Mathematics (Basic)
SET – 3 (Q.P. Code 430/2/3)

Time Allowed: 3 Hours **Maximum Marks:** 80

Note:

(i) Please check that this question paper contains 15 printed pages.

(ii) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) Please check that this question paper contains 38 questions.

(iv) Please write down the Serial Number of the question in the answer-book before attempting it.

(v) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

GENERAL INSTRUCTIONS:

Read the following instructions carefully and follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE sections - Section A, B, C, D and E.

(iii) In section A - question number 1 to 18 are multiple choice questions (MCQs) and question number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In section B-question number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.

(v) In section C-question number 26 to 31 are Short Answer (SA) type questions carrying 3:marks each.

(vi) In section D - question number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In section E - question number 36 to 38 are case based integrated units of assessment questions carrying 4 marks each. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

(x) Use of calculators is NOT allowed.

SECTION A

Section - A consists of Multiple Choice type questions of 1 mark each.

1. If the HCF of 360 and 64 is 8, then their LCM is :

(a) 2480

(b) 2780

(c) 512

(d) 2880

Answer: (d)

Explanation:

Product of two numbers = HCF x LCM

So,

360 x 64 = 8 x LCM

LCM = 2880

2. The curved surface area of a cone of radius 7 cm is 550 cm^2. Its slant height is :

(a) 24 cm

(b) 25 cm

(c) 14 cm

(d) 20 cm

Answer: (b)

Explanation:

C.S.A of cone = 550 cm^2

$\pi \times r \times l = 550\ cm^2$ (l = slant height)

$22/7 \times 7 \times l = 550$

$l = 550/22$

l = 25cm

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3. Two coins are tossed together. The probability of getting atmost two heads, is :

(a) ½

(b) ¼

(c) ¾

(d) 1

Answer: (b)
Explanation:
Two coins tossed as

(h, t) (t, t), (t, h), (h, h)

n(s) =4

n(e)=1

Probability =¼

4. If the quadratic equation $9x^2 + bx + \frac{1}{4} = 0$ has equal roots, then the value of b is :

(a) 0
(b) -3 only
(c) 3 only
(b) ± 3

Answer: (d)
Explanation:
If the roots are equal than
$b^2 - 4ac = 0$
$b^2 - 4 \times 9 \times \frac{1}{4} = 0$
$b^2 = 9$
$b = \pm 3$

5. If $tanA = \frac{2}{5}$, then the value of $\frac{1-cos^2 A}{1-sin^2 A}$ is :

(a) $\frac{25}{4}$

(b) $\frac{4}{25}$

(c) $\frac{4}{5}$

(d) $\frac{5}{4}$

Answer: (b)

Explanation:

Given $tanA = \frac{2}{5}$

Since $tanx = \frac{Perpendicular}{Base}$

⇒ Perpendicular $= 2$

⇒ Base $= 5$

⇒ Hypotenuse = $\sqrt{29}$

We know that $sin\theta = \frac{Perpendicular}{Hypotenuse}$

$sin^2\theta = \frac{4}{29}$

and $cos\theta = \frac{Base}{Hypotenuse}$

$\theta = \frac{25}{29}$

We find: $\frac{1-cos^2 A}{1-sin^2 A}$

$\frac{1-\frac{25}{29}}{1-\frac{4}{29}} = \frac{29-25}{29-4} = \frac{4}{25}$

6. Median and Mode of a distribution are 25 and 21 respectively. Mean of the data using empirical relationship is :

(a) 27

(b) 29

(c) 18

(d) 29/3

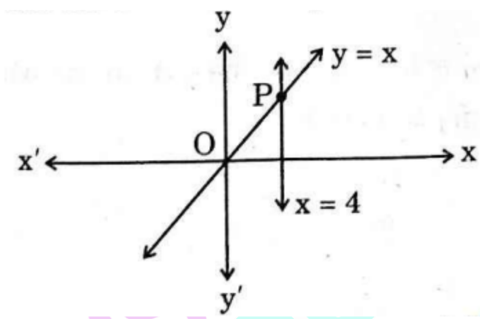
Answer: (a)

Explanation:

Ass we know Mode = 3 Median – 2 Mean
So, 21 = 3 x 25 - 2 Mean
2 Mean = 75 - 21
Mean = 54/2 = 27

7. The lines represented by the linear equations y=x and x=4 intersect at P. The coordinates of the point P are :

(a) (4,0)
(b) (4,4)
(c) (0,4)
(d) (-4,4)



Answer: (b)
Explanation:
As the value of y = x
And x = 4
So, the coordinates are (4, 4)

8. The value of $\frac{sin90^{\circ}+cos60^{\circ}}{sec45^{\circ}+tan45^{\circ}}$ is :

(a) 1
(b) $\frac{3}{2}(\sqrt{2}+1)$
(c) $\frac{3}{2}(\sqrt{2}-1)$
(d) $\frac{1+\sqrt{3}}{\sqrt{2}+1}$

Answer: (c)
Explanation:
As we know
Sin 90° = 1
Cos 60° = 1/2
Sec 45° = $\sqrt{2}$
Tan 45° = 1
Let's keep the values
$\frac{1+1/2}{\sqrt{2}+1}$

$= \frac{3}{2(\sqrt{2}+1)}$

$= \frac{3(\sqrt{2}-1)}{2(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \frac{3}{2}(\sqrt{2}-1)$

9. How many terms are there in the A.P. given below ?

$14, 19, 24, 29, ..., 119$

(a) 18
(b) 14
(c) 22
(d) 21

Answer: (c)

Explanation:

The given sequence is an A.P. with first term $a = 14$ and common difference $d = 5$.
119 be the nth term of the given sequence. Then,

$a_n = 119 \ a + (n - 1)d = 119$

$\Rightarrow 14 + (n - 1)\times 5 = 119$

$\Rightarrow n = 22$

Hence, 22^{th} term of the given sequence.

10. y-axis divides the line segment joining the points $(-6, 2)$ and $(2, -6)$ in the ratio:

(a) $1: 3$
(b) $3: 2$
(c) $3: 1$
(d) $2: 3$

Answer: (c)

Explanation:

Given the points $A(-6, 2)$ and $B(2, -6)$

$(x_1, y_1) \qquad (x_2, y_2)$

Let the coordinate of points $(0, y)$ divide the given points in the ratio m:n
We can use section formula to find the coordinates of $(0, y)$

$\Rightarrow (0, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$\Rightarrow \left(\frac{2m-6n}{m+n}, \frac{-6m+2n}{m+n}\right) = (0, y)$

Equating the x - axis co-ordinates we get,

$\therefore \frac{2m-6n}{m+n} = 0$

$\therefore 2m = 6n$

$i.e, \frac{m}{n} = \frac{3}{1}$

Required ratio $= 3:1$

11. $9sec^2 A - 9tan^2 A$ is equal to :

(a) 9

(b) 0

(c) 8

(d) $\frac{1}{9}$

Answer: (a)

Explanation:

Given, $9sec^2 A - 9tan^2 A$

= $9(sec^2 A - tan^2 A)$

As we know, $(sec^2 A - tan^2 A = 1$

So, 9 x 1 = 9

12. The string of a kite in air is $50\ m$ long and it makes an angle of 60° with the horizontal. Assuming the string to be straight, the height of the kite from the ground is :

(a) $50\sqrt{3}\ m$

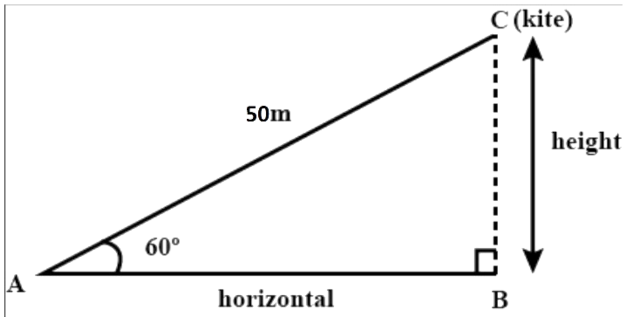
(b) $\frac{100}{\sqrt{3}}\ m$

(c) $\frac{50}{\sqrt{3}}\ m$

(d) $25\sqrt{3}\ m$

Answer: (d)

Explanation:



$sin(\theta) = \frac{Perpendicular}{hypotenuse}$

$sin\ 60° = \frac{BC}{AC} = \frac{h}{50}$

$\frac{\sqrt{3}}{2} = \frac{h}{50} \left(\because sin60° = \frac{\sqrt{3}}{2}\right)$

$h = 25\sqrt{3}\ m$

13. The area of a sector of angle α (in degrees) of a circle with radius R is :

(a) $\frac{\alpha}{180} \times 2\pi R$

(b) $\frac{\alpha}{360} \times 2\pi R$

(c) $\frac{\alpha}{180} \times \pi R^2$

(d) $\frac{\alpha}{360} \times \pi R^2$

Answer: (d)

Explanation:

Area of a sector $= \frac{\alpha}{360} \times \pi r^2$

Where θ = angle, r = radius of circle

Here, we have

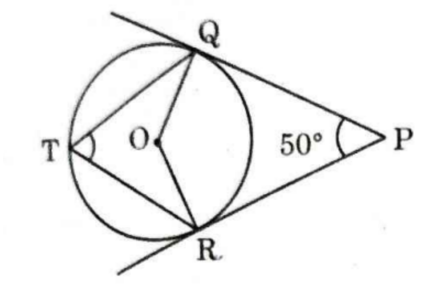
$\theta = p$ and radius $= R$

Putting these values in formula

$Area\ of\ sector = \frac{\alpha}{360} \times \pi R^2$

$= \frac{\alpha}{360} \times \pi R^2$

14. From a point P, two tangents PQ and PR are drawn to a circle with centre at O. T is a point on the major arc QR of the circle. If $\angle QPR = 50^\circ$, then $\angle QTR$ equals:



(a) 50°
(b) 130°
(c) 65°
(d) 90°

Answer: (a)
Explanation:

In quadrilateral PQOR, $\angle P + \angle PQO + \angle PRO + \angle QOR = 360^\circ$

But $\angle PQO = \angle PRO = 90^\circ$ (Tangent is perpendicular to radius at point of contact)

Thus, $50^\circ + 90^\circ + 90^\circ + \angle QOR = 360^\circ$

So, $\angle QOR = 130^\circ$

Now, $\angle QOR = 2\angle QTR$ (Measure of angles subtended to any point on the circumference of the circle from the same arc is equal to half of the angle subtended at the center by the same arc.)

$\angle QTR = \frac{130}{2} = 65^\circ$

15. The pair of linear equations $x + 2y - 5 = 0$ and $2x - 4y + 6 = 0$:
(a) is inconsistent
(b) is consistent with many solutions
(c) is consistent with a unique solution
(d) is consistent with two solutions

Answer: (c)

Explanation:

The given equations are:

$x + 2y - 5 = 0 \;\; 2x - 4y + 6 = 0$

From the given equations we have:

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{-4} = \frac{1}{-2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence the given pair of equations has unique solution.

16. Which of the following numbers cannot be the probability of an event?

(a) 0.5

(b) 5%

(c) $\frac{1}{0.5}$

(d) $\frac{0.5}{14}$

Answer: (d)

Explanation:

The probability lies between 0 and 1.

So, $\frac{0.5}{14}$ cannot be the probability of an event.

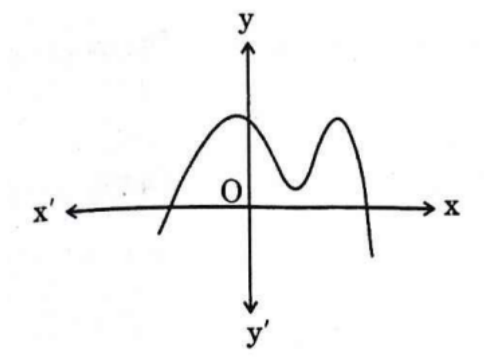
17. Graph of a polynomial is given in the figure. The number of zeroes of is :

(a) 2

(b) 3

(c) 4

(d) 5



Answer: (a)
Explanation:
Given graph is touching the x axis 2 places.
The numbers of zeros = 2

18. A solid is of the form of a cone of radius ' r ' surmounted on a hemisphere of the same radius. If the height of the cone is the same as the diameter of its base, then the volume of the solid is :

(a) πr^3

(b) $\frac{4}{3}\pi r^3$

(c) $3\pi r^3$

(d) $\frac{2}{3}\pi r^3$

Answer: (b)
Explanation:
Radius $= r\ cm$
∴ height of the cone
$h = Diameter = 2r$
$\therefore vol\ of\ cone + vol\ of\ hemisphere \quad = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$
= $= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3$
= $\frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$
= $\frac{4}{3}\pi r^3$

Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : The probability of getting a prime number, when a die is thrown once, is $\frac{2}{3}$.
Reason (R): On the faces of a die, prime numbers are $2, 3, 5$.

Answer: (d)
Explanation:
Assertion (A) is false but reason (R) is true.

When a die is thrown once, total possible outcomes $= 6$ and the prime numbers in it are $\{2, 3, 5\}$.
Total possible outcomes $= 3$
Probability of getting a prime $= 3/6 = 1/2$.

20. Assertion (A): Polynomial $x^2 + 4x$ has two real zeroes.
Reason (R): Zeroes of the polynomial $x^2 + ax$ $(a \neq 0)$ are 0 and a.

Answer: (c)
Explanation:

Assertion (A): Consider the given polynomial, $x^2 + 4x$
Because the degree of the polynomial is 2 . It is a quadratic polynomial. We know that the quadratic polynomial has at the most two zeroes.
$\therefore x^2 + 4x$ has two zeroes.
$\therefore$ Assertion is true.

Reason (R): Zeroes of the polynomial $x^2 + ax\ (a \neq 0)$ is equal to $x(x + a) = 0$ i.e.
$x = 0, x = -a$
$x = 0, \ -a$
∴ Reason is false

SECTION B

Section - B consists of Very Short Answer (VSA) type questions of 2 marks each.

21. If (-3) is one of the zeroes of the polynomial $(k - 1)x^2 + kx + 1$, find the value of k.

Answer: $4/3$

Explanation:

Given, the quadratic polynomial is $(k - 1)x^2 + kx + 1$.
One zero of the polynomial is -3 .
We have to find the value of k.
Let $f(x) = (k - 1)x^2 + kx + 1$
$f(-3) = 0$
Put $x = -3$ in the given polynomial
$(k - 1)(-3)^2 + k(-3) + 1 = 0$
$\Rightarrow (k - 1)(9) - 3k + 1 = 0$
$\Rightarrow 9k - 9 - 3k + 1 = 0$
By grouping,
$9k - 3k - 9 + 1 = 0$
$\Rightarrow 6k - 8 = 0$
$\Rightarrow 6k = 8$
$\Rightarrow k = 8/6$
$\Rightarrow k = 4/3$
Therefore, the value of k is $4/3$.

22. A box contains 90 discs, numbered from 1 to 90 . If one disc is drawn at random, then find the probability that it bears a multiple of 15 .

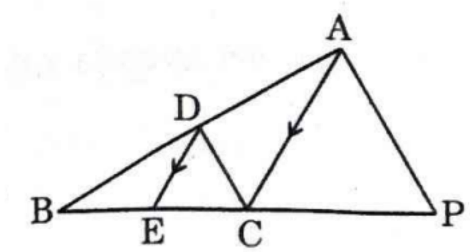
Answer: $1/15$

Explanation:

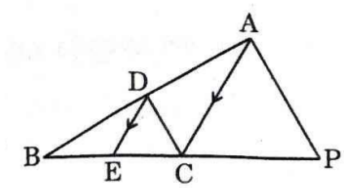
Total number of discs = 90
Multiples of 16 between 1 to 90 = 15, 30, 45, 60, 75 and 90
Number of possible outcomes = 6

Probability of getting a number multiple of 15 = Number of possible outcomes/Total number of favorable outcomes
= 6/90
= 1/15

23. In the given figure, $DE \parallel AC$ and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that $DC \parallel AP$.



Explanation:



By B.P.T, $\frac{BE}{EC} = \frac{BD}{AD}$….(1)

and, $\frac{BE}{EC} = \frac{BC}{CP}$ { given }….. (2)

from (1) & (2)

$\frac{BD}{DA} = \frac{BC}{CP}$

By converse of B.P.T

$DC \parallel AP$

Hence, proved

24. (a) Find the HCF of the numbers 540 and 630 , using prime factorization method.

Answer: 90
Explanation:

Find the prime factorization of 540
540 = 2 × 2 × 3 × 3 × 3 × 5
Find the prime factorization of 630
630 = 2 × 3 × 3 × 5 × 7
To find the HCF, multiply all the prime factors common to both numbers:
Therefore, HCF = 2 × 3 × 3 × 5
HCF = 90

OR

23(b) Show that $(15)^n$ cannot end with the digit 0 for any natural number ' n '.

Answer: 15^n *can not end with digit zero*
Explanation:

If 15^n end with digit zero,then the number should be divisible by 2 and 5 .
As $2\times5 = 10$
→ This means the prime factorization of 15^n should contain prime factors 2 and 5.
→ $15^n = (3\times5)^n$
It does not have the prime factor 2 but have 3 and 5 ,
Since 2 is not present in the prime factorization, there is no natural number nor which 15^n ends with digit zero.
So, 15^n can not end with digit zero

25 (A). Find the value(s) of ' x ' so that $PQ = QR$, where the coordinates of P, Q and R are $(6, -1)$, $(1, 3)$ and $(x, 8)$ respectively.

Answer: 5 or -3
Explanation:

Coordinate of points P, Q and R are $P(6, -1), Q(1, 3)$ and $R(x, 8)$ Given, $PQ = QR$
So by distance formula we have, Distance between two points

$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$PQ = \sqrt{(1 - 6)^2 + (3 + 1)^2} = \sqrt{25 + 16} = \sqrt{41}$$

$$\therefore PQ^2 = 41 = QR^2$$

but, $QR^2 = (x - 1)^2 + 25$
$41 = x^2 + 1 - 2x + 25$
$\Rightarrow 41 = x^2 + 1 - 2x + 25$
$\Rightarrow x^2 - 2x + 26 = 41$
$\Rightarrow x^2 - 2x - 15 = 0$
$\Rightarrow x^2 - 5x + 3x - 15 = 0$
$\Rightarrow x(x - 5) + 3(x - 5) = 0$
$\Rightarrow (x + 3)(x - 5) = 0$
$\Rightarrow x = - 3, 5$
Thus the roots of the above equation are 5 and -3 .
Hence the values of ' x ' are 5 or -3 .

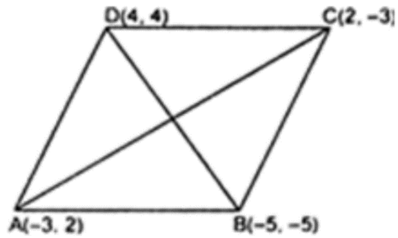
OR

25 (B). The vertices of a triangle are $(-2, 0)$, $(2, 3)$ and $(1, -3)$. Is the triangle equilateral, isosceles or scalene?
Answer: Scalene triangle
Explanation:
Given vertices are $(-2, 0)$, $(2, 3)$ and $(1, -3)$.
We know that the distance between two points (x_1, y_1) and (x_2, y_2) is
$\sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$
We compute the length of the sides as follows:
$AB = \sqrt{(2 - (-2))^2 + (3 - 0)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5$
$BC = \sqrt{(1 - 2)^2 + (-3 - 3)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$
$AC = \sqrt{(1 - (-2))^2 + (-3 - 0)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18}$
Clearly, $AB \neq BC \neq AC$.
Since none of the lengths of the sides are equal, therefore, it is a scalene triangle.

SECTION - C

Section - C consists of Short Answer (SA) type questions of 3 marks each.

26. Show that the points $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$ are vertices of a rhombus $ABCD$. Is it also a square?
Explanation:



$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{4+49} = \sqrt{53}\, units$

$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{49+4} = \sqrt{53}\, units$

$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{4+49} = \sqrt{53}\, units$

$DA = \sqrt{(-3-4)^2 + (2-4)^2} = \sqrt{4+49} = \sqrt{53}\, units$

$AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{25+25} = \sqrt{50} = \sqrt{25\times2} = 5\sqrt{2}\, units$

$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{81+81} = \sqrt{162} = \sqrt{81\times2} = 9\sqrt{2}\, units$

$\therefore AB = BC = CD = DA = \sqrt{53}\, units$

$and\ AC = 5\sqrt{2}$

$BD = 9\sqrt{2}$

$i.e.,\ AC \neq BD$

This means that $ABCD$ is a quadrilateral whose sides are equal but diagonals are not equal. Thus $ABCD$ is a rhombus.

27. (A) Find the zeroes of the polynomial $p(x) = 2x^2 - 7x - 15$ and verify the relationship between its coefficients and zeroes.

Explanation:

$2x^2 - 7x - 15 = 0$

Splitting The Middle Term we get,

$\Rightarrow 2x^2 - 10x + 3x - 15 = 0 \Rightarrow 2x(x-5) + 3(x-5) = 0 \Rightarrow (2x+3)(x-5) = 0$

Putting Both Equal to Zero now, we get,

$\Rightarrow 2x + 3 = 0 \Rightarrow 2x = -3 \Rightarrow x = (-3/2)$

OR

$\Rightarrow x - 5 = 0$
$\Rightarrow x = 5$
Hence, Zeros of The polynomial are 5 and $(-3/2)$.
Now, First Relation is :-
$\Rightarrow$ Sum of Zeros $=-$ (coefficient of x)/(coefficient of x^2)
Putting both values
$\Rightarrow(-3/2) + 5 =- (-7)/2$
$\Rightarrow(-3 + 10)/2 = (7)/2$
$\Rightarrow (7/2) = (7/2)$
Hence Verified.
Second Relation :-
$\Rightarrow$ Product Of Zeros = Constant Term /(coefficient of x^2)
Putting both Values,
$\Rightarrow (-3/2) * 5 = (-15)/(2)$
$\Rightarrow (-15)/2 = (-15)/2$
Hence Verified.

28(a). Prove that:

$$\frac{1-cos\theta}{1+cos\theta} = (cosec\theta - cot\theta)^2$$

Explanation:
We have,

$$(cosec\theta - cot\theta)^2$$

$$= \left(\frac{1}{sin\theta} - \frac{cos\theta}{sin\theta}\right)^2$$

$$= \left(\frac{1-cos\theta}{sin\theta}\right)^2$$

$$= \frac{(1-cos\theta)^2}{sin^2\theta}$$

$$= \frac{(1-cos\theta)^2}{1-cos^2\theta} \left[\because sin^2\theta = 1 - cos^2\theta\right]$$

$$= \frac{(1-cos\theta)^2}{(1-cos\theta)(1+cos\theta)}$$

$= \frac{1-\cos\theta}{1+\cos\theta}$

OR

28(b). Prove that:

$$\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$$

Explanation:

We have

$$LHS = \left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)$$

$$= \left(1+\cot^2 A\right)\left(1+\tan^2 A\right)$$

$$= cosec^2 A \cdot \sec^2 A$$

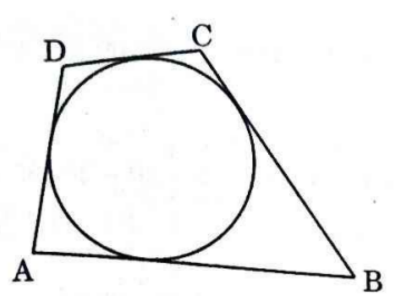
$$= \frac{1}{\sin^2 A}\cdot\frac{1}{\cos^2 A} = \frac{1}{\sin^2 A\cos^2 A}$$

$$= \frac{1}{\sin^2 A\left(1-\sin^2 A\right)}$$

$$= \frac{1}{\left(\sin^2 A-\sin^4 A\right)} = RHS.$$

$\therefore LHS = RHS$

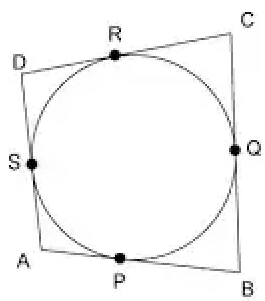
29(A). A quadrilateral $ABCD$ is drawn to circumscribe a circle, as shown in the figure. Prove that $AB + CD = AD + BC$.



Explanation:

Given : Let $ABCD$ be the quadrilateral circumscribing the circle with centre O. The quadrilateral touches the circle at points P, Q, R and S
To prove: $AB + CD = AD + BC$

Proof:
From theorem we know, lengths of tangents drawn from external point are equal



Hence, $AP = AS$______(1)

$BP = BQ$______(2)

$CR = CQ$______(3)

$DR = DS$______(4)

Adding (1) + $(2) + (3) + (4)$

$AP + BP + CR + DR = AS + BQ + CQ + DS$

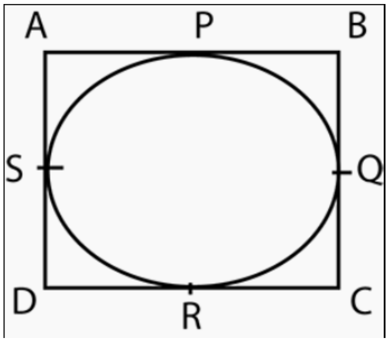
$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$ $AB + CD = AD + BC$

Hence proved

OR

29(B). Prove that the parallelogram circumscribing a circle is a rhombus.

Explanation:



Since ABCD is a parallelogram circumscribed in a circle

$AB = CD$__________(1)

$BC = AD$__________(2)

$DR = DS$ (Tangents on the circle from same point D)
$CR = CQ$ (Tangent on the circle from same point C)
$BP = BQ$ (Tangent on the circle from same point B)
$AP = AS$ (Tangents on the circle from same point A)

Adding all these equations we get

$DR + CR + BP + AP = DS + CQ + BQ + AS$

$(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$

$CD + AB = AD + BC$

Putting the value of equation 1 and 2 in the above equation we get

$2AB = 2BC$

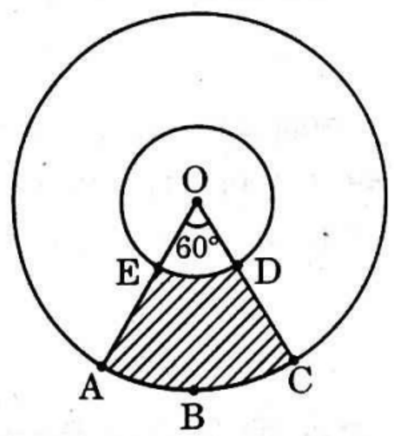
$AB = BC$__________(3)

From equation (1), (2) and (3) we get

$AB = BC = CD = DA$

$\therefore ABCD$ is a Rhombus

30. In the given figure, two concentric circles with centre O are shown. Radii of the circles are $2\ cm$ and $5\ cm$ respectively. Find the area of the shaded region.



Explanation:

Given:
Radii of inner circle $= 2\ cm = r$
Radii of outer circle $= 5\ cm = R$
$\angle AOB = \theta = 60^{\circ}$

Also,
Area of a sector ABCO $= \frac{\theta}{360}\pi R^2$

Area of a sector EDO $= \frac{\theta}{360}\pi r^2$

The area of shaded region = Area of a sector ABCO - Area of a sector EDO

$= \frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2$

$= \frac{\theta}{360}\pi(R^2 - r^2)$

$= \frac{60}{360} \times \frac{22}{7}(5^2 - 2^2) = 11\ cm^2$

31. Prove that $10 + 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
Explanation:

Given $\sqrt{3}$ is irrational number

Let $10 + 2\sqrt{3}$ is rational number.

then $10 + 2\sqrt{3} = \frac{p}{q}$

$\sqrt{3} = \frac{p-10q}{2q}$

Since $\sqrt{3}$ is irrational number but $\frac{p-10q}{2q}$ is rational it mean irrational = rational which contradict,

∴ our assumption is wrong and

$10 + 2\sqrt{3}$ is irrational number.

32. Find the mean and the median of the marks of 100 students of a class, given in the following table:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Number of students	4	11	13	15	31	26

Answer: $Mean\ (\bar{x}) = 19.3$ and $Median\ (M) = 21.129$

Explanation:

Class (1)	Frequency (f) (2)	Mid value (x) (3)	$d = \frac{x-A}{h} = \frac{x-17.5}{5}$ $A = 17.5, h = 5$ (4)	$f \cdot d$ (5) = (2) × (4)	cf (7)
$0-5$	4	2.5	-3	-12	4
$5-10$	11	7.5	-2	-22	15
$10-15$	13	12.5	-1	-13	28
$15-20$	15	$17.5 = A$	0	0	43
$20-25$	31	22.5	1	31	74
$25-30$	26	27.5	2	52	100
	$n = 100$	----	----	$\sum f \cdot d = 36$	---.

$Mean\ \bar{x} = A + \frac{\sum fd}{n} \cdot h$

$= 17.5 + \frac{36}{100} \cdot 5$

$= 17.5 + 0.36 \cdot 5$

$= 17.5 + 1.8$

$Mean\ (\bar{x}) = 19.3$

To find Median Class

= value of $\left(\frac{n}{2}\right)^{th}$ observation

= value of $\left(\frac{100}{2}\right)^{th}$ observation

= value of 50^{th} observation

From the column of cumulative frequency cf, we find that the 50^{th} observation lies in the class $20 - 25$.

∴ The median class is $20 - 25$.

Now,

$\therefore L = lower\ boundary\ point\ of\ median\ class = 20$

$\therefore n = Total\ frequency = 100$

$\therefore cf = Cumulative\ frequency\ of\ the\ class\ preceding\ the\ median\ class = 43$

$\therefore f = Frequency\ of\ the\ median\ class = 31$

$\therefore c = class\ length\ of\ median\ class = 5$

$Median\ (M) = L + \frac{\frac{n}{2} - cf}{f} \cdot c$
$= 20 + \frac{50-43}{31} \cdot 5$
$= 20 + \frac{7}{31} \cdot 5$
$= 20 + 1.129$
$= 21.129$
$Median\ (M) = 21.129$

SECTION - D

Section - D consists of Long Answer (LA) type questions of 5 marks each.

33(A). The sum of the first 8 terms of an A.P. is 100 and the sum of its first 19 terms is 551 . Find the sum of its first ' n ' terms.

Explanation:

First we will list the data we are given.
The sum of the first 8 terms of the A.P is $= 100$.
The sum of the first 19 terms is $= 551$.
Now Let The First term of the A.P be a, and the common difference be d.
So, According to the problem,
$\frac{8}{2}[a + a + (8 - 1)d] = 100$
$\Rightarrow 4[2a + 7d] = 100 \Rightarrow 2a + 7d = 25$
And, $\frac{19}{2}[a + a + (19 - 1)d] = 551$
$\Rightarrow \frac{19}{2}[2a + 18d] = 551$
$\Rightarrow 19[a + 9d] = 551$
$\Rightarrow a + 9d = 29$
Now, Just Solve For a and d.
FIrst, Multiply eq(ii) with 2 .
So, we get,
$2a + 18d = 58$
Now. Subtract eq(i) from eq(iii).
So, We get,
$2a + 18d - 2a - 7d = 58 - 25$
$\Rightarrow 11d = 33 \Rightarrow d = 3$
Now, Substitute $d = 3$ in eq(i).
So, We get,

$2a + 7\cdot3 = 25 \Rightarrow 2a + 21 = 25$
$\Rightarrow 2a = 25 - 21 \Rightarrow 2a = 4$
$\Rightarrow a = 2$
So, the sum of its first ' n ' terms = $\frac{n}{2}[2a + (n - 1)d]$
$= \frac{n}{2}[2 . 2 + (n - 1)3]$
$= \frac{n}{2}[4 + 3n - 3]$
$= \frac{n}{2}[3n + 1]$

OR

33(B). If the sum of the first p terms of an A.P. is the same as the sum of its first q terms, $(p \neq q)$, then show that the sum of its first $(p + q)$ terms is zero.

Explanation:

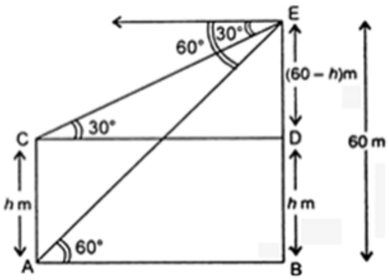
$Given\ S_p = S_q$
$\Rightarrow \frac{p}{2}(2a + (p - 1)d) = \frac{q}{2}(2a + (q - 1)d)$
$\Rightarrow p(2a + (p - 1)d) = q(2a + (q - 1)d)$
$\Rightarrow 2ap + p^2d - pd = 2aq + q^2d - qd$
$\Rightarrow 2a(p - q) + (p + q)(p - q)d - d(p - q) = 0$
$\Rightarrow (p - q)[2a + (p + q)d - d] = 0$
$\Rightarrow 2a + (p + q)d - d = 0$
$\Rightarrow 2a + ((p + q) - 1)d = 0$
$\Rightarrow S_{p+q} = 0$

34(a). From the top of a building $60\ m$ high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower. Also, find the distance between the building and the tower. (Use $\sqrt{3} = 1.732$)

Answer: $40\ m\ and\ 34.641\ m$

Explanation:

Let AC be the tower and BE be the building. Let height of the tower be hm. It is given that the angles of depression of the top C and bottom A of the tower, observed from top of the building be 30° and 60° respectively.



In right triangle CDE, we have

$tan30^{\circ} = \frac{DE}{CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{CD}$ $[DE = BE - BD = BE - AC]$

$\Rightarrow CD = \sqrt{3}(60 - h)$..... (1)

In right triangle ABE, we have

$tan60^{\circ} = \frac{BE}{AB}$

$\Rightarrow \sqrt{3} = \frac{60}{CD} (AB = CD)$

$\Rightarrow CD = \frac{60}{\sqrt{3}}$.... (2)

Comparing (1) and (2), we get

$\sqrt{3}(60 - h) = \frac{60}{\sqrt{3}}$

$\Rightarrow 3(60 - h) = 60$

$\Rightarrow 180 - 3h = 60$

$\Rightarrow 3h = 120$

$\Rightarrow h = 40$

Hence, the height of the tower is $40\ m$.

Now, from ΔABE,

$\tan tan\ 60° = \frac{BE}{AB}$

$\Rightarrow \sqrt{3} = \frac{60}{AB}$

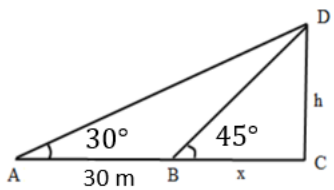
$\Rightarrow AB = \frac{60}{\sqrt{3}} = \frac{60}{1.732} = 34.641$

Hence, the distance between the building and the tower is $34.641\ m$

OR

34(b). The angle of elevation of the top of a building from a point A on the ground is 30°. On moving a distance of $30\ m$ towards its base to the point B, the angle of elevation

changes to 45°. Find the height of the building and the distance of its base from point A.
(Use $\sqrt{3} = 1{\cdot}732$)
Answer: $40.98\,m$ and $70.98\,m$
Explanation:



Let the height of the tower be $h\,m$.
In right $\triangle DCB$,
$tan45 = \frac{DC}{CB}$
$1 = \frac{h}{x}$
$x = h$.....(1)
Again in triangle DCA,
$tan30 = \frac{DC}{CA}$
$\frac{1}{\sqrt{3}} = \frac{h}{x+30}$
$\frac{1}{\sqrt{3}} = \frac{h}{h+30}$ (from eq(1)
$h + 30 = h\sqrt{3}$
$30 = h\sqrt{3} - h = h(\sqrt{3} - 1)$
$h = \frac{30}{\sqrt{3}-1} = 40.98\,m$
Hence, the height of the building is $40.98\,m$
$x = h = 40.98\,m$
Thus, distance of tower from point $A = x + 30$
$= 40.98 + 30 = 70.98\,m$

35. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

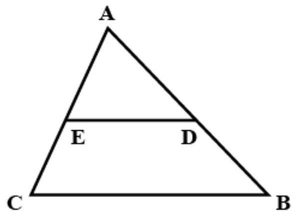
Explanation:
Given:
$DE \parallel BC$
To prove that:

$\frac{EC}{AE} = \frac{BD}{AD}$

Proof:

$\angle AED = \angle ACB$ (*Corresponding angles*)

$\angle ADE = \angle ABC$ (*Corresponding angles*)



$\angle EAD$ is common to both the triangles

$\Rightarrow \Delta AED \sim \Delta ACB$ (by AAA similarity)

$\Rightarrow \frac{AC}{AE} = \frac{AB}{AD}$

$\Rightarrow \frac{AE+EC}{AE} = \frac{AD+BD}{AD}$

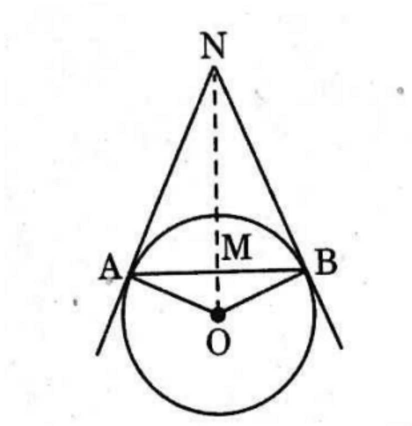
$\Rightarrow \frac{EC}{AE} = \frac{BD}{AD}$

Hence proved

SECTION - E

Section - E consists of three Case Study Based questions of 4 marks each.

36. Circles play an important part in our life. When a circular object is hung on the wall with a cord at nail N, the cords NA and NB work like tangents. Observe the figure, given that $\angle ANO = 30^\circ$ and $OA = 5\ cm$.



Based on the above, answer the following questions:
(i) Find the distance AN.
(ii) Find the measure of $\angle AOB$.
(iii) (a) Find the total length of cords NA, NB and the chord AB.

OR

(iii) (b) If $\angle ANO$ is 45°, then name the type of quadrilateral OANB. Justify your answer.

Solution:

(i) Given $\angle ANO = 30^{\circ}$

$\angle OAM = 30^{\circ}$

In right angle $\triangle OMA$

$cos30^{\circ} = \frac{AM}{OA}$

$\frac{\sqrt{3}}{2} = \frac{AM}{5}$

$AM = \frac{5\sqrt{3}}{2}$

Now in right angle $\triangle AMN$

$sin30^{\circ} = \frac{AM}{AN}$

$\frac{1}{2} = \frac{5\sqrt{3}}{2\cdot AN}$

$AN = 5\sqrt{3}$

(ii) $\angle OAB = \angle OBA$[Angles opposite to equal side of triangle are equal]

$[\because OA = OB = radius\ of\ \ circle\ ]$

$\angle OAB + \angle OBA + \angle BOA = 180^{\circ}$

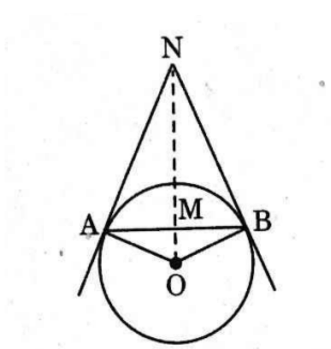
$30^{\circ} + 30^{\circ} + \angle BOA = 180^{\circ}$

$\angle BOA = 60^{\circ}$

(iii) (a) Length of $AB = 2\times AM = 5\sqrt{3}$
Length of $NA = NB = 5\sqrt{3}$
Total length of $NA + NB + AB = 5\sqrt{3} + 5\sqrt{3} + 5\sqrt{3}$
$NA + NB + AB = 15\sqrt{3}\ cm$

OR

(iii) (b)



Given that $\angle ANO = 45^{\circ}$

Thus, $\angle BNO = \angle ANO = 45^\circ$

i.e. $\angle ANB = \angle BNO + \angle ANO = 90^\circ$

Also, $\angle NAO = \angle NBO = 90^\circ$ (The tangent to the circle is perpendicular to the radius of the circle at the point of contact.)

Also, AO = BO (radii of the circle)

In Quadrilateral OANB,

$\angle BOA + \angle NAO + \angle ANB + \angle NBO = 360^\circ$

$\angle BOA = 360^\circ - (90^\circ + 90^\circ + 90^\circ) = 90^\circ$

Now, $\angle AMO = \angle BMO = 90^\circ$ and AM = MB (A perpendicular dropped from the center of the circle to a chord bisects it.)

We have a quadrilateral in which adjacent sides are equal and all the angles are 90°. Diagonals of this quadrilateral bisect each other perpendicularly.

Hence, Quadrilateral OANB is a square.

37. A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions $14\ cm \times 17\ cm \times 4\ cm$. On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height $3\ cm$ and radius $2\ cm$.

Based on the above, answer the following questions :

(i) Find the volume of wood carved out to make one cylindrical hollow.

(ii) Find the lateral surface area of the cuboid to paint it with green colour.

(iii) (a) Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows.

OR

(iii) (b) Find the surface area of the top surface of the cuboid to be painted yellow.

Explanation:

(i) Volume of cylinder $=\pi r^2 h$
Given that,
r = 2 cm
h = 3 cm

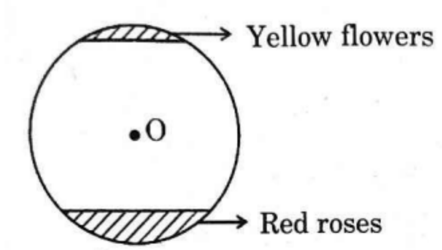
The volume of wood carved out to make one cylindrical hollow = $\pi.\,2^2.\,3$
$= \frac{22}{7} \times 2 \times 2 \times 3$
= 37.7 cm^3
(ii) Lateral surface area of the cuboid = 2h(l + b)
= 2 x 14 (17 + 4)
= 28 x 21
= 588 cm^2

(iii) (a) Volume of Cuboid = lbh
= 14 x 17 x 4 = 952 cm^3
Volume of wood carved out to make one cylindrical hollow = 37.7 cm^3
So, Volume of wood carved out to make 7 cylindrical hollow = 7 x 37.7 = 263.9 cm^3
The volume of wood in the remaining cuboid after carving out seven cylindrical hollows = 952 - 263.9 = 688.1 cm^3

OR

(b) Surface area of the top surface of the cuboid = $14\ cm \times 17\ cm$ = 238 cm^2

38. Flower beds look beautiful growing in gardens. One such circular park of radius ' r ' m, has two segments with flowers. One segment which subtends an angle of 90° at the centre is full of red roses, while the other segment with central angle 60° is full of yellow coloured flowers. [See figure]



It is given that the combined area of the two segments (of flowers) is $256\frac{2}{3}\, sq\ m$.

Based on the above, answer the following questions :

(i) Write an equation representing the total area of the two segments in terms of ' r '.

(ii) Find the value of ' r '.

(iii) (a) Find the area of the segment with red roses.

OR

(iii) (b) Find the area of the segment with yellow flowers.

Explanation:

Let the radius of the circular park be 'r' meters.

(i) The total area of the two segments can be represented as the sum of the areas of the segment with red roses and the segment with yellow flowers:

Total area = Area of red rose segment + Area of yellow flower segment

We know that the area of each segment is given by:

Area of segment = $(\theta/360)\pi r^2 - 1/2r^2\sin(\theta)$

where θ is the central angle of the segment.

Using this formula, we can write the equation for the total area of the two segments as:

$(90/360)\pi r^2 - 1/2r^2\sin(90°) + (60/360)\pi r^2 - 1/2r^2\sin(60°) = 256\frac{2}{3}$

Simplifying this equation, we get:

$(1/4)\pi r^2 - (½)r^2 + (1/6)\pi r^2 - (1/4)r^2\sqrt{3} = 256.66$

(ii) To find the value of 'r', we need to solve the quadratic equation obtained in part (i) for 'r'. After simplifying using quadratic formula and solving for 'r', we get:

$r \cong 26.13$ meters

(iii) (a) To find the area of the segment with red roses, we can use the formula for the area of a segment with central angle θ:

Area of red rose segment = $(90/360)\pi r^2 - 1/2r^2\sin(90°)$

Substituting the value of 'r' obtained in part (ii), we get:

Area of red rose segment $\cong (90/360)\pi(26.13)^2 - 1/2(26.13)^2\sin(90°)$
$\cong 535.98 - 341.39$

$\cong 194.59$ sqm.

Hence, the area of the segment with red roses is 194.59 sqm.

OR

(iii) (b) To find the area of the segment with yellow flowers, we can use the formula for the area of a segment with central angle θ:

Area of yellow flower segment = $(60/360)\pi r^2 - 1/2r^2\sin(60°)$

Substituting the value of 'r' obtained in part (ii), we get:

Area of yellow flower segment $\cong (60/360)\pi(26.13)^2 - 1/2(26.13)^2\sin(60°)$

$\cong 357.32 - 295.65$
$\cong 61.67$ sqm.

Hence, the area of the segment with yellow flowers is 61.67 sqm.