Regd. Office: Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

AMTI – NMTC - 2023 Jan. - PRIMARY – FINAL

Instructions:

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagrams and justification should be given wherever necessary.
4. Before answering, fill in the FACE SLIP completely.
5. Your 'rough work' should be in the answer sheet itself.
6. The maximum time allowed is THREE hours.

1. Three skilled workers Akbar, Baskar and Charles are employed by a person to do three different jobs. After completion of the work the total fee the person gave to the three workers is $Rs\ 6000$. It is found that $Rs.\ 400$ more than $\frac{2}{5}$ of Akbar's share, $Rs.200$ more than $\frac{2}{7}$ of Baskar's share and $Rs\ 100$ more than $\frac{9}{17}$ of Charles' share all equal. How much did each get?

Answer (b) Akbar's Share = Rs. 1500, Baskar's Share = Rs. 2800 , Charle's share = Rs. 1700

Sol. Let us Assume that Akbar's share be `A', Baskar's share be 'B' and charle's share be 'C'.

According to the question,

$\frac{2}{5}A + 400 = \frac{2}{7}B + 200 = \frac{9}{17}C + 100 = K$ …(1)

As per the question, Total fee to the three persons $= Rs\ 6000$

So, $A + B + C = 6000$ …(2)

From equation (1), we can write, $\frac{2}{5}A + 400 = k \Rightarrow A = \frac{5}{2}(k - 400)$ …(3)

$\frac{2}{7}B + 200 = k \Rightarrow B = \frac{7}{2}(k - 200)$ …(4)

$\frac{9}{17}C + 100 = k \Rightarrow C = \frac{17}{9}(k - 100)$ …(5)

Substitute (3),(4) & (5) eq in eq (2)

$$\frac{5}{2}(k - 400) + \frac{7}{2}(k - 200) + \frac{17}{9}(k - 100) = 6000$$

$\Rightarrow \frac{45(k-400)+63(k-200)+34(k-100)}{18} = 6000$

$\Rightarrow 45k - 18000 + 63k - 12600 + 34k - 3400 = 6000 \times 18$

$\Rightarrow 142k - 34000 = 108000$

$\Rightarrow 142k = 142000$

$\Rightarrow K = \frac{142000}{142} = 1000$

$\Rightarrow K = 1000$

Akbar's Share $= \frac{5}{2}(k - 400) = \frac{5}{2}(1000 - 400) = \frac{5}{2} \times 600 = Rs.1500$

Baskar's Share $= \frac{7}{2}(k - 200) = \frac{7}{2}(1000 - 200) = \frac{7}{2} \times 800 = Rs.2800$

Charle's Share $= \frac{17}{9}(k - 100) = \frac{17}{9}(1000 - 100) = \frac{17}{9} \times 900 = Rs.1700$

2. A teacher of a primary class asked his students to calculate

$$2\frac{3}{7} \text{ of } \frac{\left(13\frac{1}{2}-9\frac{2}{3}\right)}{\left(15\frac{1}{5}-11\frac{7}{30}\right)} = A$$

The teacher has $49A$ chocolates with him. He distributed equal number of chocolates (more than one chocolate) to each student of his class irrespective of whether the students got the correct answer or not. After the distribution the teacher is left with only one chocolate and he took it. Find the maximum strength of the class.

Answer 57

Sol. Given, $A = 2\frac{3}{7}$ of $\frac{\left(13\frac{1}{2}-9\frac{2}{3}\right)}{\left(15\frac{1}{5}-11\frac{7}{30}\right)}$

$\Rightarrow A = \frac{17}{7} \times \frac{\left(\frac{27}{2}-\frac{29}{3}\right)}{\left(\frac{76}{5}-\frac{337}{30}\right)}$

Taking LCM of 2 and 3 as 6 and 30 and 5 as 30, we get

$\Rightarrow A = \frac{17}{7} \times \frac{\left(\frac{81-58}{6}\right)}{\left(\frac{456-337}{30}\right)}$

$\Rightarrow A = \frac{17}{7} \times \frac{\frac{23}{6}}{\frac{119}{30}} = \frac{17}{7} \times \frac{23}{6} \times \frac{30}{119}$

$\Rightarrow A = \frac{17}{7} \times \frac{690}{714} = \frac{115}{49}$

According to the question,

Number of chocolates $= 49A = 49 \times \frac{115}{49} = 115$

Number of chocolates distributed $= 115 - 1 = 114$ …(1)

It is given that, more than one chocolate should be distributed to each student

Thus, in that case teacher should distribute at least two chocolates

Thus, maximum strength $= \frac{No.of\ chocolates\ distributed}{chocolates\ distributed\ to\ 1\ student}$

$\Rightarrow$ Maximum strength $= \frac{114}{2} = 57$

3. In a forest, Foxes always tell the truth and jackals always lie. When seen in poor light, they are indistinguishable. A person meets three of them A, B and C, in such a poor light. He asks A, "Are you a jackal?" Although A answers his question, he could not hear it clearly. B tells him that A denied being a Jackal and C says that A really is a jackal. Among the three, how many are jackals?

Answer 1 Jackal

Sol. Given, Foxes always tell the truth.

Jackals always lie.

Two cases are possible

Case I: 'A' is a fox.

$\Rightarrow$ 'A' will deny being jackal (truth)

$\Rightarrow$ `B' says 'A' denied being jackal, which is the truth.

So, 'B' is a fox.

$\Rightarrow$ 'C' said 'A' really is jackal, which is a lie.

So, `C' is a Jackal.

Thus, their are 1 Jackal and 2 foxes.

Case II: 'A' is Jackal.

$\Rightarrow$ 'A' will say that he is not a jackal. (lie)

$\Rightarrow$ `B' says 'A' denied being jackal, which is the truth.

So, 'B' is a fox.
$\Rightarrow$ 'C' said 'A' really is jackal, which is the truth.
So, 'C' is a fox.
Thus, agin their are 1 Jackal and 2 foxes.

4. Consider a natural number n. If n is less than 10 times the product of the digits, then n is called a $dwarf$ number. Find the number of dwarf numbers between 10 and 200.

Answer 120

Sol. For two digit number $(10 - 99)$
Let us solve for 10; $n = 10$

Product of digits $= 10(1 \times 0) = 0$
$n > 0$ (10 is not a $dwarf$ number)

Similarly, 11 is not a $dwarf$ number.
Now, $n = 12$

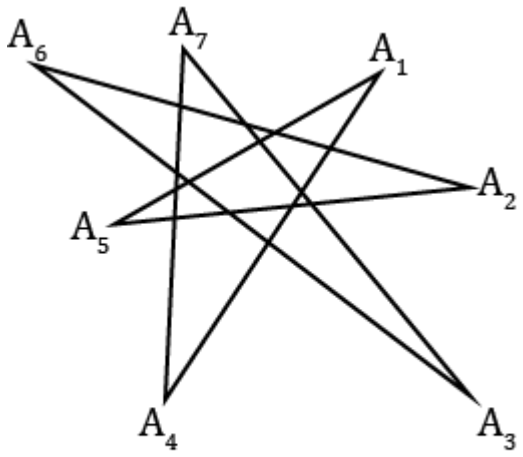
Product of digits $= 10(1 \times 2) = 20$
$n < 20$ (12 is a $dwarf$ number)

Let us draw a series for $dwarf$ number

$$9 \text{ rows} \begin{cases} 12,13,14 \ldots\ldots 19 \text{ (8 numbers)} \\ 22,23,24 \ldots\ldots 29 \text{ (8 numbers)} \\ 32,33,34 \ldots\ldots 39 \text{ (8 numbers)} \\ 42,43,44 \ldots\ldots 49 \text{ (8 numbers)} \\ \vdots \quad \vdots \\ 92,93,94 \ldots\ldots 99 \text{ (8 numbers)} \end{cases}$$

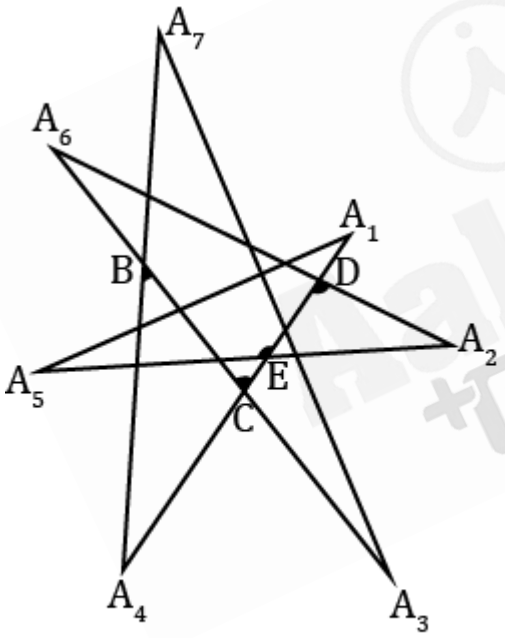
Total $2 -$ digit $dwarf$ numbers $= 9 \times 8 = 72$
For $3 -$ digit numbers $(100 - 200)$
From 100 to 109, no number is $dwarf$ number
As every number's digit's product is 0 and 10 times is also 0. Thus $0 < (100 \ldots 109)$
Similarly, $110, 111, \ldots, 119$ are also not $dwarf$ number
From 120 to $129 \Rightarrow 127, 128, 129$ (3 numbers)
From 130 to $139 \Rightarrow 135, 136, \ldots, 139$ (5 numbers)
From 140 to $149 \Rightarrow 144, 145, \ldots, 149$ (6 numbers)
From 150 to $159 \Rightarrow 154, 155, \ldots, 159$ (6 numbers)
From 160 to $169 \Rightarrow 163, 164, \ldots 169$ (7 numbers)
From 170 to $179 \Rightarrow 173, 174, \ldots, 179$ (7 numbers)
From 180 to $189 \Rightarrow 183, 184, \ldots\ldots 189$ (7 numbers)
From 190 to $199 \Rightarrow 173, 174, \ldots 179$ (7 numbers)
Thus, $3 -$ digit $dwarf$ no. $= 3 + 5 + 6 + 6 + 7 + 7 + 7 + 7$
$= 48$
Total number of $dwarf$ numbers from 10 to $200 = 72 + 48 = 120$.

5. In the given figure, $A_1A_2A_3A_4A_5A_6A_7$ is a 7 – pointed star. Find the value of
$\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5 + \angle A_6 + \angle A_7$



Answer 180^0

Sol.



We know that an exterior angle of a triangle is equal to the sum of the two opposite interior angles.

$\because \angle B$ is ext. angle of ΔBA_7A_3

$\therefore \angle B = \angle A_7 + \angle A_3$

$\because \angle C$ is ext. angle of ΔCA_4B

$\therefore \angle C = \angle A_4 + \angle B = \angle A_4 + \angle A_3 + \angle A_7$

$\because \angle D$ is ext. angle of ΔA_6CD

$\therefore \angle D = \angle C + \angle A_6 = \angle A_3 + \angle A_4 + \angle A_7 + \angle A_6$

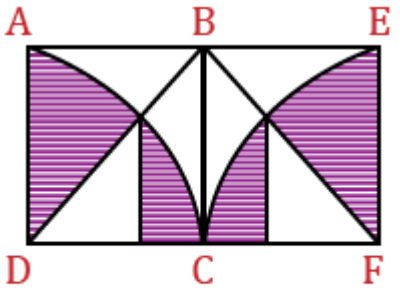
$\because \angle E$ is ext. angle of ΔDEA_2

$\therefore \angle E = \angle A_2 + D = \angle A_2 + \angle A_3 + \angle A_4 + \angle A_6 + \angle A_7$

Now, in $\Delta A_1A_5E, \angle A_1 + \angle A_5 + \angle E = 180^0$

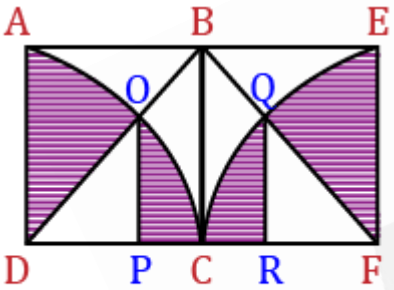
So, $\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5 + \angle A_6 + \angle A_7 = 180^0$

6. Two squares of side length $20\ cm$ are joined together as in the diagram. With D, F as centers, quadrants are drawn. Taking $\pi = 3.14$, find the area of the shaded portion. Let A be the area in cm^2 Find A.



Answer $428\ cm^2$

Sol.



⇒ In given diagram,

$AD = OD = 20\ cm$ (Radius)

In ΔDPO or $\Delta FRQ, \angle DOP = \angle ODP = 45^0$

$Sin\ 45^0 = \frac{OP}{20}$

$\frac{1}{\sqrt{2}} = \frac{OP}{20}$

$OP = \frac{20}{\sqrt{2}}$

$OP = 10\sqrt{2}\ cm$

So, $OP = QR = 10\sqrt{2}\ cm$

and $DP = RF = 10\sqrt{2}\ cm$.

Area of of shaded Region $= 2 \times$ Area of Quadrants $- 2 \times$ Area of Blank triangle

$= 2 \times \frac{1}{4}\pi \times (20)^2 - 2 \times \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}$

$= \frac{1}{2} \times 3.14 \times 400 - 200$

$= 3.14 \times 200 - 200$

$= 200(3.14 - 1)$

$= 200 \times 2.14$

$= 428\ cm^2$