JEE Main 2021 26Feb Shift 1

1. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

(1) 77

(2) 42

(3) 35

(4) 82

Ans. **(1)**

Sol. CASE-I: 1, 1, 1, 1, 1, 2, 3

Number of ways $=\frac{7!}{5!}=42$

CASE-II: 1, 1, 1, 1, 2, 2, 2

Number of ways $=\frac{7!}{4!\cdot 3!}=35$

Total number of ways $=42+35=77$

2. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x\in(0,1)$ is :

(1) $\frac{10!}{\sqrt{3}(5!)^2}$

(2) $\frac{2.10!}{3(5!)^2}$

(3) $\frac{10!}{3(5!)^2}$

(4) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

Ans. **(4)**

Sol. $T_{r+1}={}^{10}C_r(tx^{1/5})^{10-r}\left[\frac{(1-x)^{1/10}}{t}\right]^r$

$={}^{10}C_r\, t^{(10-2r)}\times x^{\frac{10-r}{5}}\times(1-x)^{\frac{r}{10}}$

$\Rightarrow 10-2r=0$

$\Rightarrow r=5$

$T_6={}^{10}C_5\, x\sqrt{1-x}$

$\frac{dT_6}{dx}={}^{10}C_5\left[\sqrt{1-x}-\frac{x}{2\sqrt{1-x}}\right]=0$

$\Rightarrow 1 - x = \frac{x}{2}$

$\Rightarrow x = \frac{2}{3}$

$\therefore \max(T_6) = \frac{10!}{5!5!} \times \frac{2}{3\sqrt{3}}$

3. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\, dx$, where $[x]$ is the greatest integer $\leq x$, is :

(1) $100\,(e-1)$

(2) $100e$

(3) $100(1-e)$

(4) $100(1+e)$

Ans. **(1)**

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\, dx$

$= \int_0^1 e^{\{x\}}dx + \int_1^2 e^{\{x\}}dx + \int_2^3 e^{\{x\}}dx + \cdots\cdots + \int_{99}^{100} e^{\{x\}}dx \quad (\because \{x\} = x - [x])$

$= e^x|_0^1 + e^{(x-1)}|_1^2 + e^{(x-2)}|_2^3 + \cdots\cdots + e^{(x-99)}|_{99}^{100}$

$= (e-1) + (e-1) + (e-1) + \cdots\cdots + (e-1)$

$= 100(e-1)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

(1) 4 (2) 2 (3) 16 (4) 8

Ans. **(1)**

Sol. Let x be the number of bacteria at time t.

$\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = \lambda x$

$\Rightarrow \int_{1000}^{x} \frac{dx}{x} = \int_0^t \lambda\, dt$

$\Rightarrow \ln x - \ln 1000 = \lambda t$

$\Rightarrow \ln\left(\frac{x}{1000}\right) = \lambda t$

Put $t = 2,\ x = 1200$

$\ln\left(\frac{12}{10}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2}\ln\frac{6}{5}$

Now, $\ln\left(\frac{x}{1000}\right) = \frac{t}{2}\ln\left(\frac{6}{5}\right)$

$\Rightarrow x = 1000e^{\frac{t}{2}\ln\left(\frac{6}{5}\right)}$

Given, $x = 2000$ at $t = \frac{k}{\log_e\left(\frac{6}{5}\right)}$

$\Rightarrow 2000 = 1000\, e^{\frac{k}{2\ln(6/5)} \times \ln(6/5)}$

$\Rightarrow 2 = e^{k/2} \quad \Rightarrow \ln 2 = \frac{k}{2}$

$\Rightarrow \frac{k}{\ln 2} = 2$
$\Rightarrow \left(\frac{k}{\ln 2}\right)^2 = 4$

5. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to :

(1) $\frac{1}{2}|\vec{a}|^4\vec{b}$

(2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^4\vec{b}$

(4) $\vec{0}$

Ans. **(3)**

Sol. $\vec{a} \times \left(\vec{a} \times \left((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}\right)\right)$

$= \vec{a} \times \left(-|\vec{a}|^2(\vec{a} \times \vec{b})\right)$

$= -|\vec{a}|^2((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2\vec{b}))$

$= -(\vec{a} \cdot \vec{b})\vec{a}|\vec{a}|^2 + |\vec{a}|^4\vec{b}$

$= |\vec{a}|^4\vec{b} \qquad (\because \vec{a} \cdot \vec{b} = 0)$

6. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 35
(2) 30
(3) 26
(4) 32

Ans **(1)**
Sol. Let a be the first term and r be the common ratio.
$ar + ar^5 = \frac{25}{2}$
and $ar^2 \times ar^4 = 25$
$\Rightarrow a^2r^6 = 25$
$\Rightarrow ar^3 = 5$
$\boxed{\Rightarrow a = \frac{5}{r^3}}$ ...(1)

$\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2}$

$\Rightarrow \frac{1}{r^2} + r^2 = \frac{5}{2}$

Put $r^2 = t$

$\frac{t^2+1}{t} = \frac{5}{2}$

$\Rightarrow 2t^2 - 5t + 2 = 0$

$\Rightarrow 2t^2 - 4t - t + 2 = 0$

$\Rightarrow (2t - 1)(t - 2) = 0$

$\Rightarrow t = \frac{1}{2}, 2 \;\; \Rightarrow \boxed{r^2 = \frac{1}{2}, 2}$

$\Rightarrow \boxed{r = \sqrt{2}}$ as the G.P. is increasing.

$ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5[1 + 2 + 4] = 35$

7. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_3 are parallel.

(2) P_2 and P_3 are parallel.

(3) P_1 and P_2 are parallel.

(4) P_1, P_2 and P_3 all are parallel.

Ans. **(1)**

Sol. $P_1 = x + 5y + 7z = 3$

$P_2 = x - 3y - z = 5$

$P_3 = x + 5y + 7z = 5/2$

$\Rightarrow P_1 || P_3$

8. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to :

(1) $\frac{9}{4}$

(2) $\frac{15}{4}$

(3) $\frac{13}{4}$

(4) $\frac{11}{4}$

Ans. **(3)**

Sol. $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$

$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \ldots\ldots \infty$

$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots\ldots \infty$

$\Rightarrow \frac{2S}{3} = \frac{4}{3} + \frac{5}{3}\left\{\frac{1/3}{1-\frac{1}{3}}\right\} = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$

$\Rightarrow \boxed{s = \frac{13}{4}}$

9. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is :

(1) -2

(2) (a+1) (a+2) (a+3)

(3) 0

(4) (a+2) (a+3) (a+4)

Ans. **(1)**

Sol. $C_1 \to C_1 - C_2$,

$$\Delta = \begin{vmatrix} (a+2)a & a+2 & 1 \\ (a+3)(a+1) & a+3 & 1 \\ (a+4)(a+2) & a+4 & 1 \end{vmatrix}$$

$C_2 \to C_2 - C_3$,

$$\Delta = \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$\Delta = \begin{vmatrix} a^2+2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$$

$= 6 - 8 = -2$

10. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

(1) $\frac{1-y^2}{2y}$

(2) $\frac{1-y^2}{1+y^2}$

(3) $1 - y^2$

(4) $\frac{1-y^2}{y\sqrt{y}}$

Ans. **(2)**

Sol. $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$

$\Rightarrow \frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\sin^{-1}x+\cos^{-1}x}{a+b}=\frac{\pi}{2(a+b)}$

Now, $\frac{\tan^{-1}y}{c}=\frac{\pi}{2(a+b)}$

$\Rightarrow 2\tan^{-1}y=\frac{\pi c}{a+b}$

$\therefore \cos\left(\frac{\pi c}{a+b}\right)=\cos\ (2\tan^{-1}y)=\frac{1-y^2}{1+y^2}$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

(1) 6
(2) 1
(3) 4
(4)12

Ans. **(3)**

Sol. Let $A=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$A^2=\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}=\begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & c^2+b^2 \end{bmatrix}$

$\Rightarrow a^2 + 2b^2 + c^2 = 1$

a = 1, b = 0, c = 0

a = 0, b = 0, c = 1

a = −1, b = 0, c = 0

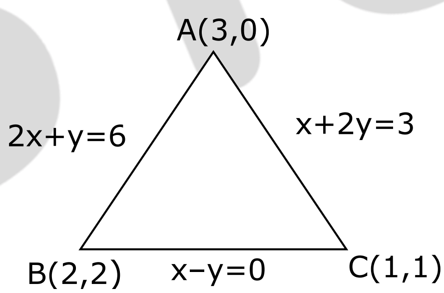
c = −1, b = 0, a = 0

12. The intersection of three lines $x-y=0,\ x+2y=3$ and $2x+y=6$ is a :

(1) Equilateral triangle
(2) Right angled triangle
(3) Isosceles triangle
(4) None of the above

Ans. **(3)**

Sol.



$AB=AC=\sqrt{5},\ BC=\sqrt{2}$

Hence, the triangle is isosceles.

13. The maximum slope of the curve y = $\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point :

(1) (2, 9)
(2) (2, 2)
(3) $\left(3,\frac{21}{2}\right)$
(4) (0, 0)

Ans. **(2)**

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$\Rightarrow \boxed{x = 2, 3}$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $\boxed{x = 2}$

So, at $\boxed{x = 2}$, slope is maximum.

$y = 8 - 40 + 72 - 38$

$= 72 - 70 = 2$

Maximum slope occurs at (2, 2)

14. Let f be any function defined on **R** and let it satisfy the condition :

$|f(x) - f(y)| \le |(x-y)^2|, \ \forall\, x, y \in \mathbf{R}$

If $f(0) = 1$, then :

(1) $f(x) < 0, \forall\, x \in \mathbf{R}$

(2) $f(x)$ can take any value in **R**

(3) $f(x) = 0, \forall\, x \in \mathbf{R}$

(4) $f(x) > 0, \forall\, x \in \mathbf{R}$

Ans. (4)

Sol. $|f(x) - f(y)| \le |(x-y)^2|, \ \forall\, x, y \in \mathbf{R}$

$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$

$\Rightarrow \lim_{x \to y} \left|\frac{f(x)-f(y)}{x-y}\right| \le 0$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = C$

Since $f(0) = 1 \Rightarrow f(y) = 1 \ \forall\, y \in \mathbf{R}$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3^x} dx$ is :

(1) 2π

(2) 4π

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}$

Ans. **(4)**

Sol. Let I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3^x} dx$

$\Rightarrow$ I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3^{-x}} dx$

$\Rightarrow$ I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{3^x \cos^2 x}{1+3^x} dx$

2I = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x \, dx$

I = $\int_0^{\pi/2} \cos^2 x \, dx = \frac{\pi}{4}$

16. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}\right\}$ is :

(1) $\frac{3}{4}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

Ans. (3)

Sol. $\lim_{h\to 0} 2\left\{\frac{2\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{2\sqrt{3}h\left(\cos\left(h+\frac{\pi}{6}\right)\right)}\right\}$

$= \lim_{h\to 0} 2\left\{\frac{2\sin(h)}{2\sqrt{3}h\left(\cos\left(\frac{\pi}{6}\right)\right)}\right\}$

$= 2\times\frac{2}{2\sqrt{3}}\times\frac{2}{\sqrt{3}} = \frac{4}{3}$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{12}}$ (2) $\frac{15}{2^{13}}$

(3) $\frac{15}{2^{14}}$ (4) $\frac{15}{2^{8}}$

Ans. (2)

Sol. $P(x=9) = P(x=7)$

$\Rightarrow {}^{n}C_9\left(\frac{1}{2}\right)^{n-9}\times\left(\frac{1}{2}\right)^{9} = {}^{n}C_7\left(\frac{1}{2}\right)^{n-7}\times\left(\frac{1}{2}\right)^{7}$

$\Rightarrow {}^{n}C_9\left(\frac{1}{2}\right)^{n} = \left(\frac{1}{2}\right)^{n}\times{}^{n}C_7$

$\Rightarrow n = 9+7 = 16$

$P(x=2) = {}^{16}C_2\times\left(\frac{1}{2}\right)^{14}\times\left(\frac{1}{2}\right)^{2}$

$= {}^{16}C_2\times\left(\frac{1}{2}\right)^{16} = \frac{15}{2^{13}}$

18. If $(1,5,35)$, $(7,5,5)$, $(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is :

(1) $-\frac{44}{5}$ (2) $\frac{39}{5}$ (3) $-\frac{39}{5}$ (4) $\frac{44}{5}$

Ans. **(4)**

Sol. Let $P(1,5,35)$, $Q(7,5,5)$, $R(1,\lambda,7)$, $S(2\lambda,1,2)$

$\left[\vec{PQ}\ \vec{PR}\ \vec{PS}\right] = 0$

$\Rightarrow \begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$

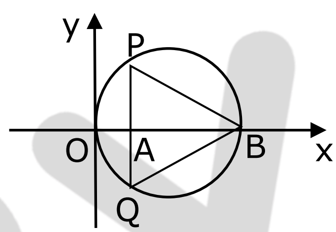
$\Rightarrow \begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$

$\Rightarrow \{-33\lambda + 165 - 112\} + 5(\lambda - 5)(2\lambda - 1) = 0$
$\Rightarrow 53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$
$\Rightarrow 10\lambda^2 - 88\lambda + 78 = 0$
$5\lambda^2 - 44\lambda + 39 = 0 <^{\lambda_1}_{\lambda_2}$
$\therefore \lambda_1 + \lambda_2 = 44/5$

19. Let R = {P,Q)|P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :
(1) S = $\{(x, y)|x^2 + y^2 = 1\}$
(2) S = $\{(x, y)|x^2 + y^2 = 4\}$
(3) S = $\{(x, y)|x^2 + y^2 = \sqrt{2}\}$
(4) S = $\{(x, y)|x^2 + y^2 = 2\}$

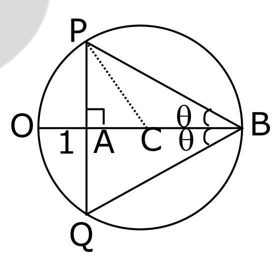
Ans. **(4)**
Sol. P(a, b) , Q(c, d), OP = OQ
$\Rightarrow a^2 + b^2 = c^2 + d^2$
R(x ,y), S = (1, –1) $\Rightarrow$ OR = OS (∵ equivalence class)
$\Rightarrow x^2 + y^2 = 2$

20. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ ⊥OB. Then, the area of the triangle PQB (in square units) is :



(1) $26\sqrt{3}$
(2) $24\sqrt{2}$
(3) $24\sqrt{3}$
(4) $26\sqrt{2}$

Ans. **(3)**
Sol.



OC = $\frac{13}{2} = 6.5$
$AC = CO - AO$
$= 6.5 - 1$
$= 5.5$
In ΔPAC
PA = $\sqrt{6.5^2 - 5.5^2}$

PA = $\sqrt{12}$

$\Rightarrow$ PQ = $2PA = 2\sqrt{12}$

Now, area of $\Delta PQB = \frac{1}{2} \times PQ \times AB$

$= \frac{1}{2} \times 2\sqrt{12} \times 12$

$= 12\sqrt{12}$

$= 24\sqrt{3}$

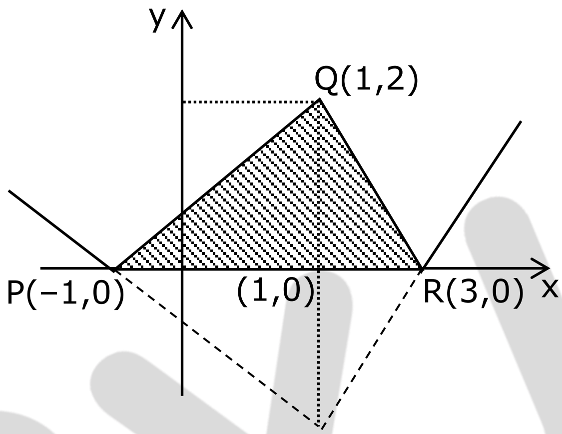
Section-B

1. The area bounded by the lines y = $||x-1|-2|$ is _____.

Ans. **(4)**

NTA Ans. (8)

Sol.



Required area = area of ΔPQR

Area = $\frac{1}{2} \times 4 \times 2 = 4$

This is a **bonus** question as the second curve is also not given.

If the second curve is x −axis, then answer will be 4.

2. The number of integral values of 'k' for which the equation 3sin x + 4cos x = k + 1 has a solution, $k \in \mathbf{R}$ is _____.

Ans. **(11)**

Sol. 3 sin x + 4cos x = k + 1

$\Rightarrow -5 \le k+1 \le 5$

$\Rightarrow -6 \le k \le 4$

$\boxed{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4} \Rightarrow$ 11 integral values

3. Let $m, n \in \mathbf{N}$ and gcd (2, n) = 1. If $30\binom{30}{0} + 29\binom{30}{1} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$, then $n + m$ is equal to _____.

$\left(\text{Here, } \binom{n}{k} = {}^nC_k\right)$

Ans. **(45)**

Sol. Let $S = \sum_{r=0}^{30} (30-r)\ ^{30}C_r$

$= 30\sum_{r=0}^{30}\ ^{30}C_r - \sum_{r=0}^{30} r\ ^{30}C_r$

$= 30 \times 2^{30} - \sum_{r=1}^{30} r.\frac{30}{r}.\ ^{29}C_{r-1}$

$= 30 \times 2^{30} - 30.2^{29}$

$= 30 \cdot 2^{29}(2-1)$

$= 15 \cdot 2^{30}$

$\therefore n = 15$ and $m = 30$

$n + m = 45$

4. If $y = y(x)$ is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$, $y(0) = 0$; then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to _____.

Ans. **(1)**

Sol. $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$

Put $e^{\sin y} = t$

$e^{\sin y} \times \cos y\frac{dy}{dx} = \frac{dt}{dx}$

Then, $\frac{dt}{dx} + t\cos x = \cos x$

I.F. = $e^{\int \cos x\, dx} = e^{\sin x}$

Solution of differential equation :

$t.e^{\sin x} = \int e^{\sin x}.\cos x\, dx$

$\Rightarrow e^{\sin y}.e^{\sin x} = e^{\sin x} + C$

At $x = 0, y = 0$

$\Rightarrow 1 = 1 + C \quad \Rightarrow C = 0$

$\sin y + \sin x = \sin x$

$\Rightarrow y = 0$

$\Rightarrow y\left(\frac{\pi}{6}\right) = 0, \quad y\left(\frac{\pi}{3}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 0$

Required answer is $1 + 0 + 0 + 0 = 1$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____.

Ans. **(1)**

Sol. $\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow x - 1 = (x-3)^2$

$\Rightarrow x^2 - 6x + 9 = x - 1$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow x = 2, 5$

$x = 2$ Not possible as $\log_2(x-3)$ is not defined.

$\therefore$ Number of solution $= 1$

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ______.

Ans. **(1)**

Sol. $\sqrt{3}t^2 - (\sqrt{3}-1)t - 1 = 0$, where t = cos x

Now, $t = \frac{(\sqrt{3}-1)\pm\sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$

t = cos x = 1 or $-\frac{1}{\sqrt{3}} \rightarrow$ rejected as $x \in \left[0, \frac{\pi}{2}\right]$

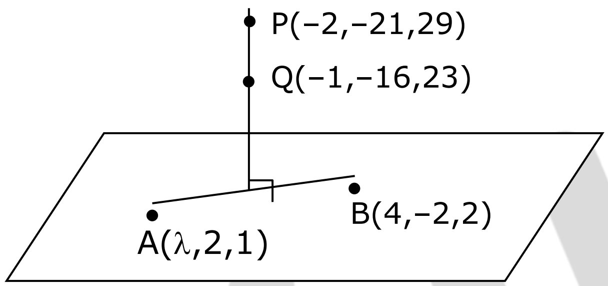
⇒ cos x = 1

⇒ Number of solution = 1

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _____.

Ans. **(8)**

Sol.



$\overrightarrow{AB} \perp \overrightarrow{PQ}$

$\Rightarrow \left[(4-\lambda)\hat{\imath} - 4\hat{\jmath} + \hat{k}\right] \cdot \left[+\hat{\imath} + 5\hat{\jmath} - 6\hat{k}\right] = 0$

$\Rightarrow 4 - \lambda - 20 - 6 = 0$

$\boxed{\lambda = -22}$

Now, $\frac{\lambda}{11} = -2$

$\therefore \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4 = 4 + 8 - 4 = 8$

8. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _____.

Ans. **(2)**

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$

Differentiating w.r.t. x,

$2yy' = a$

$y^2 = 2yy'\left(x + \frac{\sqrt{2yy'}}{2}\right)$

$\Rightarrow y = 2y'\left(x + \frac{\sqrt{yy'}}{\sqrt{2}}\right)$

$\Rightarrow y - 2xy' = \sqrt{2}y'\sqrt{yy'}$

$\Rightarrow \left(y - 2x\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right)^3$
Degree = 3 and Order = 1
Degree – Order = 3 – 1 = 2

9. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____.

Ans. **(3)**
Sol. Let roots of $x^3 - 2x^2 + 2x - 1 = 0$ be α, β, γ
$(x^3 - 1) - (2x^2 - 2x) = 0$
$\Rightarrow (x-1)(x^2 - x + 1) = 0$
$x = \underset{\alpha}{\underset{\downarrow}{1}}, \underset{\beta}{\underset{\downarrow}{-\omega}}, \underset{\gamma}{\underset{\downarrow}{-\omega^2}}$

Now, $\alpha^{162} + \beta^{162} + \gamma^{162}$
$= 1 + (\omega)^{162} + (\omega^2)^{162}$
$= 1 + (\omega^3)^{54} + (\omega^3)^{108} = 3$

10. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is ______.

Ans. **(2)**
Sol. $I = \int_0^{\pi} |\sin 2x| \, dx$
$I = 2\int_0^{\pi/2} |\sin 2x| \, dx = 2\int_0^{\pi/2} \sin 2x \, dx$
$I = 2\left[\frac{-\cos(2x)}{2}\right]_0^{\pi/2} = 2$