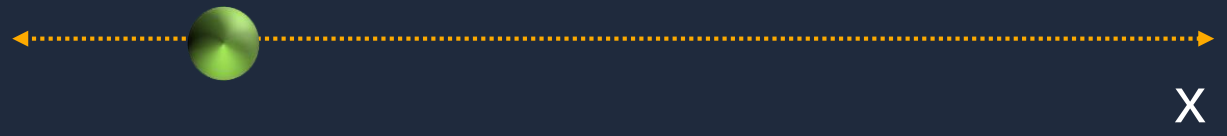
T	$\frac{2u \sin \theta}{g}$
h_{max}	$\frac{u^2 \sin^2 \theta}{2g}$

x-axis

Projectile Motion

Horizontal Range

x-axis



$$x = u_x t + \frac{1}{2} a_x t^2$$

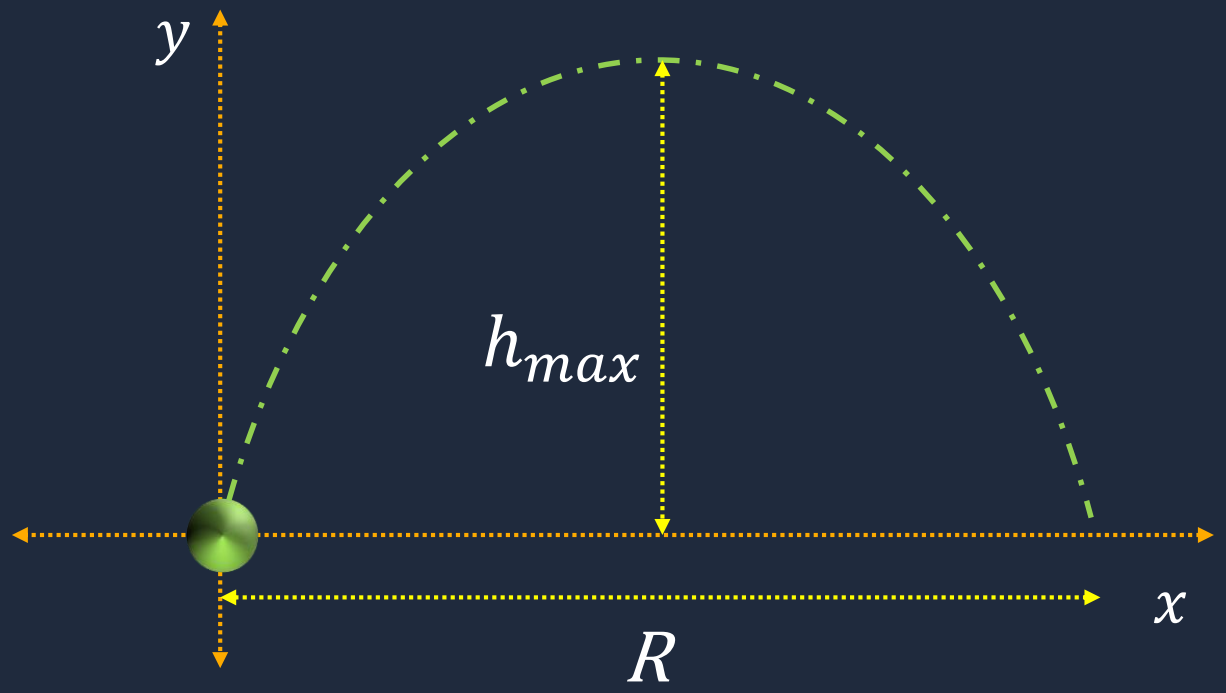
$$R = u_x T$$

$$R = \frac{(u\cos\theta)2u\sin\theta}{g}$$

$$\boldsymbol{R = \frac{2u_x u_y}{g}}$$

$$\boldsymbol{R = \frac{u^2 \sin 2\theta}{g}}$$

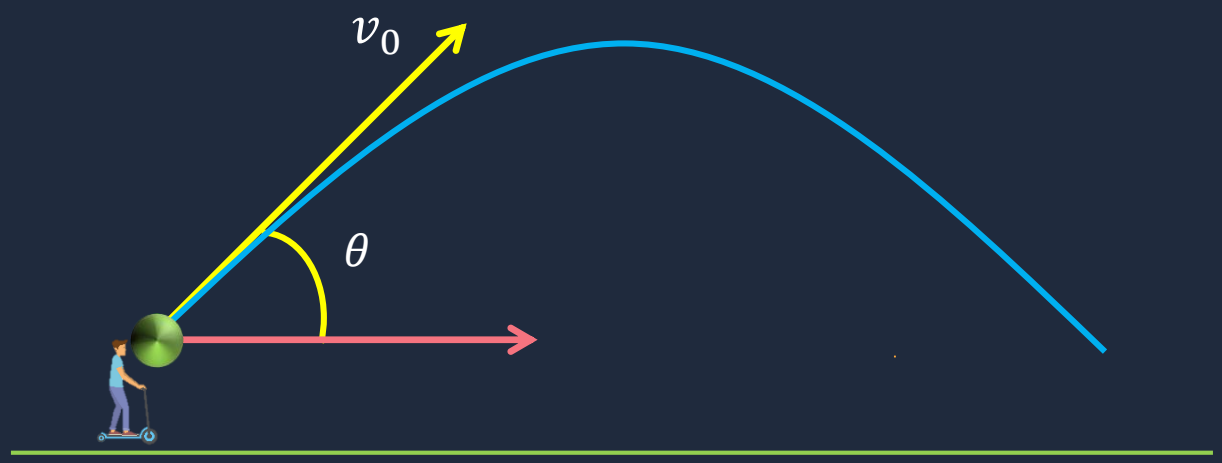
Projectile Motion



T	$\frac{2u\sin\theta}{g}$
h_{max}	$\frac{u^2\sin^2\theta}{2g}$
R	$\frac{u^2\sin 2\theta}{g}$

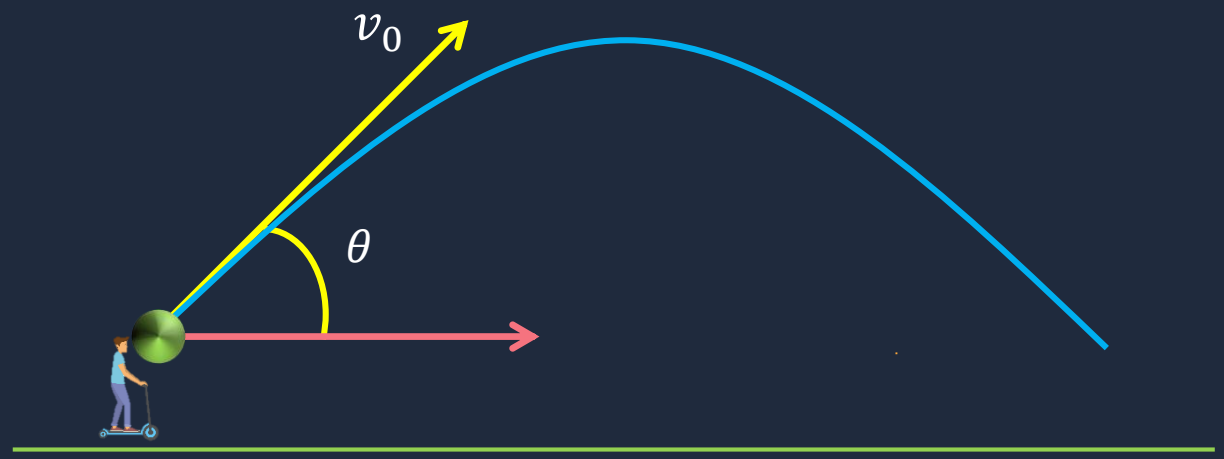
Let's solve this

A ball is thrown from a point with a speed v_0 at an angle θ with the horizontal. From the same point and at the same instant a person starts cycling with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection? ($g = 10\ ms^{-2}$)



Let's solve this

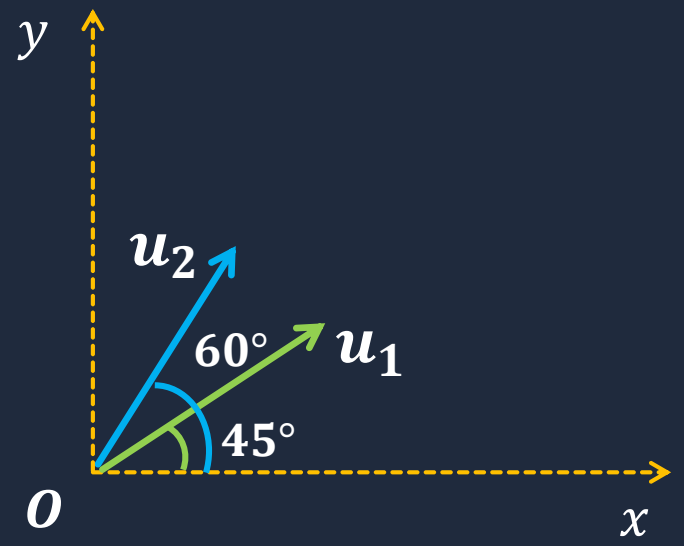
Person will catch the ball if the horizontal component of velocity of the ball is equal to the running speed of the person.



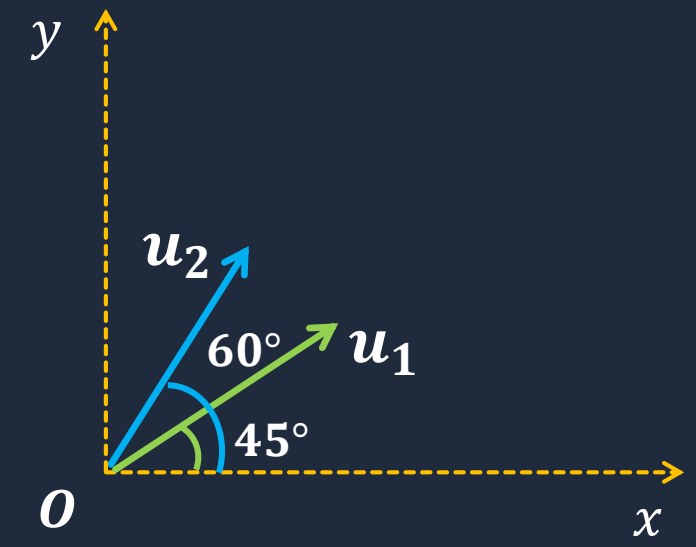
$\theta = 60°$

Let's solve this

Two bodies are thrown up at angles of $45°$ and $60°$ respectively, with the horizontal. If the bodies attain the same vertical height , then the ratio of their velocities with which they are thrown is (given $g = 10\ ms^{-2}$)

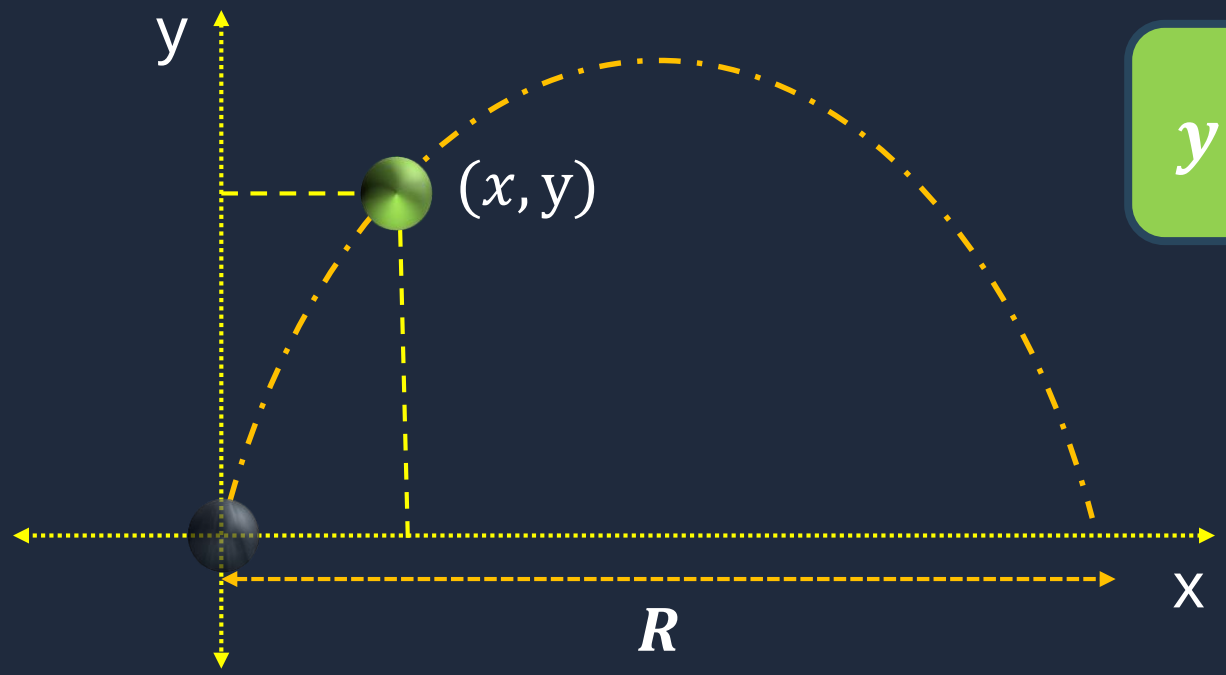


Let's solve this



$$\frac{u_1}{u_2} = \sqrt{\frac{3}{2}}$$

Equation of Trajectory of Motion



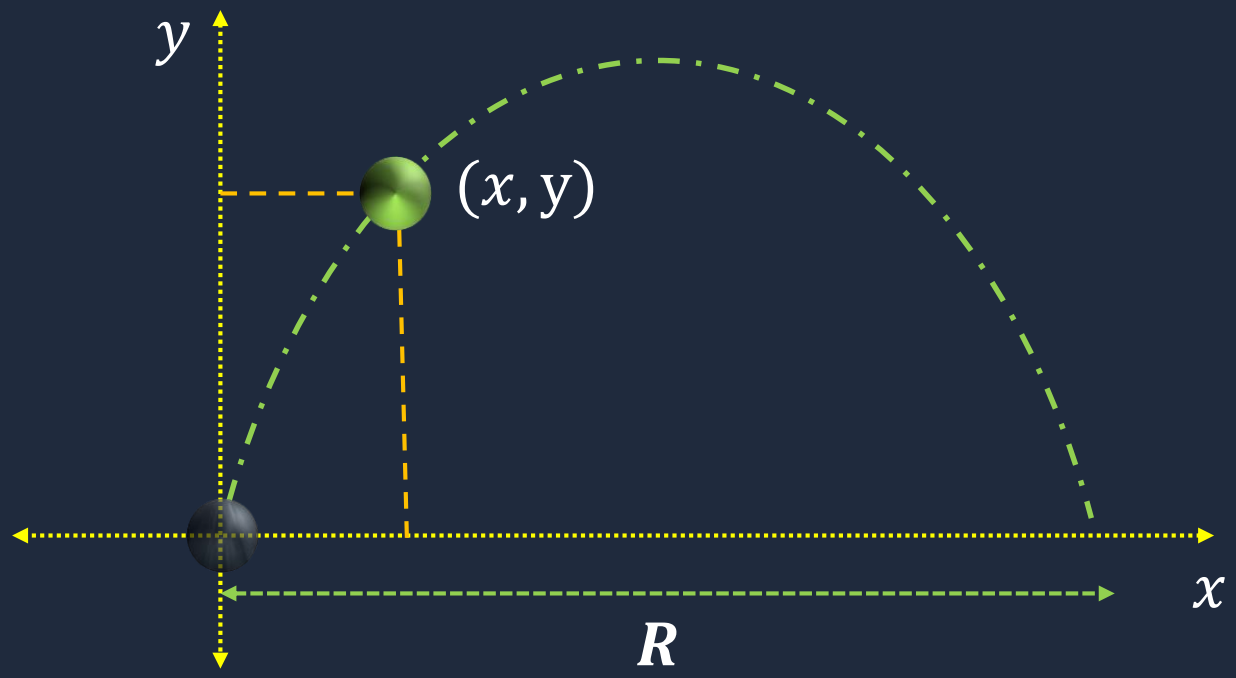
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = u\,sin\theta t - \frac{1}{2}gt^2$$

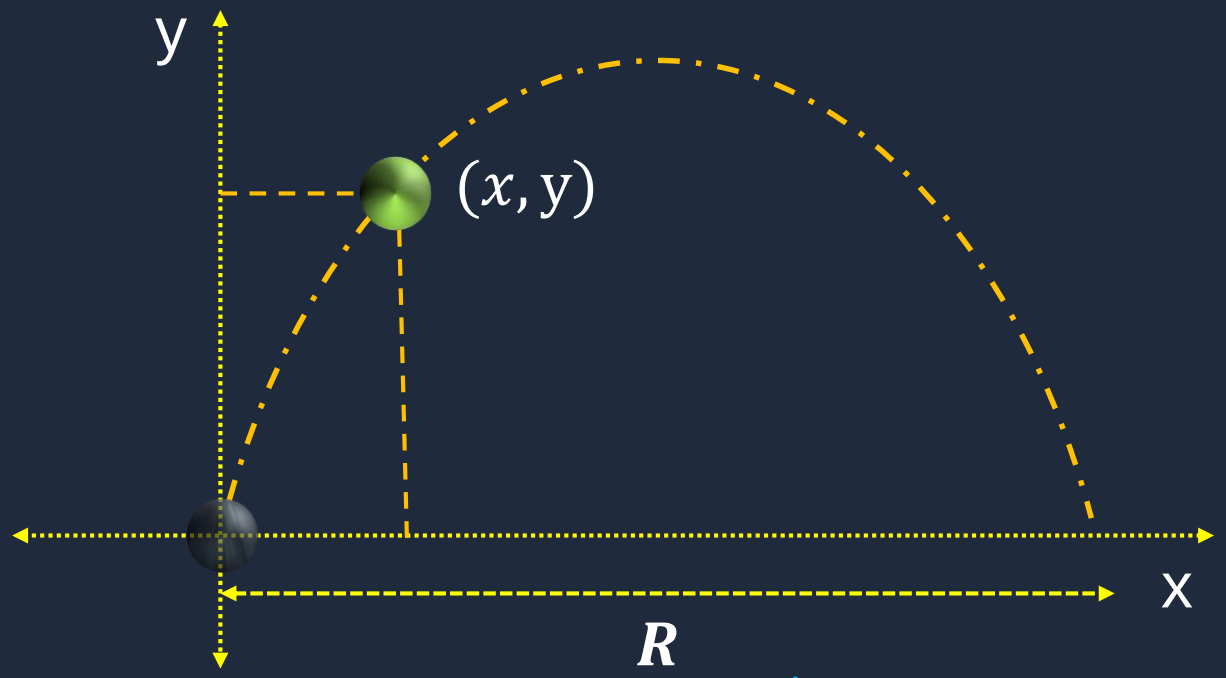
$$x = u\,cos\theta\; t$$

$$y = u\,sin\theta\,\frac{x}{u\,cos\theta} - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

Equation of Trajectory of Motion



Equation of Trajectory of Motion



$$y = x\tan\theta - \frac{x^2\tan\theta}{R}$$

$$\boldsymbol{y = x\tan\theta\left(1 - \frac{x}{R}\right)}$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$\boldsymbol{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \times \frac{\sin\theta}{\sin\theta}$$

$$y = x\tan\theta - \frac{x^2\sin\theta}{\frac{2u^2\sin\theta\cos\theta}{g}} \times \frac{1}{\cos\theta}$$

Let's solve this

A projectile is given an initial velocity of $(\hat{\imath} + 2\hat{\jmath})\ \ m/s$. Find the equation of its trajectory $(g = 10\ m/s^2)$.

Let's solve this

$$y = 2x - 5x^2$$

Let's solve this

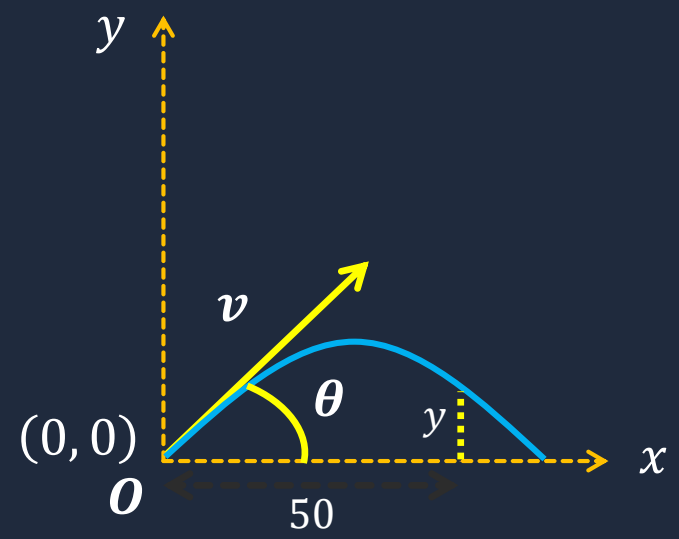
The equation of motion of a projectile is given by $x = (36t)\ m$ and $2y = (96\ t - 9.8\ t^2)\ (in\ m)$. The angle of projection is $(\ t\ in\ s)$

Let's solve this

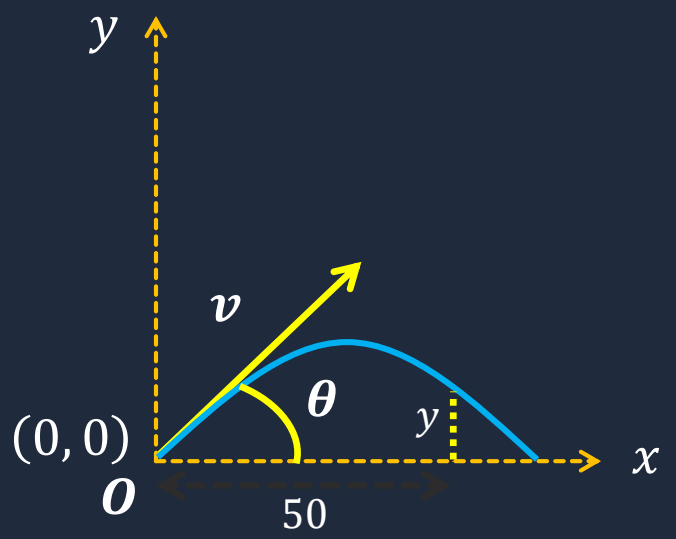
$$\theta = sin^{-1}\left(\frac{4}{5}\right)$$

Let's solve this

A cricketer hits a ball with a velocity $25\ m/s$ at $60°$ above the horizontal. How far above the ground it passes over a fielder $50\ m$ from the bat? (assume the ball is struck very close to ground and $(g = 9.8\ ms^{-2})$



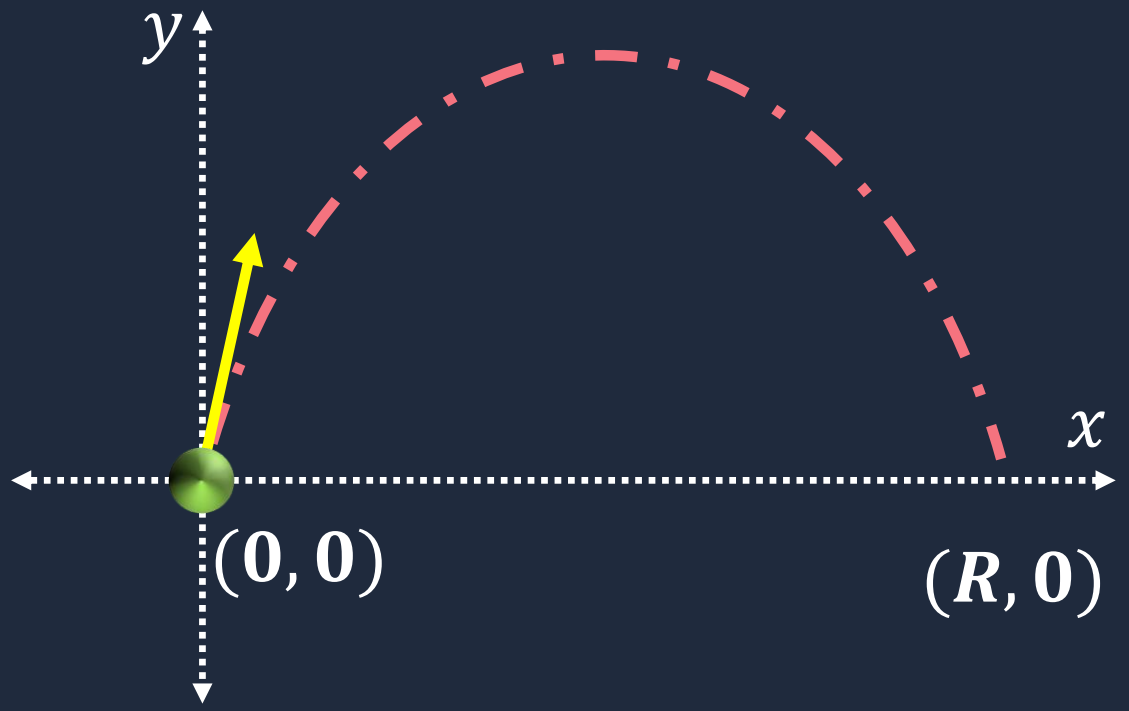
Let's solve this



$y = 8.2\ m$

Short trick

A particle is projected from the ground whose trajectory is given by $y = ax - bx^2$, where a and b are constants. Find the horizontal range of particle.

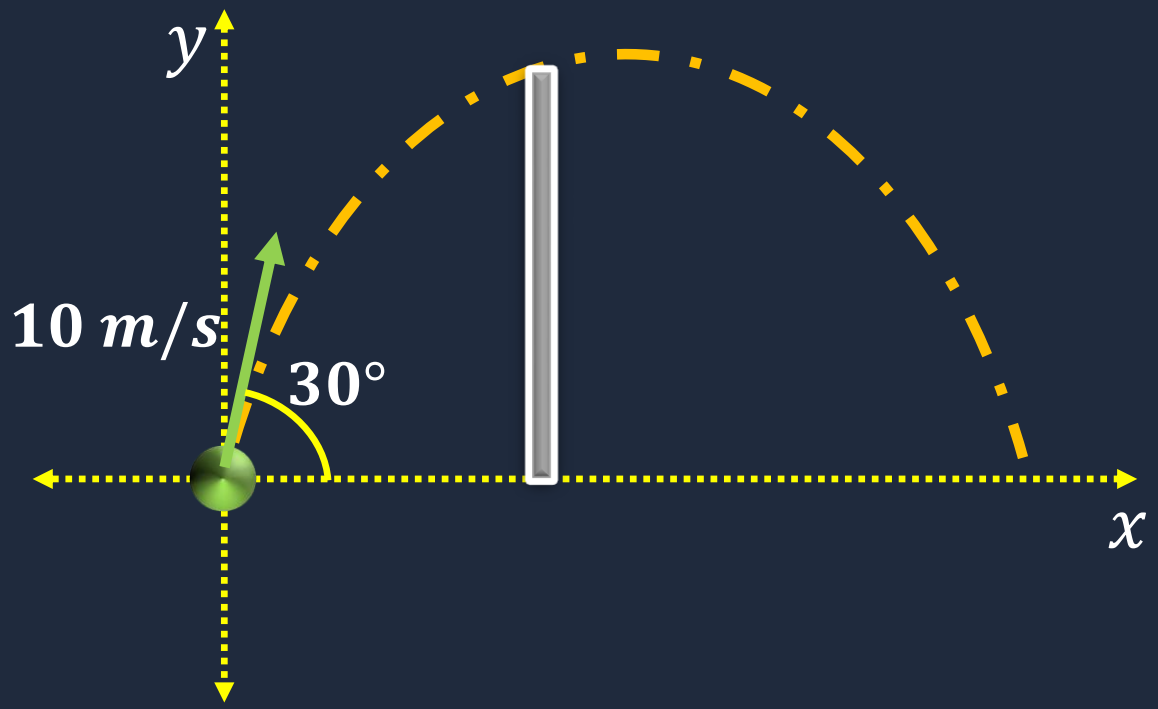


Let's solve this

$$R = \frac{a}{b}$$

Let's solve this

A particle is projected with a velocity of $10\ m/s$ at an angle of $30°$ with horizontal. A wall is present at a horizontal distance of $\sqrt{3}\ m$ from the point of projection. Find the height of the wall such that the particle just grazes the wall.

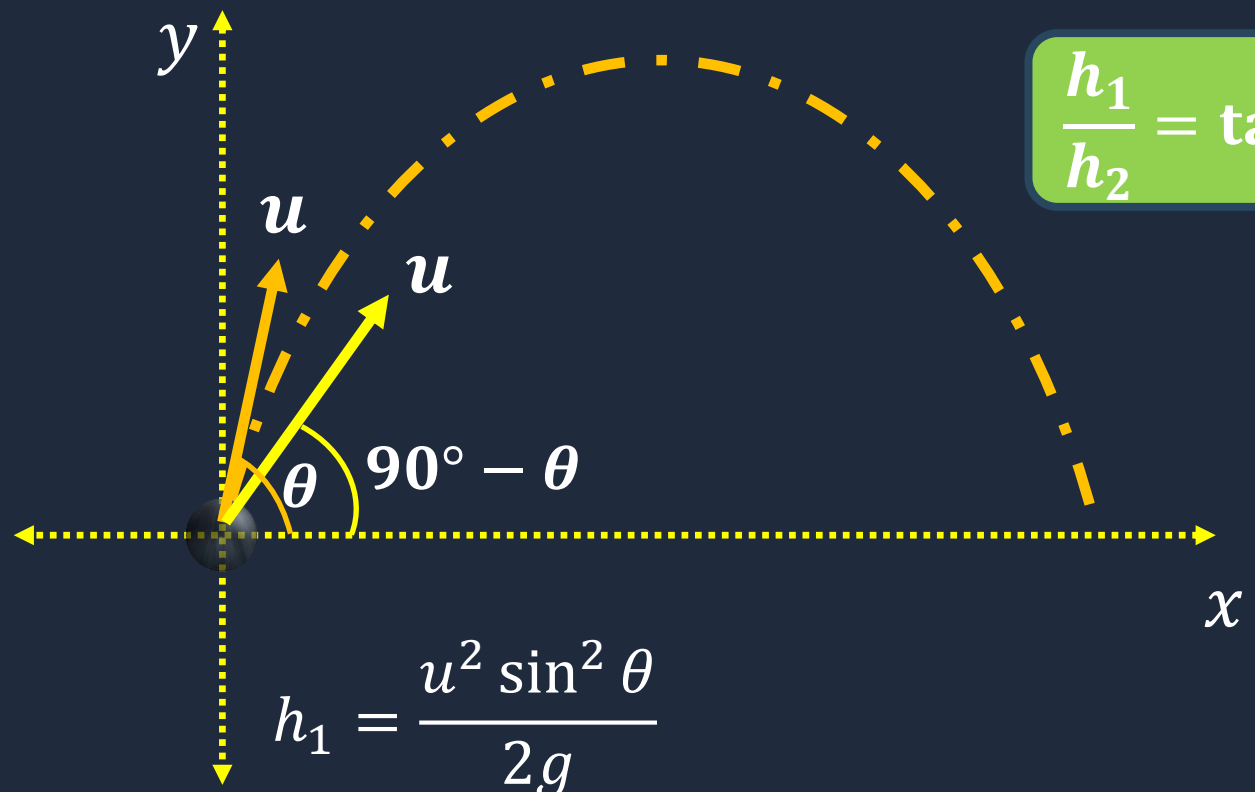


Let's solve this

$$y = 0.8\ m$$

Tips and Tricks

1) Height



$$\frac{h_1}{h_2} = \tan^2\theta$$

$$h_1 + h_2 = \frac{u^2}{2g}$$

$$h = \frac{u_y^2}{2g}$$

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$h_1 = \frac{u^2 \sin^2\theta}{2g}$$

$$h_2 = \frac{u^2 \sin^2(90-\theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

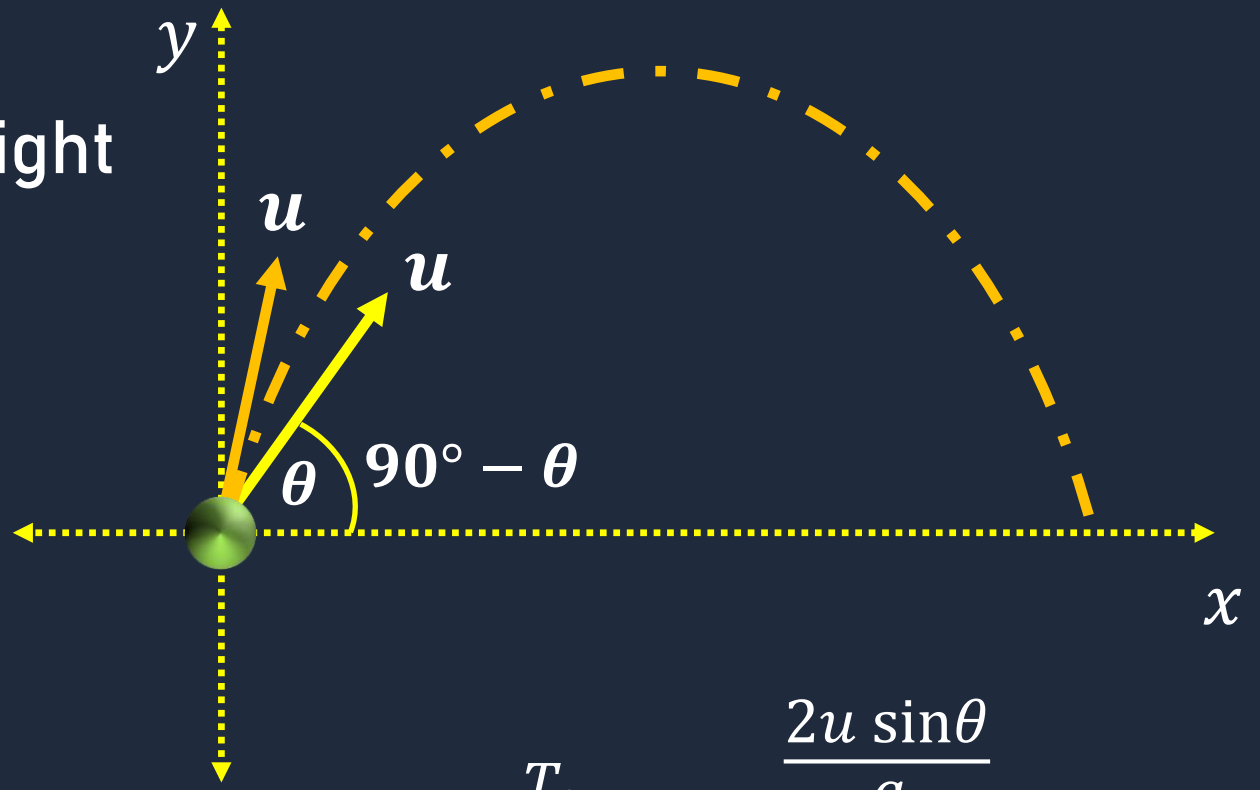
Let's solve this

Two particles are projected from ground with velocity $u = 40m/s$ at complementary angles. If difference in maximum height of both the projectiles is $50\ m$, then find the height of individual projectiles.($g = 10\ ms^{-2}$)

Let's solve this

$$h_1 = 15m,$$
$$h_2 = 65m$$

2)Time of Flight



$$T_1 \times T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2R}{g}$$

$$\frac{T_1}{T_2} = \frac{\frac{2u\sin\theta}{g}}{\frac{2u\sin(90-\theta)}{g}}$$

$$\boldsymbol{T_1 \times T_2 = \frac{2R}{g}}$$

$$\boldsymbol{\frac{T_1}{T_2} = \tan\theta}$$

Let's solve this

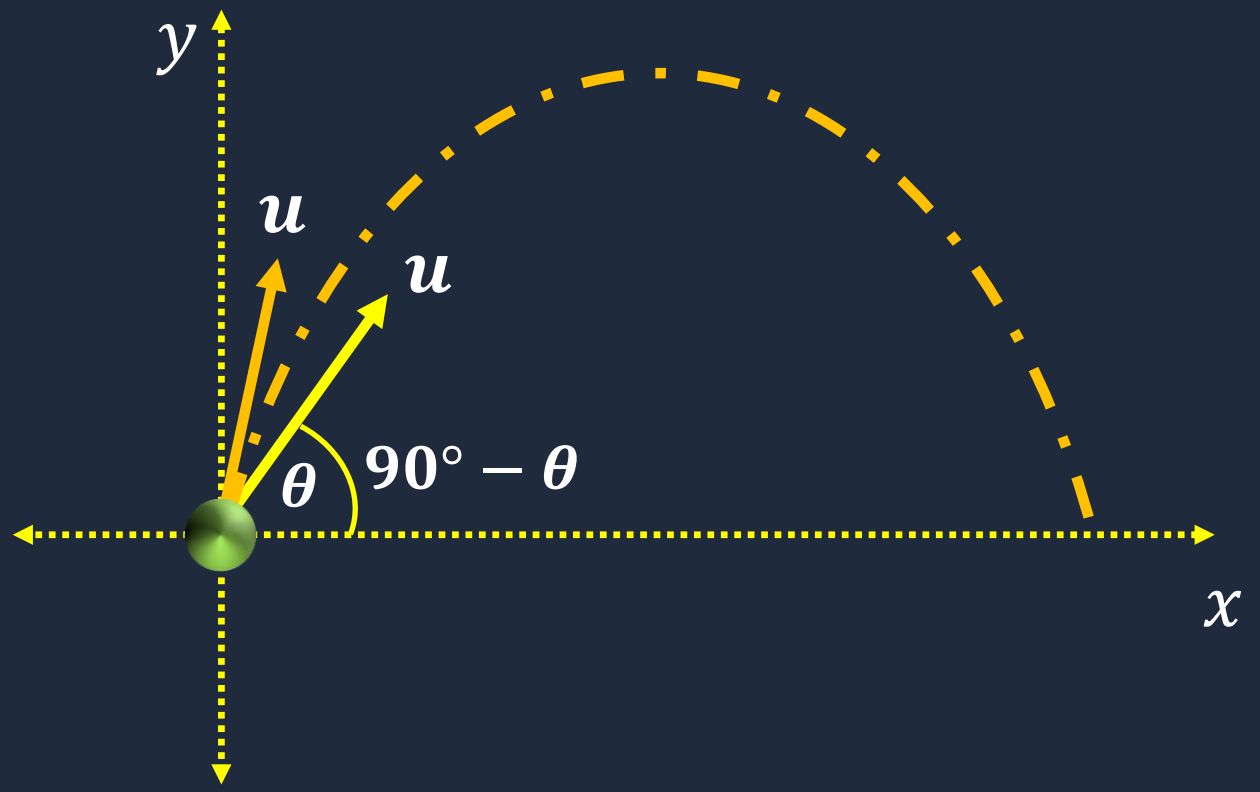
A particle is projected from ground at an angle of $30°$.Its time of flight is $5\ s$. Another particle is projected with the same speed at angle of $60°$. Find the time of flight of other particle.

Let's solve this

$$T_2 = 5\sqrt{3}s$$

Tips and Tricks

3) Range



$$\frac{R_1}{R_2} = \frac{\frac{u^2 \sin 2\theta}{g}}{\frac{u^2 \sin 2(90-\theta)}{g}} = \frac{\frac{u^2 \sin 2\theta}{g}}{\frac{u^2 \sin 2\theta}{g}}$$

$$\frac{R_1}{R_2} = 1$$

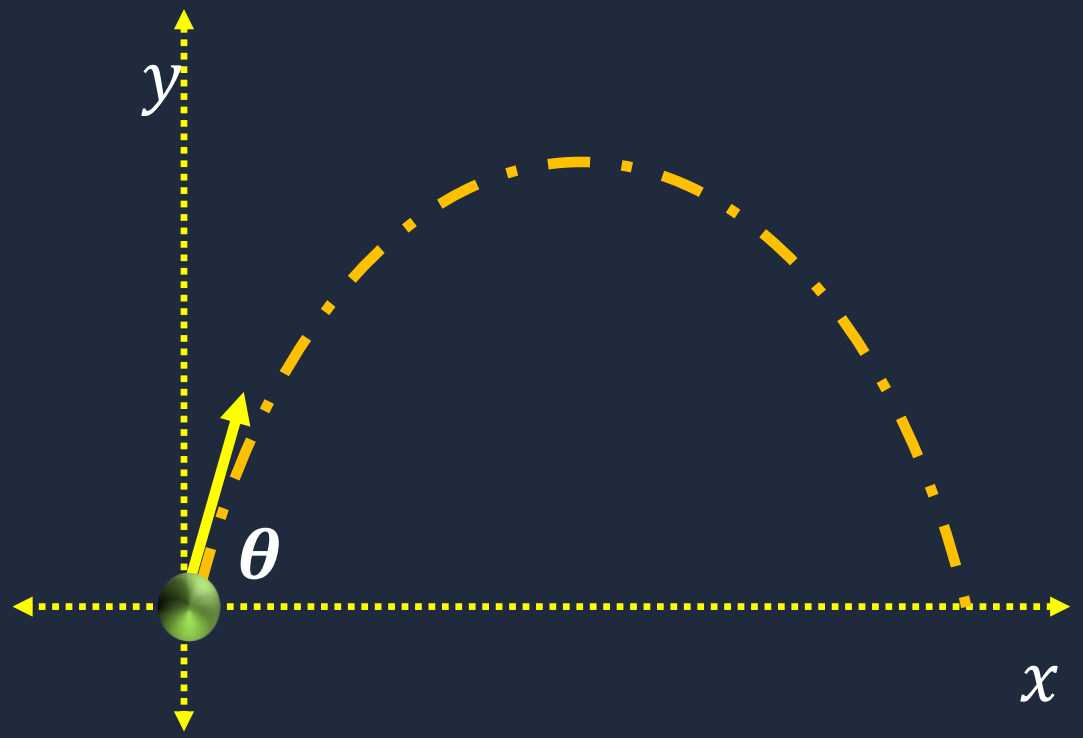
Tips and Tricks

$$R = nh_{max}$$

$$\frac{u^2 \sin 2\theta}{g} = n\left(\frac{u^2 \sin^2 \theta}{2g}\right)$$

$$2\cos\theta = \frac{n}{2}\sin\theta$$

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{4}{n}\right)}$$



Let's solve this

A particle is projected from ground with velocity u such that its range becomes 2 times that of its maximum height. Find range and time of flight of the particle.

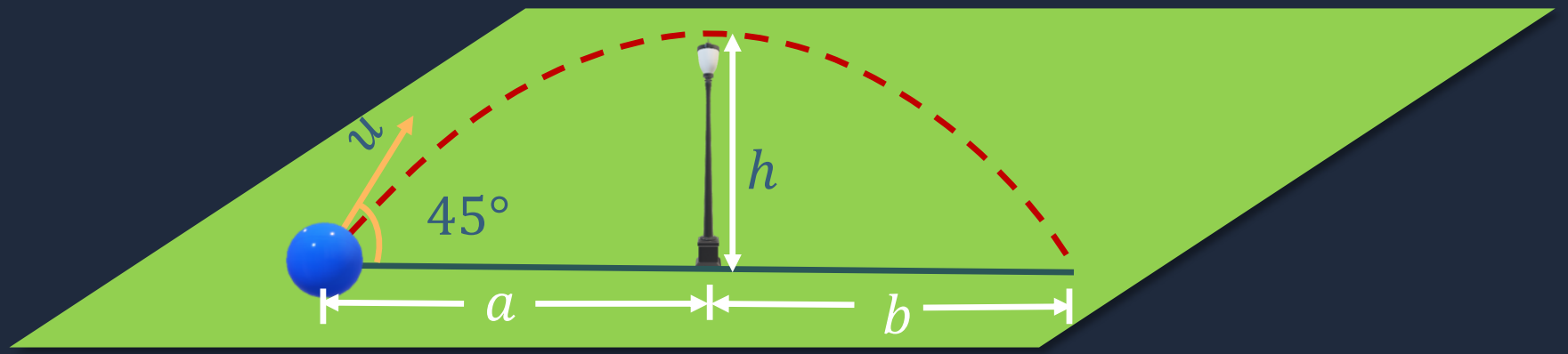
Let's solve this

A particle is projected from ground with velocity u such that its range becomes 2 times that of its maximum height. Find range and time of flight of the particle.

$$T = \frac{4u}{g\sqrt{5}}$$

Let's solve this

From a point on the ground at a distance a from the foot of a pole, a ball is thrown, at an angle $45°$ which just clears the top of pole and strikes the ground at a distance of b, on the other side of it. Height of the pole is



Let's solve this

From a point on the ground at a distance a from the foot of a pole, a ball is thrown, at an angle $45°$ which just clears the top of pole and strikes the ground at a distance of b, on the other side of it. Height of the pole is

$$h = \frac{ab}{a+b}$$

Thank You