

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The set of all values of a for which $\lim_{x \rightarrow a} ([x-5] - [2x+2]) = 0$, where $[x]$ denotes the greatest integer less than or equal to x is equal to
 (1) $[-7.5, -6.5]$ (2) $(-7.5, -6.5]$
 (3) $[-7.5, -6.5]$ (4) $(-7.5, -6.5)$

Answer (4)

Sol. $\lim_{x \rightarrow a} ([x-5] - [2x+2]) = 0$

$$\Rightarrow [x-5] = [2x+2]$$

$$\Rightarrow [x]-5 = [2x]+2$$

$$\Rightarrow [x] = [2x] + 7 \quad \dots(i)$$

if $x \in \mathbb{Z}$ we have

$$x = -7$$

also $2x \in \mathbb{Z}$ if x is of form $z \pm \frac{1}{2}$

Hence, if $x \in (-7.5, -7)$ eq. (1) become

$$-8 = -15 + 7 \Rightarrow 7 = 7$$

Similarly, if $x \in (-7, -6.5)$ in eq. (1)

$$-7 = -14 + 7 \Rightarrow 7 = 7$$

At $x = -6.5$ in eq. (1)

$$-7 = -13 + 7 \Rightarrow -14 \neq -13 \text{ not possible}$$

At $x = -7.5$ in eq. (1)

$$-8 = -15 + 7 \Rightarrow 8 = 8$$

But $x \rightarrow a \quad a \neq -6.5 \text{ or } -7.5$

$$\therefore a \in (-7.5, -6.5)$$

62. Let p and q be two statements. Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

- (1) $(\sim p) \vee q$ (2) $p \vee ((\sim p) \wedge q)$
 (3) $p \vee (p \wedge q)$ (4) $p \vee (p \wedge (\sim q))$

Answer (1)

Sol. Making truth table ($E \equiv \sim(p \wedge (p \Rightarrow \sim q))$)

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \wedge q$	$p \Rightarrow \sim q$	$p \wedge (p \Rightarrow \sim q)$	E
T	T	F	F	T	T	F	F	T
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	T
F	F	T	T	F	F	T	F	T

&

$\sim p \vee q$	$p \vee (\sim p \wedge q)$	$p \vee (p \wedge q)$	$p \vee (p \wedge \sim q)$
T	T	T	T
F	T	T	T
T	T	F	F
T	F	F	F

$\therefore \sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to $\sim p \vee q$

63. The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1 , is a circle of

radius r_i . If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then

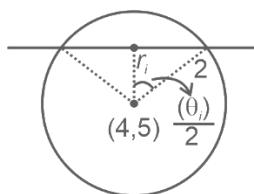
θ_2 is equal to

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

Answer (4)

Sol.



$$\therefore \cos\left(\frac{\theta_1}{2}\right) = \frac{r_i}{2} \Rightarrow r_i = 2\cos\left(\frac{\theta_i}{2}\right)$$

$$\begin{aligned}
 \text{Sol. } & \sum_{r=1}^{30} r \cdot \binom{30}{r}^2 = \sum_{r=1}^{30} 30 \cdot \binom{29}{r-1} \cdot \binom{30}{r} \\
 &= \sum_{r=1}^{30} 30 \cdot \binom{29}{r-1} \cdot \binom{30}{30-r} \\
 &= 30 \cdot \binom{59}{30} \\
 &= 30 \cdot \frac{59!}{30! \cdot 29!} \cdot \frac{30}{30} \\
 &= \frac{15 \cdot 60!}{(30!)^2}
 \end{aligned}$$

68. If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to
 (1) -1 (2) 1
 (3) 3 (4) 5

Answer (4)

Sol. Direction of line

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix} \\
 = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3) \\
 = -5\hat{i} + \hat{j} + 3\hat{k}$$

Equation of line

$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3}$$

Let foot of perpendicular be $= (-5k+3, k+2, 3k+1)$
 $\Rightarrow (-5k+2)(-5) + (k-7)(1) + (3k-6)(3) = 0$

Or $25k - 10 + k - 7 + 9k - 18 = 0$

Or $k = 1$

$\alpha + \beta + \gamma = -k + 6 = 5$

69. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}(\text{adj} A)))| = 12^4$. Then $|A^{-1} \text{adj } A|$ is equal to

- (1) 12 (2) $2\sqrt{3}$
 (3) $\sqrt{6}$ (4) 1

Answer (2)

$$\text{Sol. } |A|^{(n-1)^3} = 12^4$$

$$|A|^8 = 12^4$$

$$|A| = \sqrt{12}$$

$$|A^{-1} \text{adj } A| = |A^{-1}| \cdot |A|^2$$

$$= |A|$$

70. The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$ is

- (1) $\frac{1}{2}(\sqrt{3} + i)$ (2) $-\frac{1}{2}(1 - i\sqrt{3})$
 (3) $\frac{1}{2}(1 - i\sqrt{3})$ (4) $-\frac{1}{2}(\sqrt{3} - i)$

Answer (4)

$$\begin{aligned}
 \text{Sol. } & z = \left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3 \\
 & 1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9} = 1 + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18} \\
 & = 1 + 2 \cos^2 \frac{5\pi}{36} - 1 + 2i \sin \frac{5\pi}{36} \cos \frac{5\pi}{36} \\
 & = 2 \cos \frac{5\pi}{36} \left(\cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36} \right) = 2 \cos \frac{5\pi}{36} e^{i \frac{5\pi}{36}} \\
 & \Rightarrow z = \left(\frac{2 \cos \left(\frac{5\pi}{36} \right) e^{i \frac{5\pi}{36}}}{2 \cos \left(\frac{5\pi}{36} \right) e^{-i \frac{5\pi}{36}}} \right)^3 = e^{i \frac{5\pi}{6}} \\
 & z = -\frac{\sqrt{3}}{2} + \frac{1}{2}i = \frac{1}{2}(i - \sqrt{3}) = -\frac{1}{2}(\sqrt{3} - i)
 \end{aligned}$$

71. The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

- (1) 120 (2) 225
 (3) 150 (4) 125

Answer (1)

82. If $\frac{1^3 + 2^3 + 3^3 + \dots \text{ up to } n \text{ terms}}{1.3 + 2.5 + 3.7 + \dots \text{ up to } n \text{ terms}} = \frac{9}{5}$, then the value of n is

Answer (05)

Sol. Given $\frac{1^3 + 2^3 + 3^3 + \dots \text{ up to } n \text{ terms}}{1.3 + 2.5 + 3.7 + \dots \text{ up to } n \text{ terms}} = \frac{9}{5} \quad \dots(1)$

Now

Let $S = 1.3 + 2.5 + 3.7 + \dots$

$$T_n = n \cdot (2n+1)$$

$$\therefore S = \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$\Rightarrow \frac{\left(\frac{n(n+1)}{2}\right)^2}{n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]} = \frac{9}{5}$$

$$\Rightarrow 5n^2 - 19n - 30 = 0$$

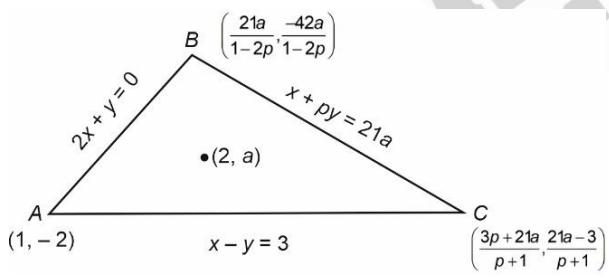
$$\Rightarrow (5n+6)(n-5) = 0$$

$$\therefore \boxed{n=5}$$

83. The equations of the sides AB , BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, ($a \neq 0$) and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of $\triangle ABC$. Then $(BC)^2$ is equal to

Answer (122)

Sol.



$$\therefore \frac{21a}{1-2p} + 1 + \frac{3p+21a}{p+1} = 6$$

$$\therefore 4p^2 - 21ap + 8p + 42a - 5 = 0 \quad \dots(1)$$

$$\text{And } \frac{-42a}{1-2p} - 2 + \frac{21a-3}{p+1} = 3a$$

$$\therefore 4p^2 - 81ap + 6ap^2 - 24a + 8p - 5 = 0 \quad \dots(2)$$

From equation (1) – equation (2) we get;

$$60ap + 66a - 6ap^2 = 0$$

$$\therefore a \neq 0 \Rightarrow p^2 - 10p - 11 = 0$$

$$p = -1 \text{ or } 11 \Rightarrow p = 11.$$

When $p = 11$ then $a = 3$

Coordinate of $B = (-3, 6)$

And coordinate of $C = (8, 5)$

$$\therefore BC^2 = 122$$

84. The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is _____.

Answer (13)

Sol. $R = \{(a, b), (b, c), (b, d)\}$

$$S : \{a, b, c, d\}$$

Adding $(a, a), (b, b), (c, c), (d, d)$ make reflexive.

Adding $(b, a), (c, b), (d, b)$ make Symmetric

And adding $(a, d), (a, c)$ to make transitive

Further (d, a) & (c, a) to be added to make Symmetry.

Further (c, d) & (d, c) also be added.

So total 13 elements to be added to make equivalence.

85. Let $S = \{\theta \in [0, 2\pi] : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to _____.

Answer (02)

Sol. $S = \{\theta \in [0, \pi] : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$

$$\tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0$$

$$\tan(\pi \cos \theta) = \tan(-\pi \sin \theta)$$

$$\pi \cos \theta = n\pi - \pi \sin \theta \quad n \in I$$

$$\sin \theta + \cos \theta = n$$

$$\therefore \sin \theta + \cos \theta = \{-1, 0, 1\}$$

$$\therefore \theta = 0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \pi, \frac{3\pi}{2}$$

$$\text{Now } \sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$$

$$= \sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4} + \frac{\pi}{4}\right)$$

$$+ \sin^2\left(\frac{7\pi}{4} + \frac{\pi}{4}\right) + \sin^2\left(\pi + \frac{\pi}{4}\right) + \sin^2\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)$$

$$= \frac{1}{2} + \frac{1}{2} + 0 + 0 + \frac{1}{2} + \frac{1}{2}$$

$$= 2$$

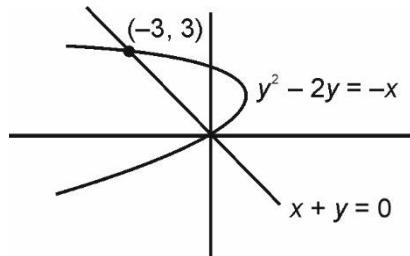
86. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then $8A$ is equal to _____.

Answer (36)

Sol. Area enclosed by

$$y^2 - 2y = -x$$

$$x + y = 0$$



$$\text{Area} = \int_0^3 (2y - y^2) - (-y) dy$$

$$= \int_0^3 (3y - y^2) dy$$

$$= \left[\frac{3y^2}{2} - \frac{y^3}{3} \right]_0^3$$

$$= \frac{27}{2} - 9$$

$$= \frac{27 - 18}{2} = \frac{9}{2} = A$$

$$8A = \frac{9}{2} \times 8 = 36 \text{ sq. units}$$

87. If the shortest distance between the lines

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4} \text{ and}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5}$$

is 6, then the square of sum of all possible values of λ is

Answer (384)

Sol. Shortest distance between

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4} \text{ and}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5} \text{ is } 6$$

$$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = (\lambda + \sqrt{6})\hat{i} + \sqrt{6}\hat{j} - 3\sqrt{6}\hat{k}$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = 6$$

$$\frac{|\lambda + \sqrt{6} + 2\sqrt{6} + 3\sqrt{6}|}{\sqrt{6}} = 6$$

$$|\lambda + 4\sqrt{6}| = 6\sqrt{6}$$

$$-\lambda + 4\sqrt{6} = \pm 6\sqrt{6}$$

$$-\lambda + 4\sqrt{6} = 6\sqrt{6} \quad | \quad -\lambda + 4\sqrt{6} = -6\sqrt{6}$$

$$\lambda_1 = -2\sqrt{6} \quad | \quad \lambda_2 = 10\sqrt{6}$$

$$(\lambda_1 + \lambda_2)^2 = (8\sqrt{6})^2$$

$$= 384$$

88. Let the sum of the coefficients of the first three

terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$. $n \in \mathbb{N}$,

be 376. Then the coefficient of x^4 is _____.

Answer (405)

$$\text{Sol. } S = 1 - 3n + \frac{9n(n-1)}{2} = 376$$

$$3n^2 - 5n - 250 = 0$$

$$n = 10, \frac{-25}{3} \text{ (Rejected)}$$

$$T_{r+1} = {}^nC_r \cdot x^{n-r} \left(\frac{-3}{x^2}\right)^r$$

$$= {}^nC_r \cdot x^{n-3r} (-3)^r$$

$$= {}^{10}C_r \cdot x^{10-3r} (-3)^r$$

Here $r = 2$

$$\text{Required coefficient} = {}^{10}C_2 (-3)^2$$

$$= 45 \times 9$$

$$= 405$$

89. Three urns A , B and C contain 4 red, 6 black; 5 red, 5 black, and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is

Answer (432)

Sol. E_1 : Ball is drawn from urn A ($4R + 6B$)

E_2 : Ball is drawn from urn B ($5R + 5B$)

E_3 : Ball is drawn from urn C ($\lambda R + 4B$)

$A \rightarrow$ Ball drawn is red.

$$\text{Required probability} = P\left(\frac{E_3}{A}\right)$$

$$= \frac{\frac{1}{3} \times \frac{\lambda}{\lambda+4}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{\lambda}{\lambda+4}} = \frac{2}{5}$$

$$\Rightarrow \frac{10\lambda}{19\lambda+36} = \frac{2}{5}$$

$$\Rightarrow \lambda = 6$$

Parabola: $y^2 = 6x = 4ax$

Let length of side = l

Point $\left(\frac{\sqrt{3}}{2}l, \frac{l}{2}\right)$ lies on parabola

$$\frac{l^2}{4} = 4a\left(\frac{\sqrt{3}}{2}l\right)$$

$$\Rightarrow l = 8a\sqrt{3}$$

$$l = 12\sqrt{3}$$

$$\rho = 432$$

90. Let

$$\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a} \cdot \vec{c} = 7, 2\vec{b} \cdot \vec{c} + 43 = 0,$$

$\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $|\vec{a} \cdot \vec{b}|$ is equal to

Answer (08)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$

$$\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$$

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$$

$$\Rightarrow (\vec{a} - \vec{b}) \times \vec{c} = 0$$

$$\Rightarrow \vec{c} \parallel \vec{a} - \vec{b}$$

$$\Rightarrow \vec{c} = \alpha(\vec{a} - \vec{b})$$

$$\therefore \vec{c} = \alpha(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$$

$$\vec{a} \cdot \vec{c} = \alpha(12 + 2\lambda^2) = 7 \quad \dots(i)$$

$$\vec{b} \cdot \vec{c} = \alpha(-41 - 2\lambda^2) = \frac{-43}{2} \quad \dots(ii)$$

(i) and (ii)

$$\Rightarrow \frac{12 + 2\lambda^2}{41 + 2\lambda^2} = \frac{14}{43}$$

$$\Rightarrow \lambda^2 = 1$$

$$|\vec{a} \cdot \vec{b}| = |3 - 10 - \lambda^2| = 8$$

