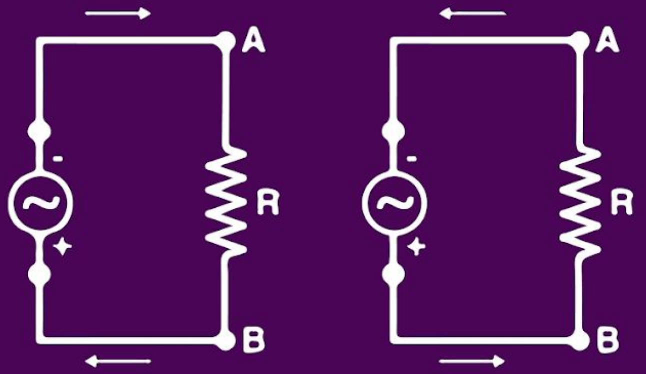
BONUS SESSION

ALTERNATING CURRENT - L2



PHYSICS

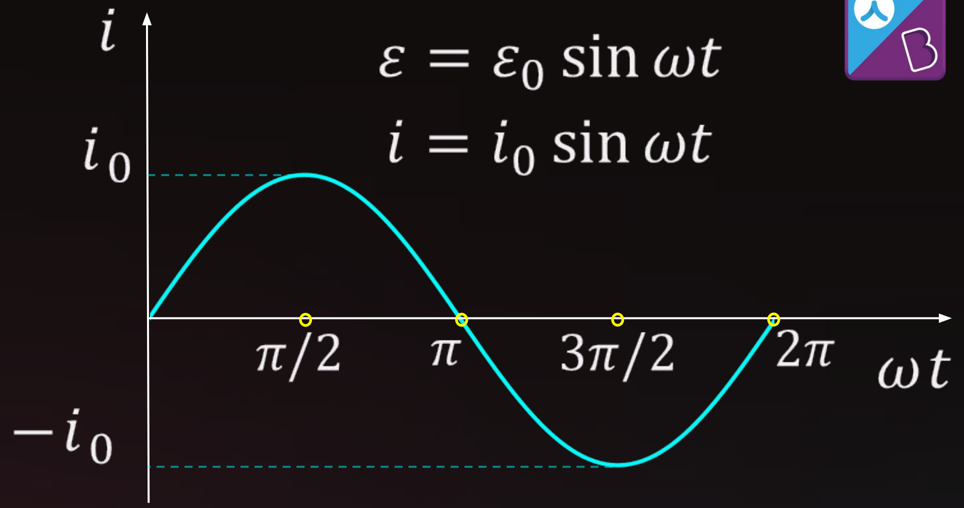
https://t.me/neetaakashdigital

CONTENTS

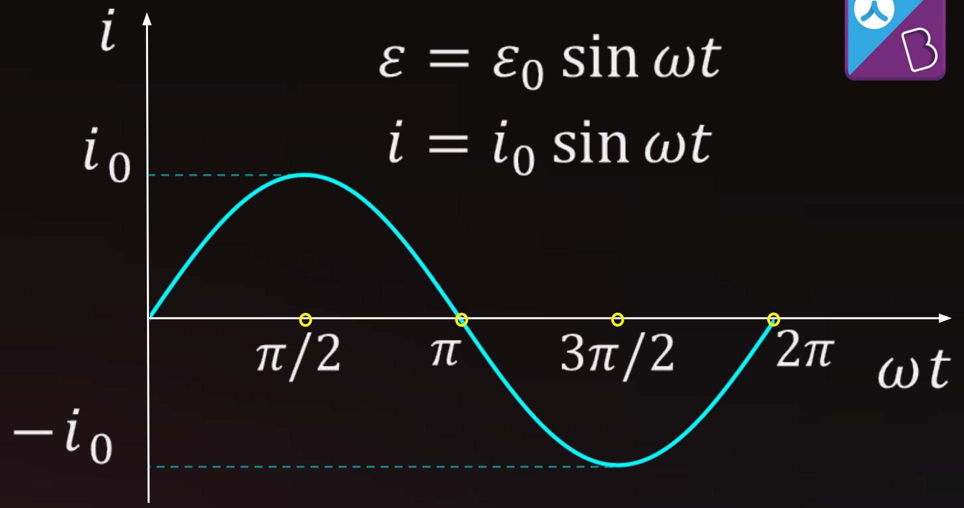
- Rms value of A sinusoidal AC
- Phasor diagram
- Pure resistive ac circuit
- Pure Inductive ac circuit
- Pure capacitive ac circuit

AC

$$i_{rms} = \sqrt{< i^2 >}$$



AC



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \omega t$$

$$i_{rms} = \sqrt{< i^2 >}$$

$$i_{rms} = \sqrt{< i_0^2 \sin^2 \omega t >}$$

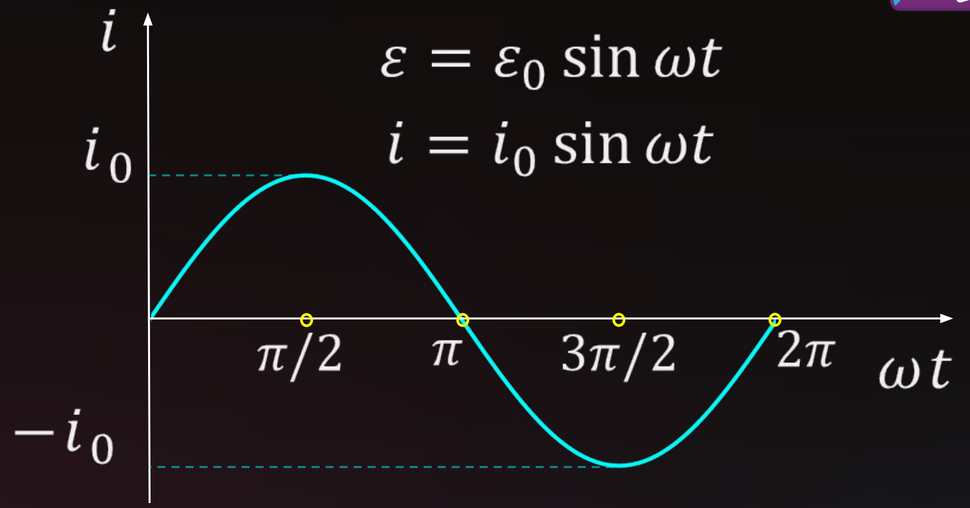
$$i_{rms}^2 = \frac{1}{T}\int_0^T i_0^2 \sin^2 \omega t \; dt = \frac{i_0^2}{T}\int_0^T \sin^2 \omega t \; dt$$

$$i_{rms}^2 = \frac{i_0^2}{T}\int_0^T \frac{1-\cos 2\omega t}{2} \, dt \qquad \left(\sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$i_{rms}^2 = \frac{i_0^2}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T$$

AC

$$i_{rms} = \sqrt{< i^2 >}$$



AC

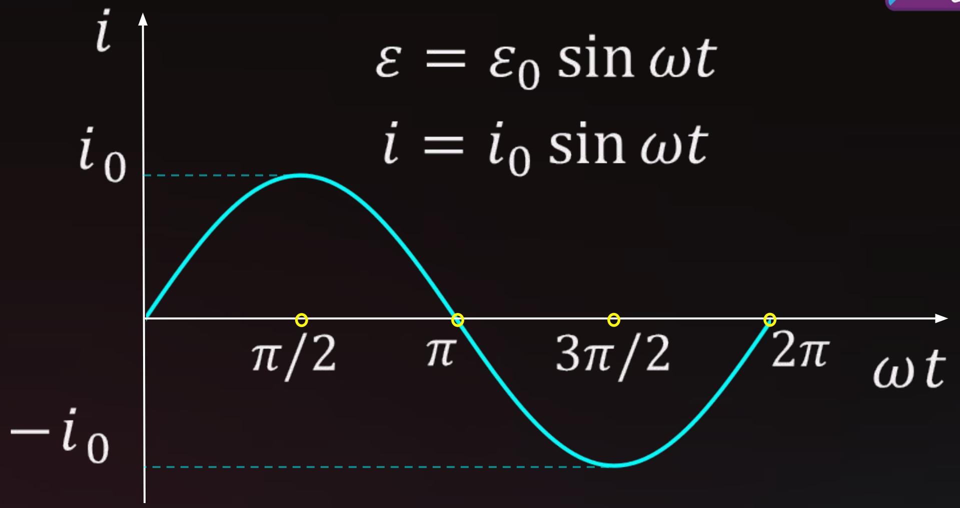
$$i_{rms} = \sqrt{< i^2 >}$$

$$i_{rms}^2 = \frac{i_0^2}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T \qquad \left(T = \frac{2\pi}{\omega}\right)$$

$$i_{rms}^2 = \frac{i_0^2}{2T}\left(\left(T - \frac{\sin 2\omega\left(\frac{2\pi}{\omega}\right)}{2\omega}\right) - (0 - \sin 0)\right) = \frac{i_0^2}{2T}\text{T}$$

$$i_{rms}^2 = \frac{i_0^2}{2}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}}$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \omega t$$

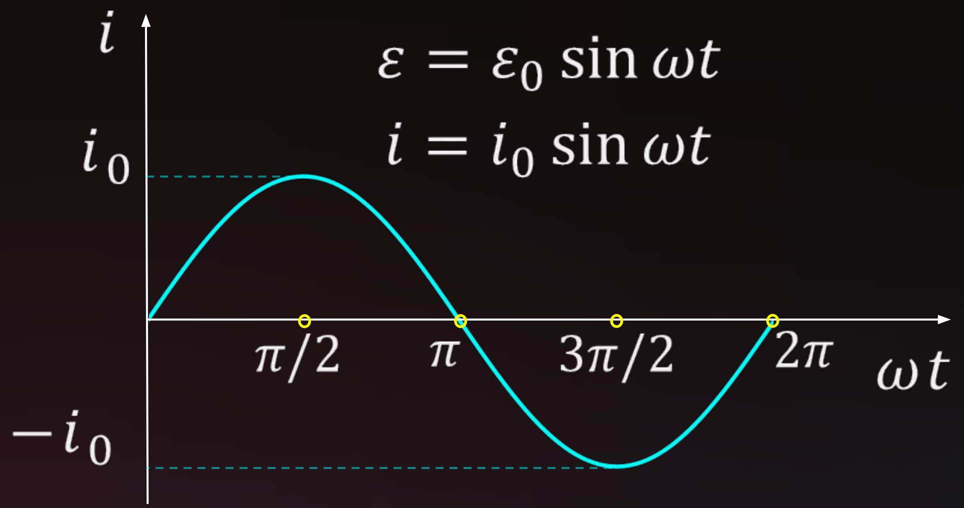
AC

$$i_{rms} = \frac{i_0}{\sqrt{2}}$$

$$(i_{av})_{half\ cycle} = \frac{2i_0}{\pi}$$

$$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}}$$

$$(\varepsilon_{av})_{half\ cycle} = \frac{2\varepsilon_0}{\pi}$$



Household current $\rightarrow$ sinusoidal AC ($\varepsilon = \varepsilon_0 \sin \omega t$)

$220\ V, 50\ Hz$

Significance OF Rms value

Household current → sinusoidal AC ($\varepsilon = \varepsilon_0 \sin \omega t$)

$220\, V, 50\, Hz$

$\varepsilon_{rms} = 220\, V$

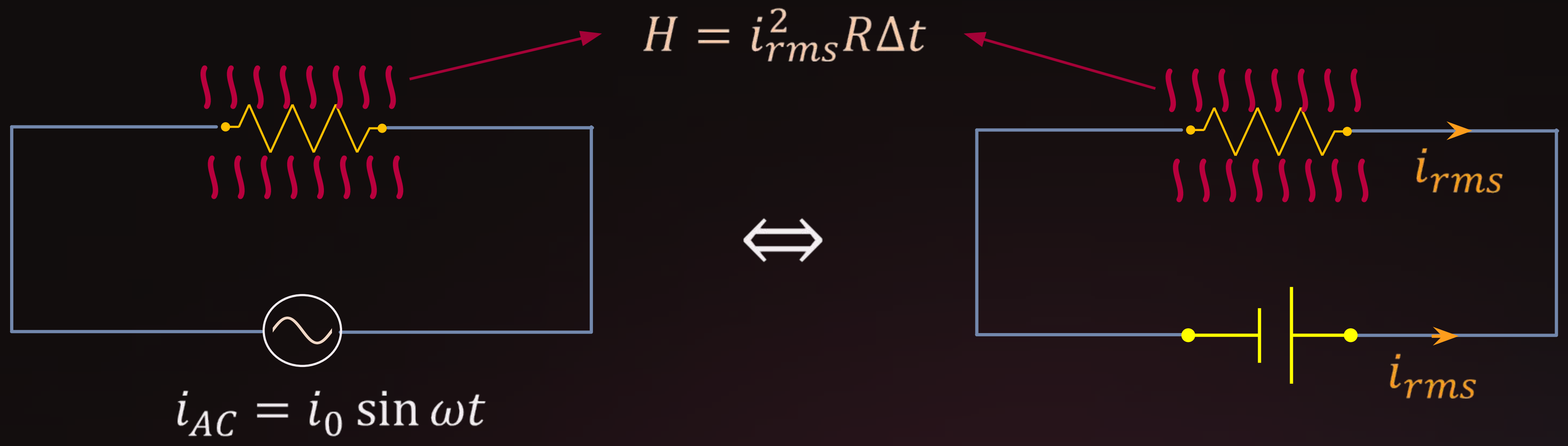
$\varepsilon_{av} = 0\, V$

$\varepsilon_0 = \sqrt{2}\varepsilon_{rms}$

$\varepsilon_0 = \sqrt{2} \times 220 = 311.12\, V \approx 311\, V$

If problem states only ε (not $\varepsilon_0, \varepsilon_{rms}, \varepsilon_{av}$) then, consider it as ε_{rms}

Significance OF Rms value

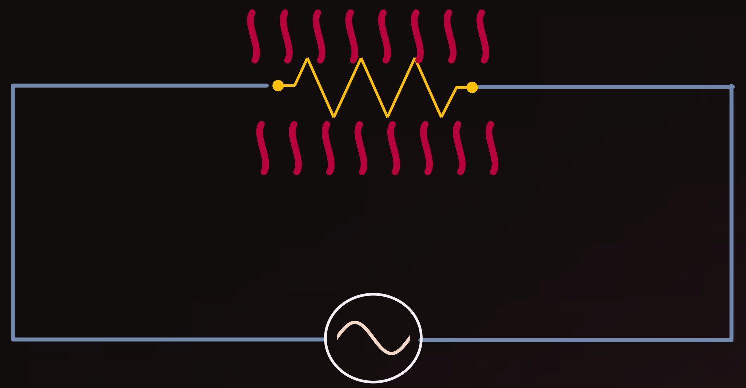


RMS value of a given AC can be defined as that DC value which produces same heat in a resistance which the AC produces in that resistance in same duration.

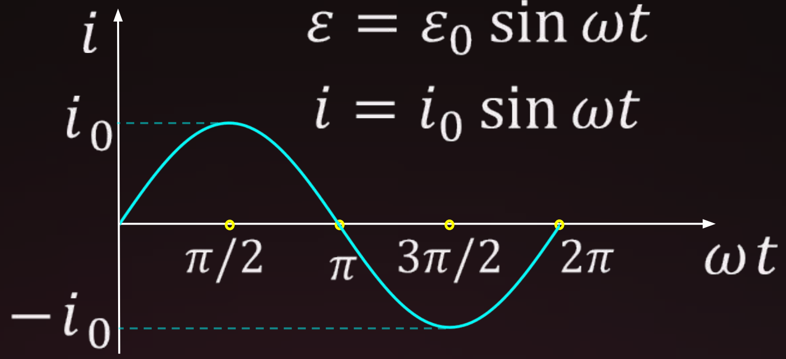
i_{rms} is the effective DC value of a given AC

Significance OF Rms value

$$H = i_{rms}^2 R\Delta t$$



$$i_{AC} = i_0 \sin \omega t$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin \omega t$$

- DC devices cannot measure alternating current or emf. Normal Ammeter, Voltmeter will show only zero for AC

- Hot Wire Ammeter & Hot Wire Voltmeter are used to measure AC. They measure RMS value of i & ε.

Question

If the voltage of a source in an AC circuit is represented by the equation, $E = 220\sqrt{2}\sin(314t)$. Calculate the peak value of the current if the net resistance of the circuit is $220\ \Omega$. Take $\sqrt{2} = 1.4$

a) $1.8\ A$

b) $1.6\ A$

c) $1.4\ A$

d) $1.2\ A$

DISCUSSION

Given,

Voltage, $\varepsilon = 220\sqrt{2}\sin(314t)$

DISCUSSION

Given,

Voltage, $\varepsilon = 220\sqrt{2}\sin(314t)$

Comparing with $\varepsilon = \varepsilon_o \sin \omega t$, we get,

$$\varepsilon_o = 220\sqrt{2}\ V$$

So, peak value of current

$$i_o = \frac{\varepsilon_o}{R} = \frac{220\sqrt{2}}{220}$$

$$i_o = 1.4\ A$$

ANSWER

If the voltage of a source in an AC circuit is represented by the equation, $E = 220\sqrt{2}\sin(314t)$. Calculate the peak value of the current if the net resistance of the circuit is $220\ \Omega$. Take $\sqrt{2} = 1.4$

a) $1.8\ A$

b) $1.6\ A$

c) $1.4\ A$

d) $1.2\ A$

Example

Find I_{rms} for the following :-

(i) $i = i_o + i_o \sin \omega t$

Shortcut To Find RMS Value

For Full Cycle

$$\overline{\sin \omega t} = 0$$

$$\overline{\cos \omega t} = 0$$

$$\overline{\sin^2 \omega t} = \frac{1}{2}$$

$$\overline{\cos^2 \omega t} = \frac{1}{2}$$

Example

Find I_{rms} for the following :-

(i) $i = i_o + i_o \sin \omega t$

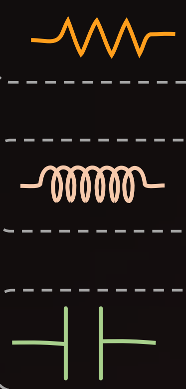
Homework

Find I_{rms} for the following :-

(ii) $i = i_1 \sin \omega t + i_2 \cos \omega t$

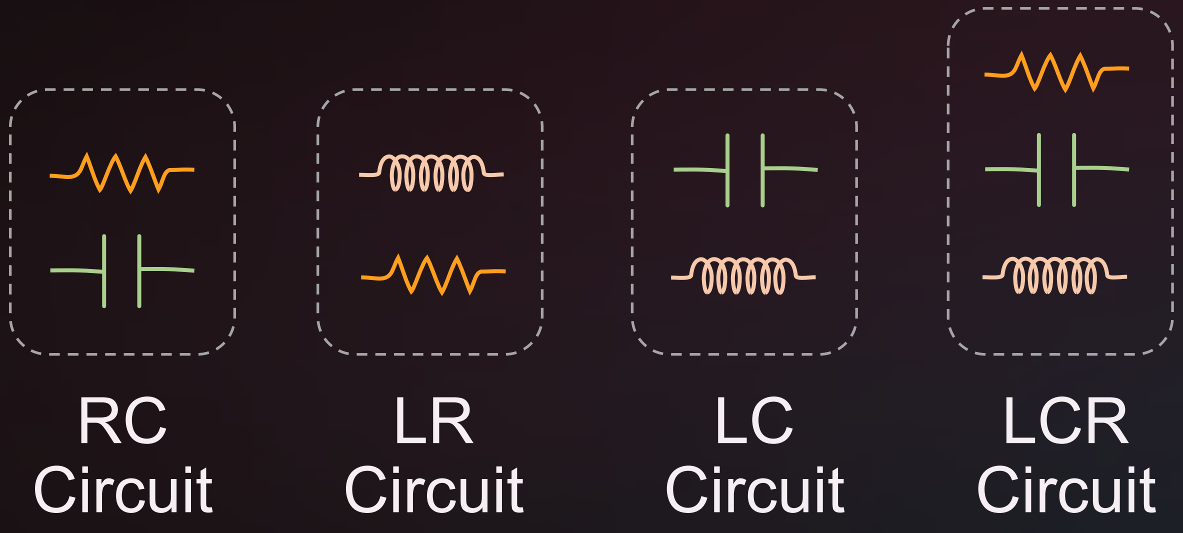
CIRCUITS

I. An AC source connected only to:



- A Resistor
- An Inductor
- A Capacitor

II. An AC source connected to more than one element.

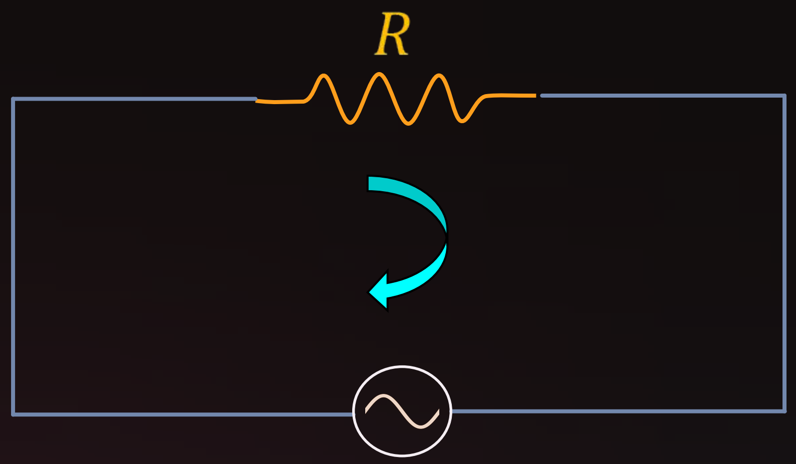


Circuit

Apply KVL ;

$$\varepsilon - iR = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{R}$$

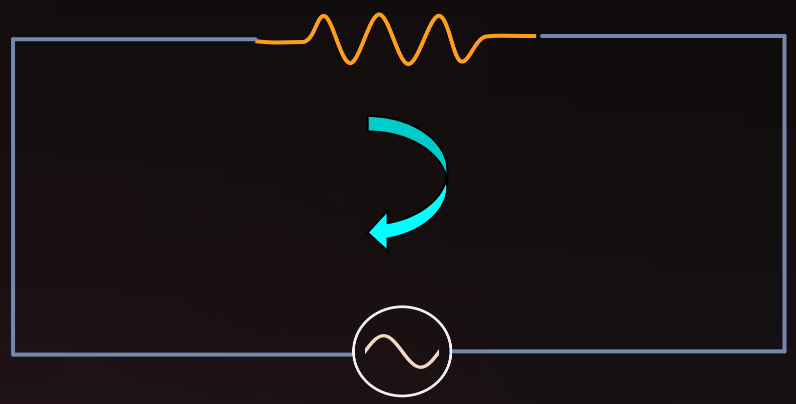
$$i = \frac{\varepsilon_0 \sin \omega t}{R}$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

Circuit

Apply KVL ;



Peak voltage

Peak current

Current is in phase with potential

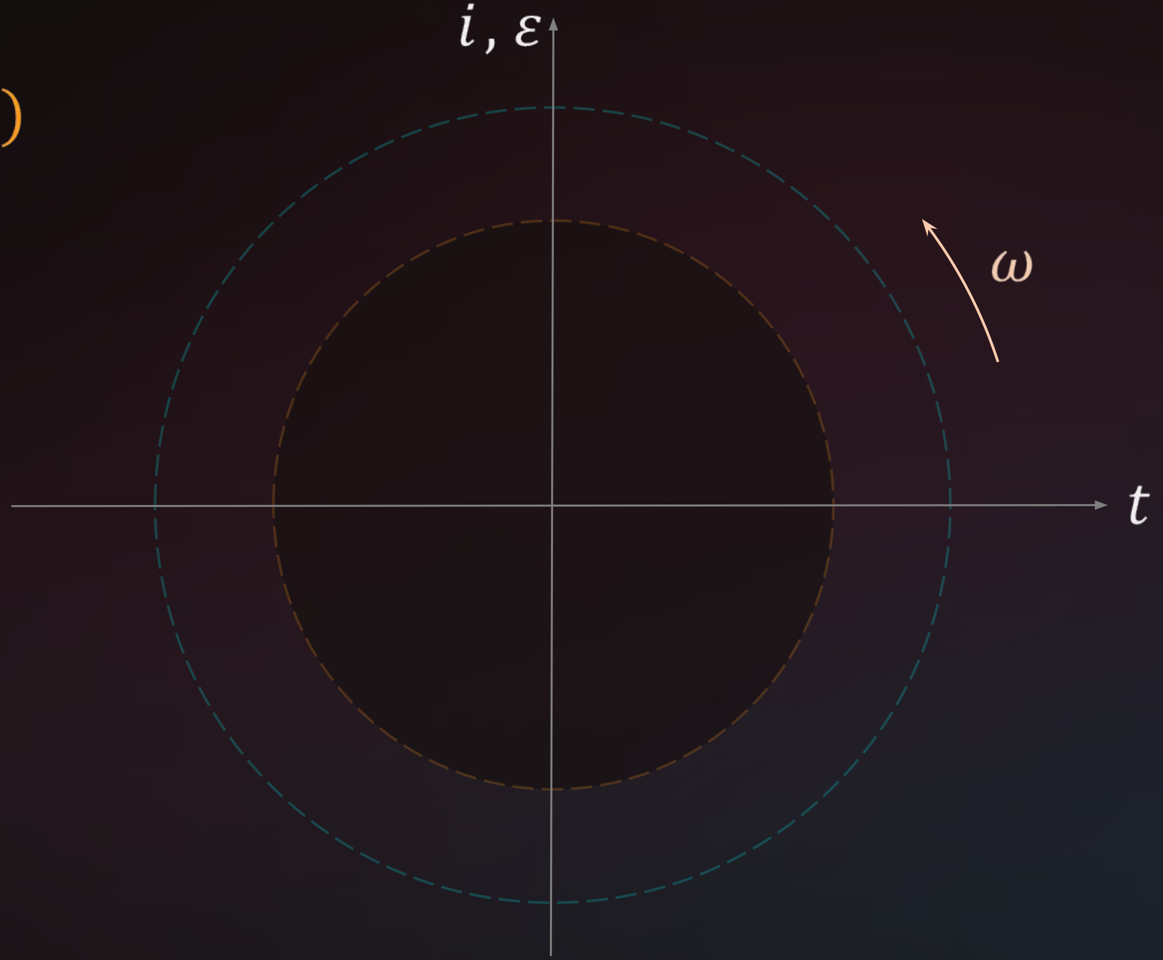
diagram

A diagram that represents AC and voltage of same frequency as rotating vectors (phasors)

along with proper phase angle between them.

$\varepsilon = \varepsilon_0 \sin \omega t$

$i = i_0 \sin(\omega t + \phi)$

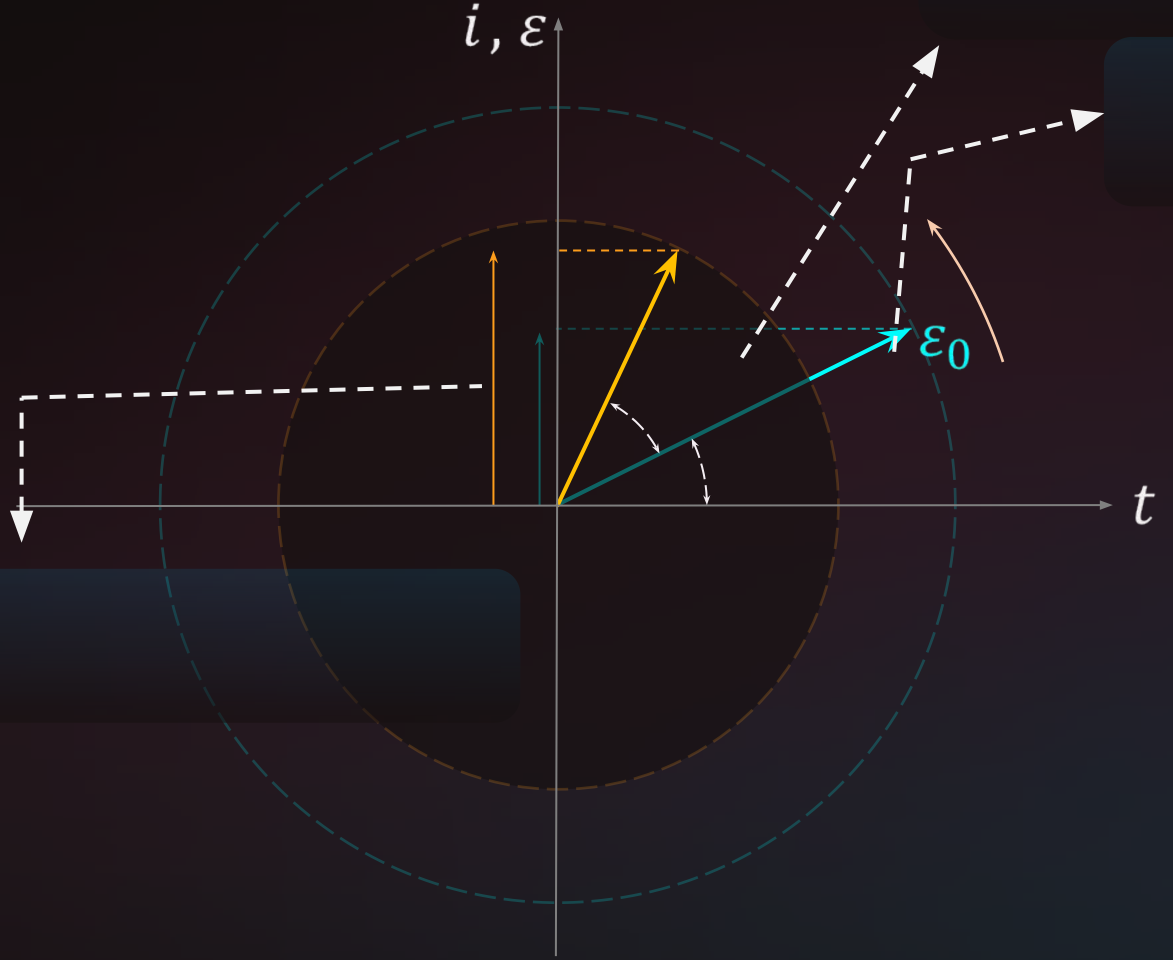


diagram

A diagram that represents AC and voltage of same frequency as rotating vectors (phasors)

along with proper phase angle between them.

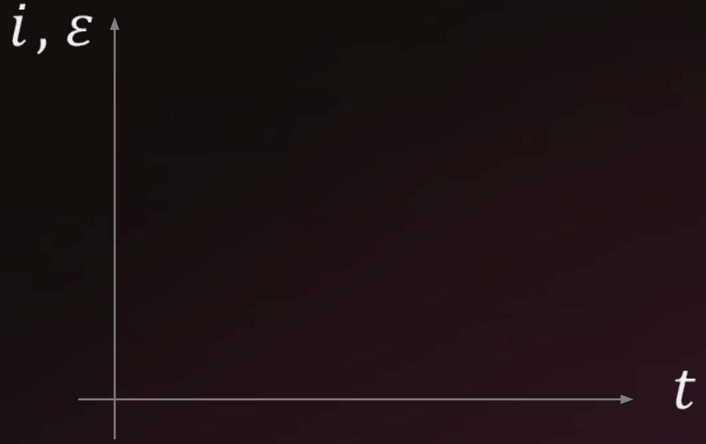
Phase difference



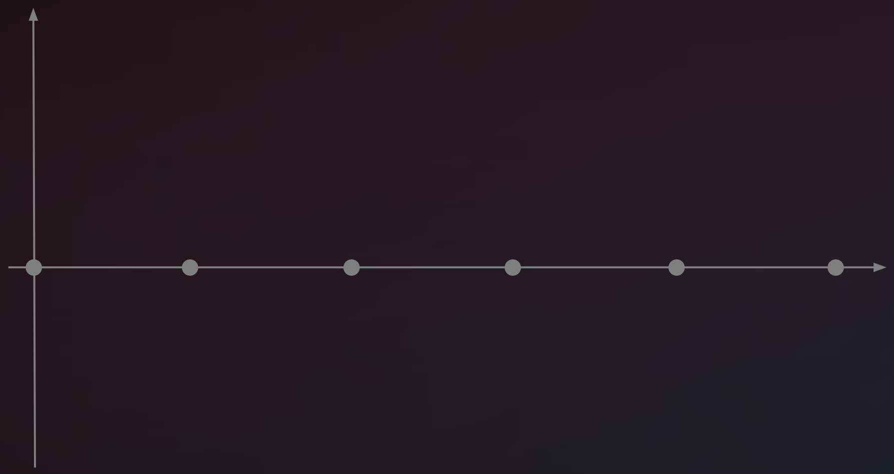
Circuit

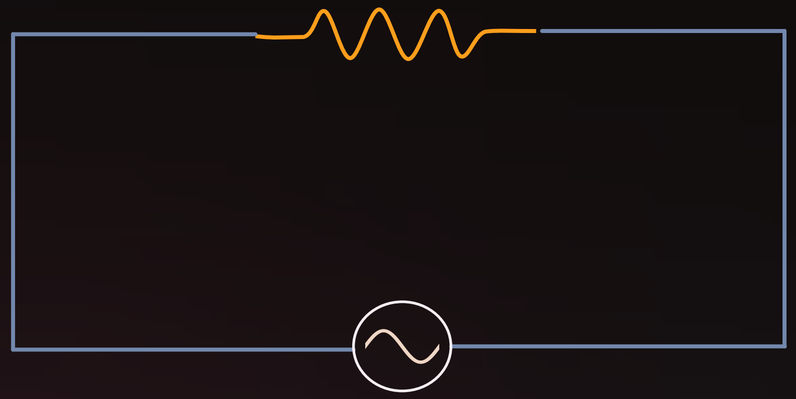
Current is in phase with potential

Phasor diagram



Wave diagram

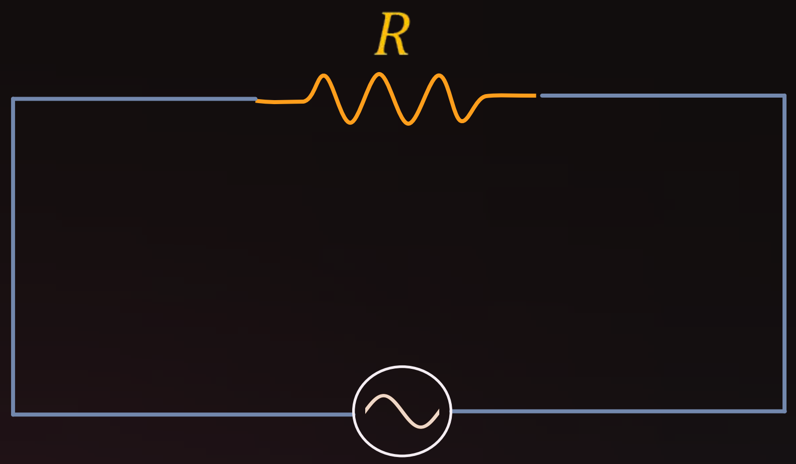




$$i = i_0 \sin \omega t$$

Circuit

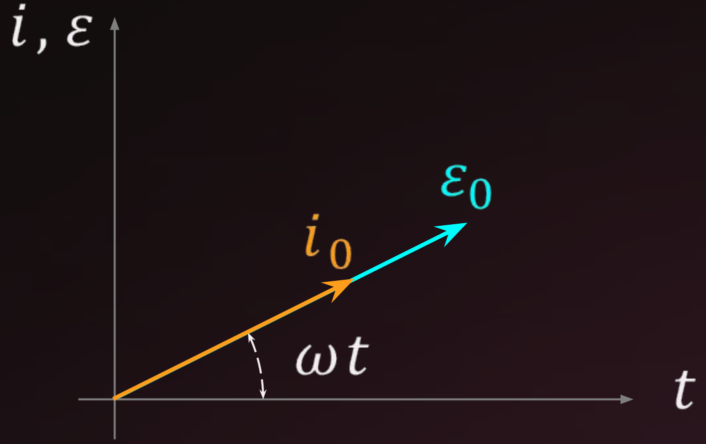
Current is in phase with potential



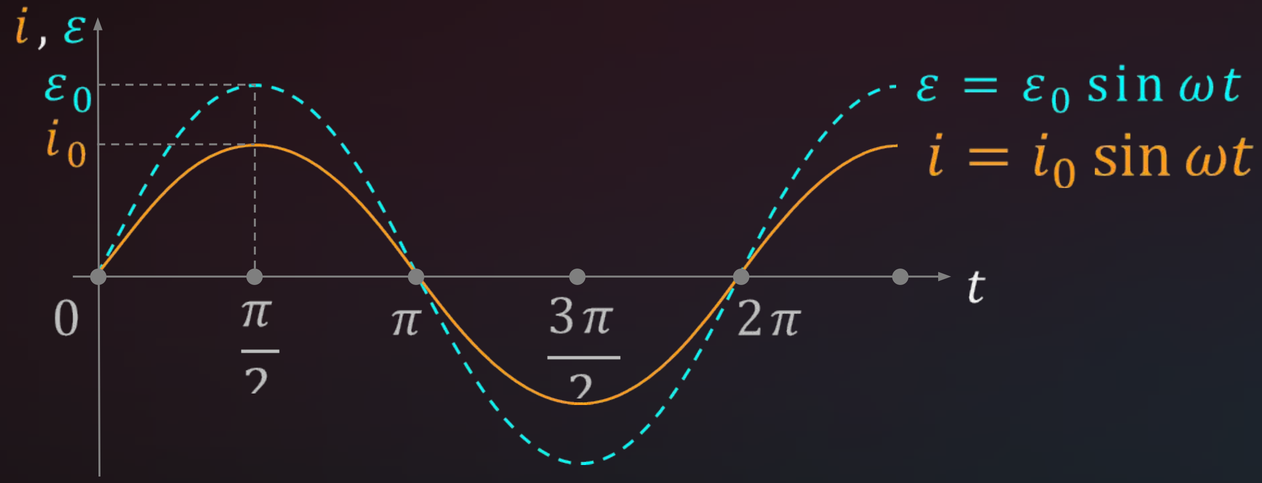
$\varepsilon = \varepsilon_0 \sin \omega t$

$i = i_0 \sin \omega t$

Phasor diagram



Wave diagram

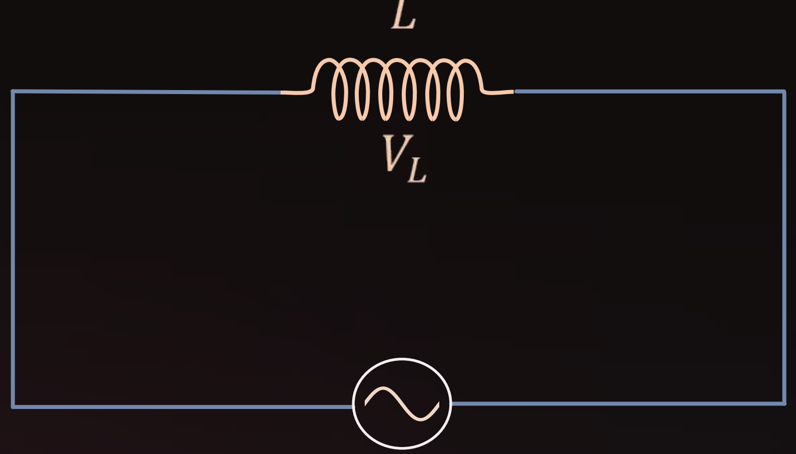


Circuit

Potential drop across inductance,

$$V_L = L\frac{di}{dt}$$

Apply KVL ;



$$\varepsilon = \varepsilon_0 \sin \omega t$$

Circuit

Potential drop across inductance,

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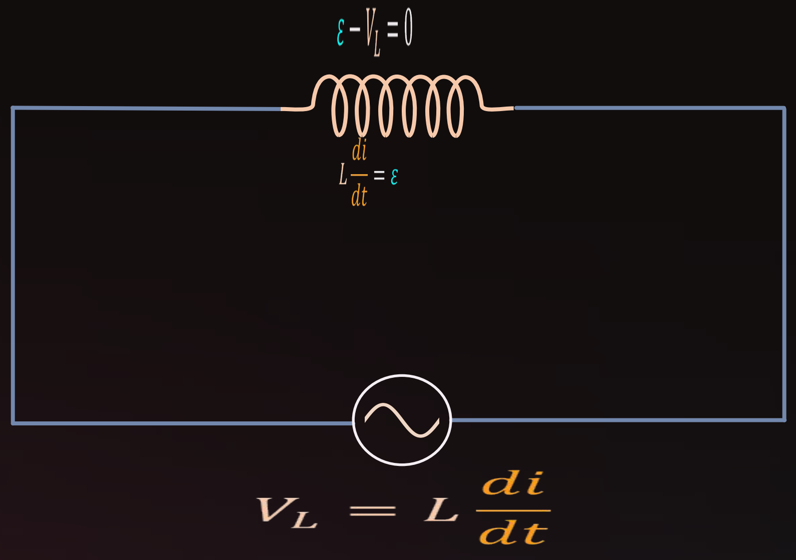


$$\varepsilon - V_L = 0$$

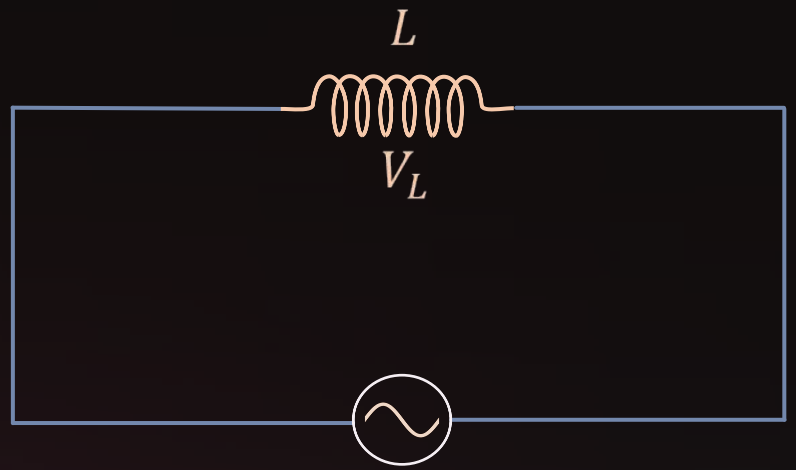
$$L\frac{di}{dt} = \varepsilon \quad \Rightarrow \quad di = \frac{\varepsilon}{L}dt$$

$$i = \int di = \int \frac{\varepsilon_0 \sin \omega t}{L} dt$$

$$i = \frac{\varepsilon_0}{L}\frac{(-\cos \omega t)}{\omega} = -\frac{\varepsilon_0}{L\omega} \cos \omega t$$

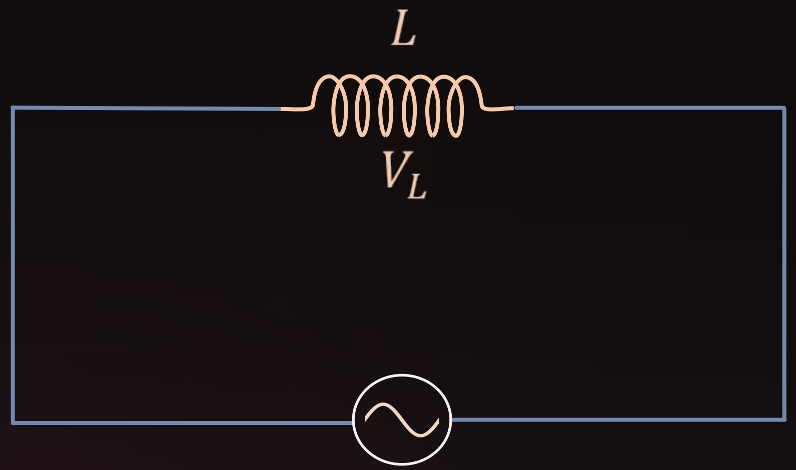
Circuit

$$i = \frac{\varepsilon_0}{L}\frac{(-\cos\omega t)}{\omega} = -\frac{\varepsilon_0}{L\omega}\cos\omega t$$



$\varepsilon = \varepsilon_0 \sin\omega t$

Circuit



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = \frac{\varepsilon_0}{L} \frac{(-\cos \omega t)}{\omega} = -\frac{\varepsilon_0}{L\omega} \cos \omega t$$

$$i = -\frac{\varepsilon_0}{L\omega} \sin\left(\frac{\pi}{2} - \omega t\right) = \frac{\varepsilon_0}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right)$$

i_0

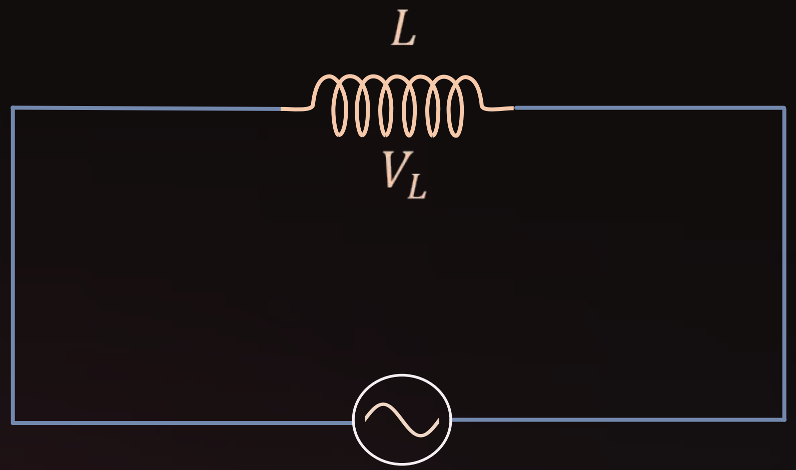
$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_0 = \frac{\varepsilon_0}{L\omega} = \frac{\varepsilon_0}{X_L}$$

$X_L = L\omega$ Inductive reactance

SI Unit : Ohm (Ω)

Circuit



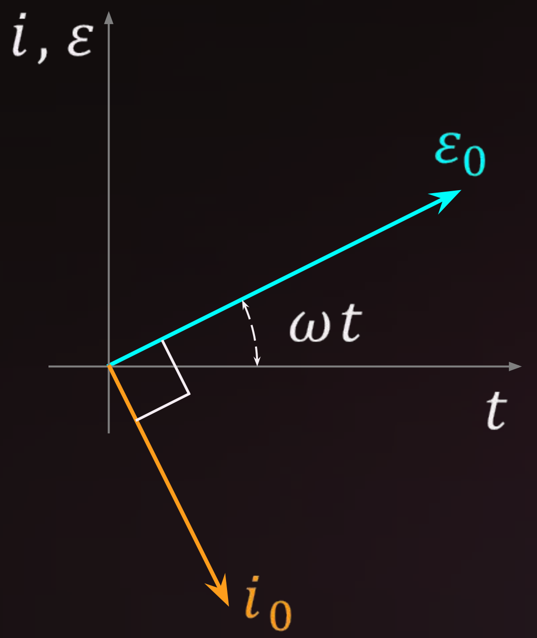
$$\text{Phase difference } (\phi) = \left(\omega t - \frac{\pi}{2}\right) - \omega t = -\frac{\pi}{2}$$

Current lags potential by $90°$

$$\varepsilon = \varepsilon_0 \sin \omega t$$

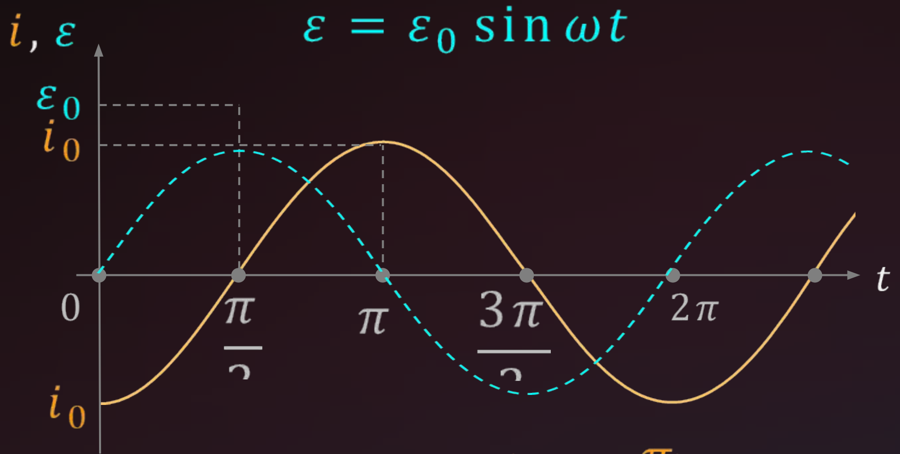
$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor diagram



Wave diagram

$$\varepsilon = \varepsilon_0 \sin \omega t$$

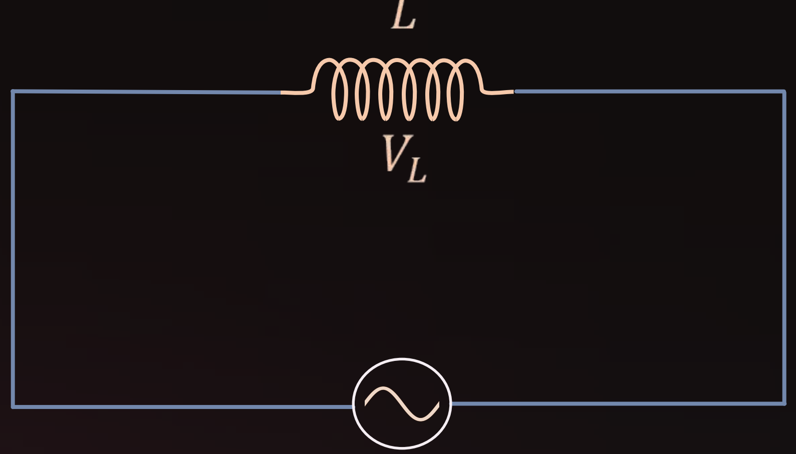


$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Circuit

X_L v/s frequency (f)

$X_L = L\omega$



$\varepsilon = \varepsilon_0 \sin \omega t$

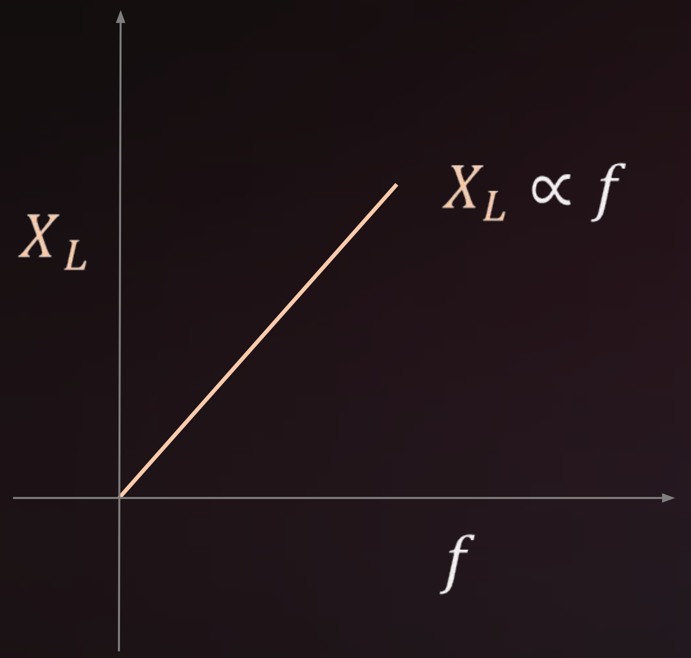
$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

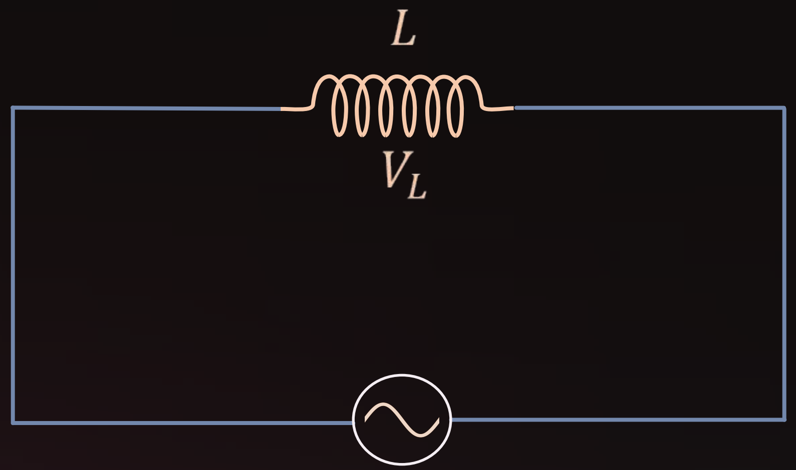
Circuit

X_L v/s frequency (f)

$$X_L = L\omega$$

$$X_L = L \times 2\pi f \quad (\because \omega = 2\pi f)$$





$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Question

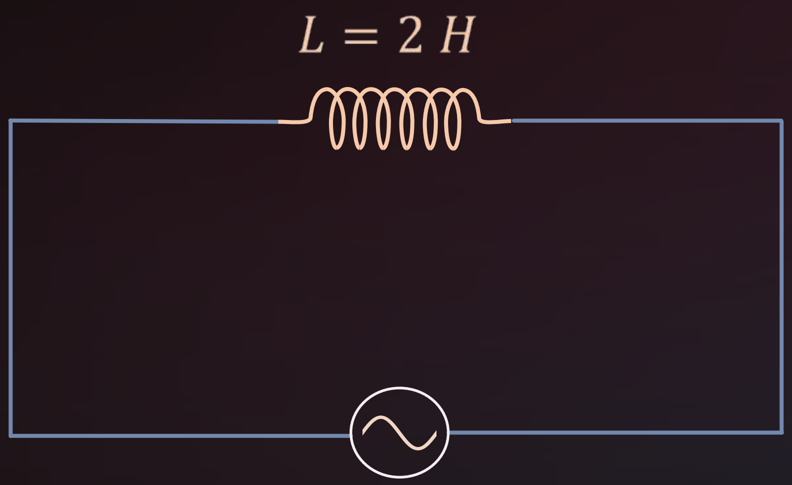
Find i_{avg} and i_{rms} of given circuit.

a $i_{avg} = 0$, $i_{rms} = 0$

b $i_{avg} = \frac{1}{\pi} A$, $i_{rms} = \frac{0.5}{\sqrt{2}} A$

c $i_{avg} = 0$, $i_{rms} = \frac{0.5}{\sqrt{2}} A$

d $i_{avg} = \frac{2}{\pi} A$, $i_{rms} = 0$



$$\varepsilon = 10\sin(10t + 30°)$$

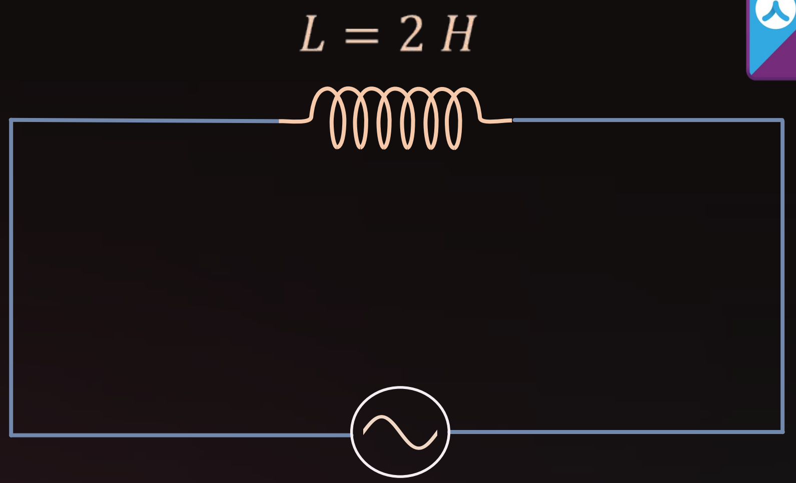
SUMMARY

$$\varepsilon = \varepsilon_0 \sin(\omega t + \phi)$$

$$\varepsilon_0 = 10\ V$$

$$\omega = 10\ s^{-1}$$

$$i_0 = \frac{\varepsilon_0}{X_L}$$



$$\varepsilon = 10 \sin(10t + 30°)$$

Y

$$\varepsilon = \varepsilon_0 \sin(\omega t + \phi)$$

$$\varepsilon_0 = 10\ V$$

$$\omega = 10\ s^{-1}$$

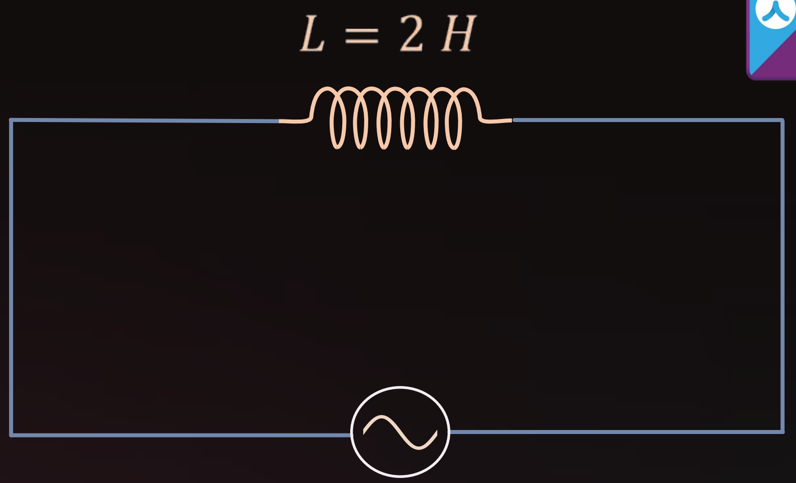
$$i_0 = \frac{\varepsilon_0}{X_L}$$

$$i_0 = \frac{\varepsilon_0}{L\omega}$$

$$i_0 = \frac{10}{2 \times 10} = 0.5\ A$$

$i_{avg} = 0$ (For full cycle of AC $i_{av} = 0$)

$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{0.5}{\sqrt{2}}\ A$$



$$\varepsilon = 10 \sin(10t + 30°)$$

ANSWER

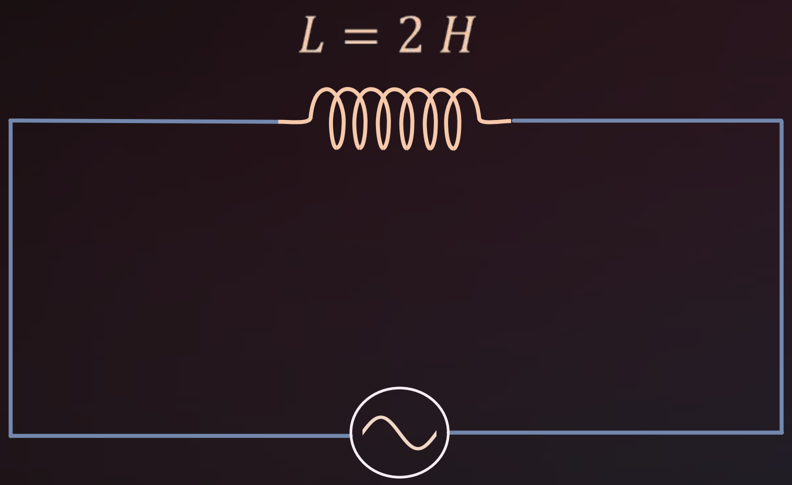
Find i_{avg} and i_{rms} of given circuit.

a
$i_{avg} = 0$
$i_{rms} = 0$

b
$i_{avg} = \frac{1}{\pi} A$
$i_{rms} = \frac{0.5}{\sqrt{2}} A$

c
$i_{avg} = 0$
$i_{rms} = \frac{0.5}{\sqrt{2}} A$

d
$i_{avg} = \frac{2}{\pi} A$
$i_{rms} = 0$

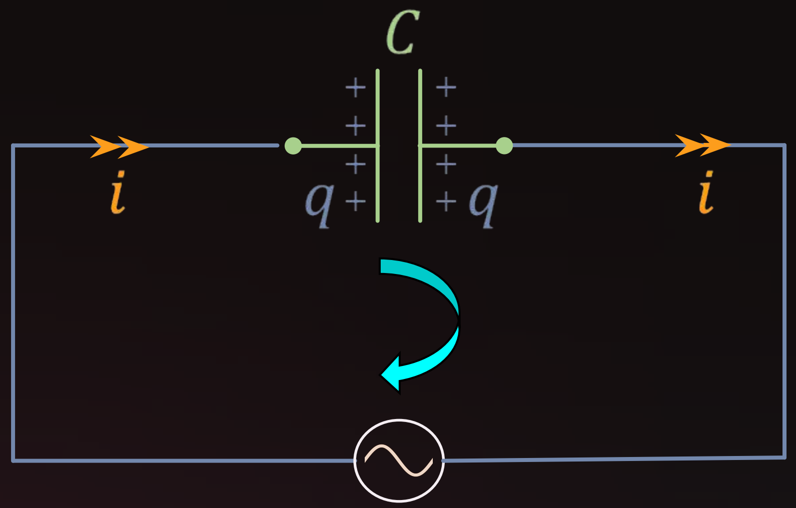


$\varepsilon = 10 \sin(10t + 30°)$

PURE CAPACITIVE AC Circuit

Potential drop across capacitance, $V_C = \frac{q}{C}$

Apply KVL ;

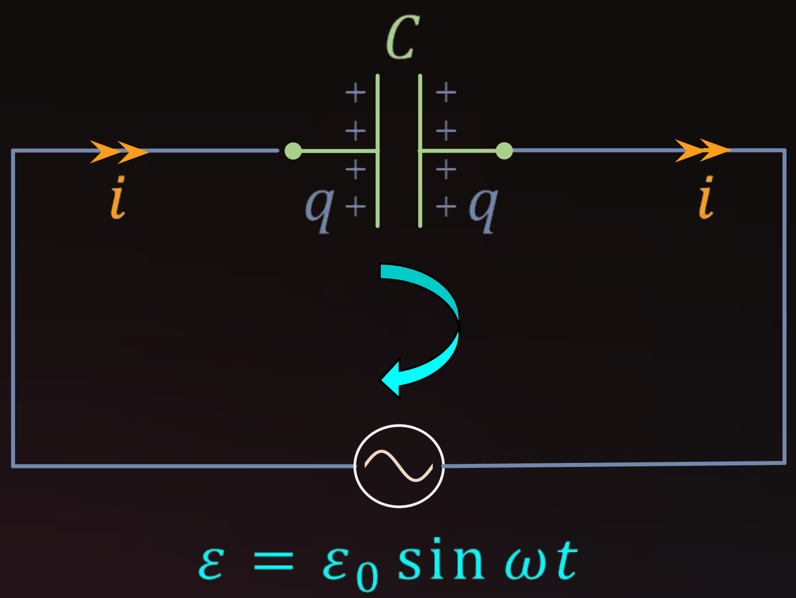


$\varepsilon = \varepsilon_0 \sin \omega t$

PURE CAPACITIVE AC Circuit

Potential drop across capacitance, $V_C = \frac{q}{C}$

Apply KVL ;



$$\varepsilon - V_C = 0$$

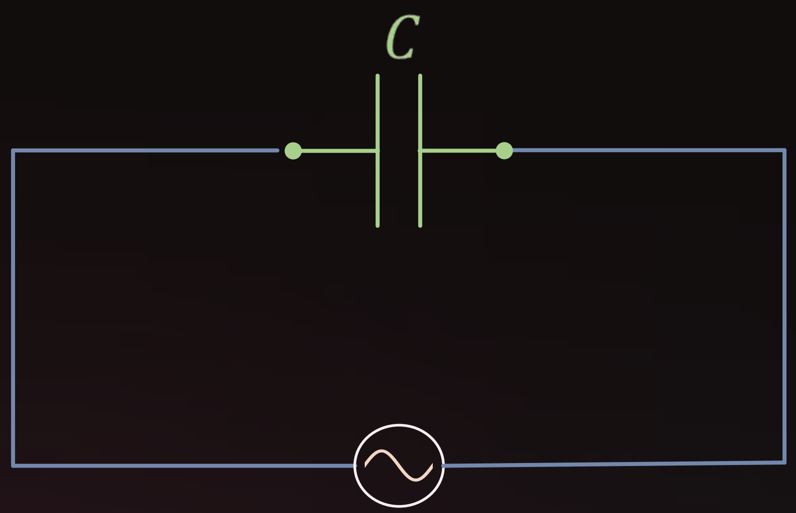
$$\frac{q}{C} = \varepsilon \quad \Rightarrow \quad q = C\varepsilon_0 \sin \omega t$$

$$i = \frac{dq}{dt} = \frac{d(C\varepsilon_0 \sin \omega t)}{dt}$$

$$i = C\omega\varepsilon_0 \cos \omega t$$

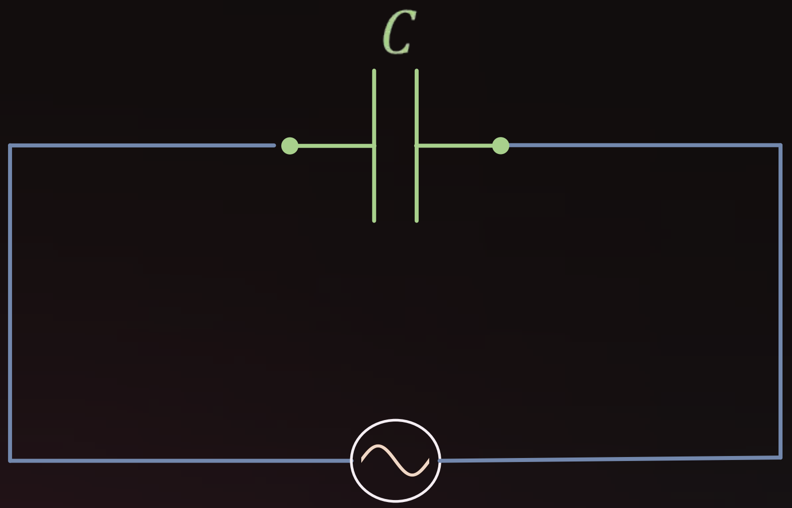
PURE CAPACITIVE AC Circuit

$$i = C\omega\varepsilon_0 \cos \omega t$$



$$\varepsilon = \varepsilon_0 \sin \omega t$$

PURE CAPACITIVE AC Circuit



$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = C\omega\varepsilon_0 \cos \omega t$$

$$i = \frac{\varepsilon_0}{\frac{1}{C\omega}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

i_0

$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

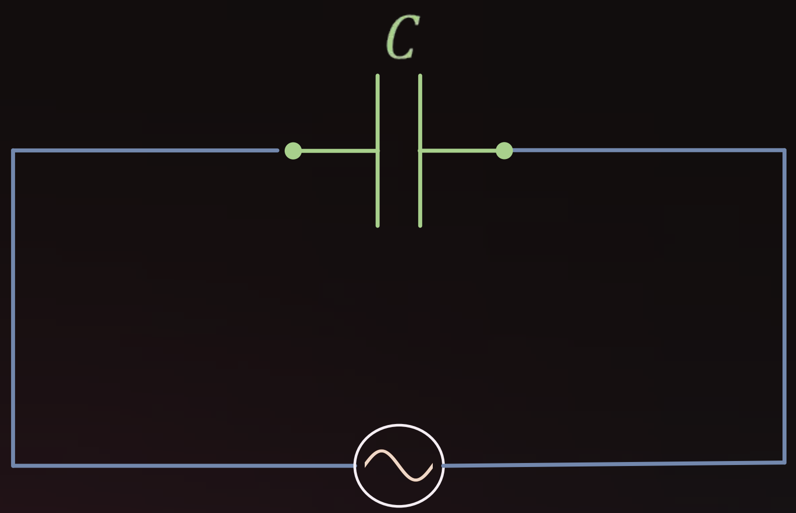
$$i_0 = \frac{\varepsilon_0}{\frac{1}{C\omega}} = \frac{\varepsilon_0}{X_C}$$

$$X_C = \frac{1}{C\omega}$$

Capacitive reactance

SI Unit : Ohm Ω

PURE CAPACITIVE AC Circuit



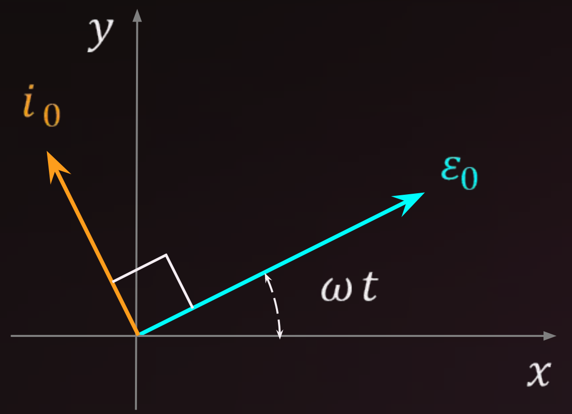
Phase difference $(\phi) = \left(\omega t + \frac{\pi}{2}\right) - \omega t = \frac{\pi}{2}$

Current leads potential by 90°

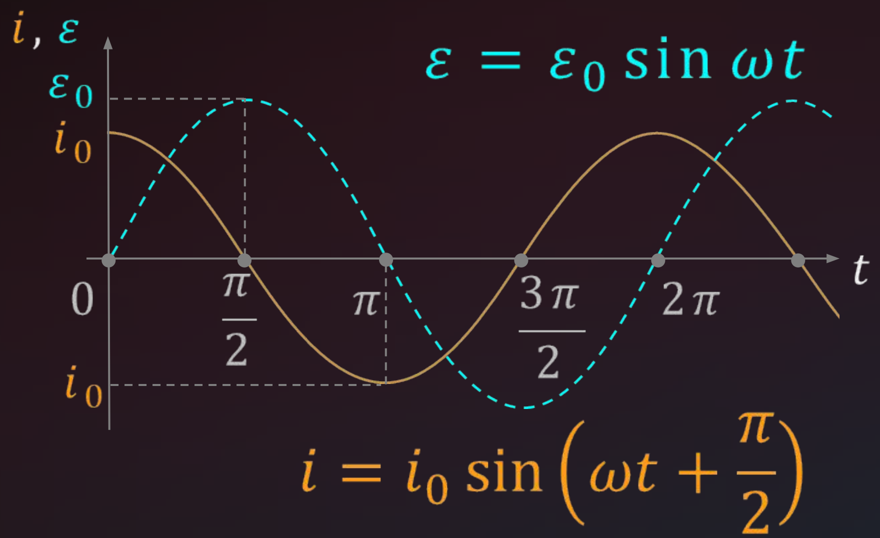
$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Phasor diagram

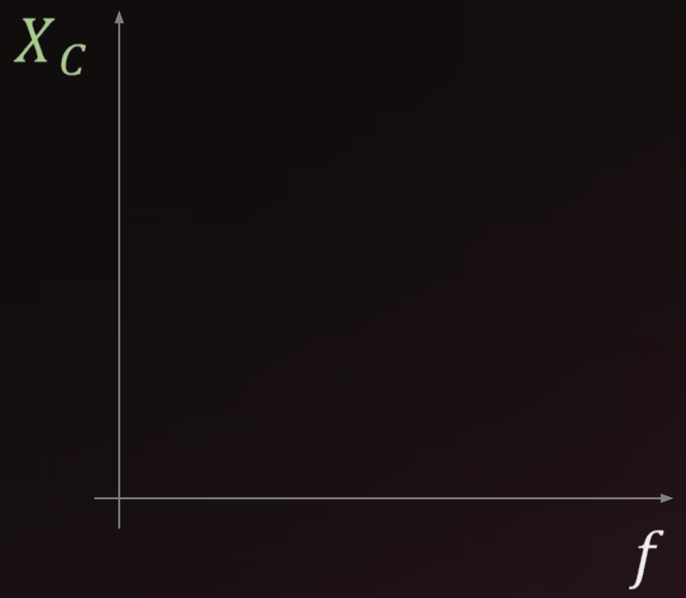


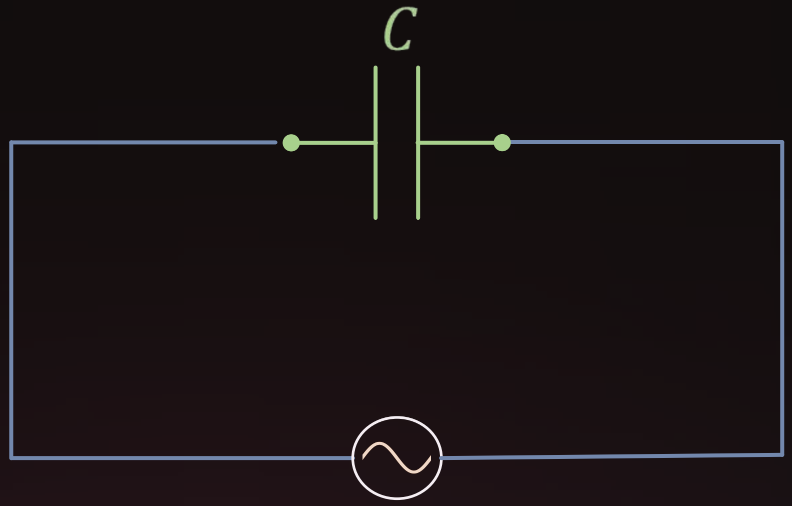
Wave diagram



PURE CAPACITIVE AC Circuit

X_C v/s frequency (f)





$$\varepsilon = \varepsilon_0 \sin \omega t$$

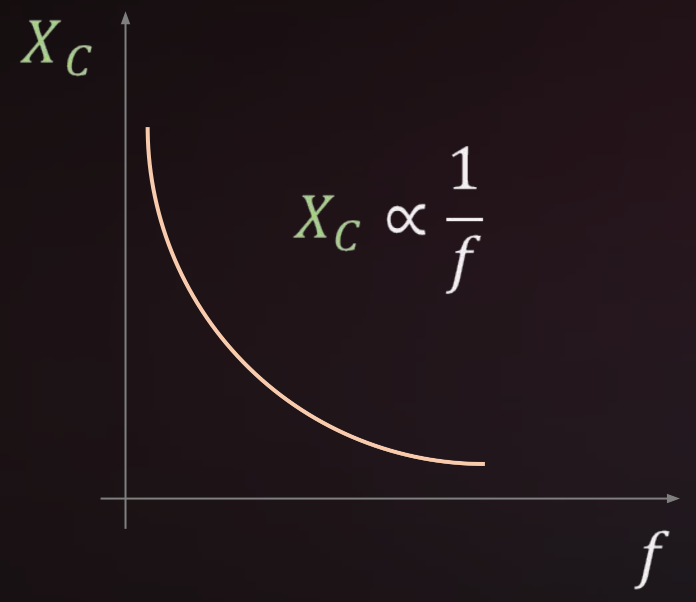
$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

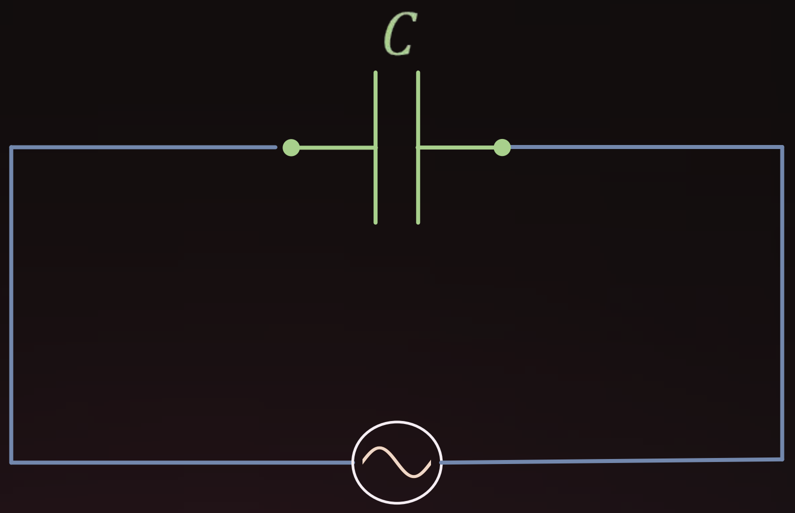
PURE CAPACITIVE AC Circuit

X_C v/s frequency (f)

$$X_C = \frac{1}{C\omega} \quad (\because \omega = 2\pi f)$$

$$X_C = \frac{1}{C \times 2\pi f}$$





$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

CIRCUITS

Element	Current (emf: $\varepsilon = \varepsilon_0 \sin \omega t$)	ϕ	X (Reactance)
Resistor	$\frac{\varepsilon_0}{R} \sin \omega t$	0	R
Inductor	$\frac{\varepsilon_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$	$-\frac{\pi}{2}$	ωL
Capacitor	$\frac{\varepsilon_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$	$\frac{\pi}{2}$	$\frac{1}{\omega C}$

Question

Current in a pure capacitive circuit of $C = 5\ \mu F$ is $5\sin(50t + 30°)$. Find the equation for emf.

a) $2 \times 10^6 \sin(50t)$

b) $2 \times 10^4 \sin(50t - 60°)$

c) $2 \times 10^4 \sin(50t + 90°)$

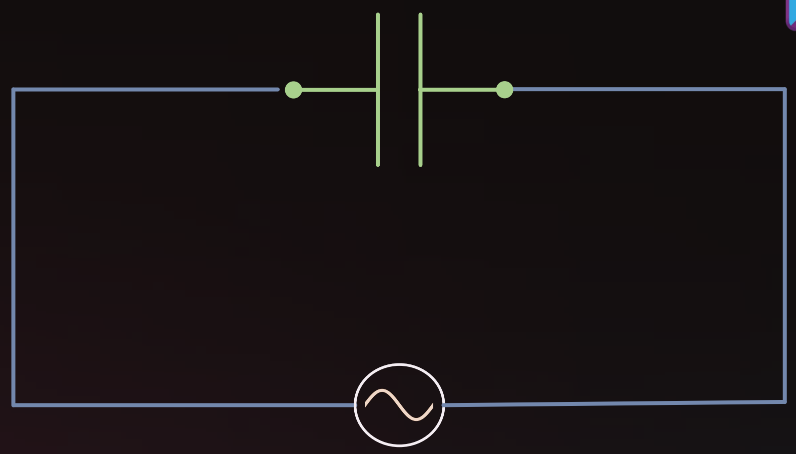
d) $2 \times 10^4 \sin(50t)$

Y

$i = i_0 \sin(\omega t + \phi)$

$i_0 = 5\ A$

$\omega = 50\ s^{-1}$



$i = 5 \sin(50t + 30°)$

Y

$$i = i_0 \sin(\omega t + \phi)$$

$$i_0 = 5\ A$$

$$\omega = 50\ s^{-1}$$

$$\varepsilon_0 = i_0 X_C = i_0 \frac{1}{\omega C}$$

$$\varepsilon_0 = 5 \frac{1}{50 \times 5 \times 10^{-6}}$$

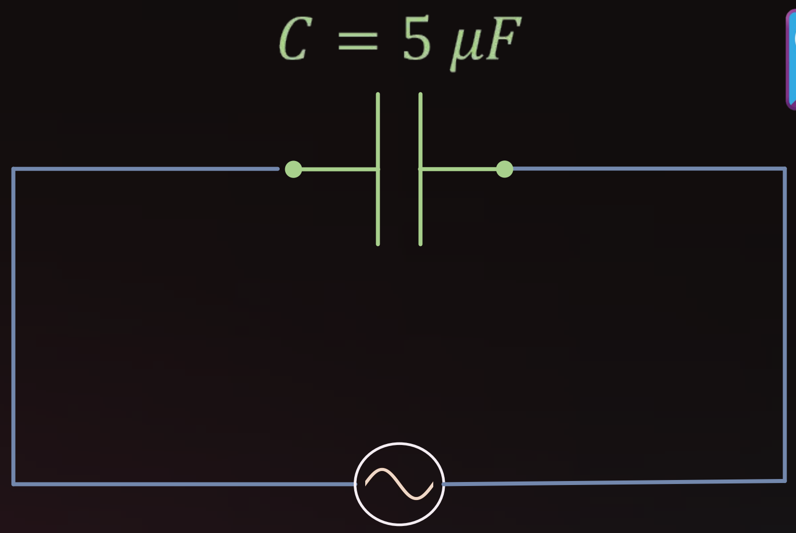
$$\varepsilon_0 = 2 \times 10^4\ V$$

Current leads potential by 90°

$$\varepsilon = \varepsilon_0 \sin(\omega t + \phi - 90°)$$

$$\varepsilon = 2 \times 10^4 \sin(50t + 30 - 90°)$$

$$\varepsilon = 2 \times 10^4 \sin(50t - 60°)$$



$$i = 5 \sin(50t + 30°)$$

ANSWER

Current in a pure capacitive circuit of $C = 5\ \mu F$ is $5\sin(50t + 30°)$. Find the equation for emf.

(a) $2 \times 10^6 \sin(50t)$

(b) $2 \times 10^4 \sin(50t - 60°)$

(c) $2 \times 10^4 \sin(50t + 90°)$

(d) $2 \times 10^4 \sin(50t)$