Mathematics (Standard)
SET – 1 (Q.P. Code 30/4/1)

Time Allowed: 3 Hours **Maximum Marks:** 80

Note:

(i) Please check that this question paper contains 15 printed pages.

(ii) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) Please check that this question paper contains 38 questions.

(iv) Please write down the Serial Number of the question in the answer-book before attempting it.

(v) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

GENERAL INSTRUCTIONS:

Read the following instructions carefully and follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE sections - Section A, B, C, D and E.

(iii) In section A - question number 1 to 18 are multiple choice questions (MCQs) and question number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In section B-question number 21 to 25 are Very Short Answer (VSA) type questions of 2 marks each.

(v) In section C-question number 26 to 31 are Short Answer (SA) type questions carrying 3:marks each.

(vi) In section D - question number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In section E - question number 36 to 38 are case based integrated units of assessment questions carrying 4 marks each. Internal choice is provided in 2 marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section $C, 2$ questions in Section D and 3 questions in Section E.

(ix) Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

(x) Use of calculators is NOT allowed.

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SECTION A

Section - A consists of Multiple Choice type questions of 1 mark each.

1. The ratio of HCF to LCM of the least composite number and the least prime number is :

(a) 1:2

(b) $2:1$

(c) $1:1$

(d) $1:3$

Answer: (a)

Explanation:

The least composite number is 4, which has prime factors 2 and 2. The least prime number is 2.

The HCF (Highest Common Factor) of 4 and 2 is 2, because that is the largest number that divides both 4 and 2.

The LCM (Least Common Multiple) of 4 and 2 is 4, because that is the smallest number that is a multiple of both 4 and 2.

Therefore, the ratio of HCF to LCM of 4 and 2 is 2:4 or 1:2.

So the answer is (a) 1:2.

2. The roots of the equation $x^2 + 3x - 10 = 0$ are :

(a) $2, -5$

(b) $-2, 5$

(c) 2,5

(d) $-2, -5$

Answer: (a)

Explanation:

Given, $x^2 + 3x - 10 = 0$

$x^2 + 5x - 2x - 10 = 0$

$x(x + 5) - 2(x + 5) = 0$

$(x + 5)(x - 2) = 0$

$x = 2, -5$ are the roots of the given equation.

3. The next term of the A.P. : $\sqrt{6}, \sqrt{24}, \sqrt{54}$ is :

(a) $\sqrt{60}$

(b) $\sqrt{96}$

(c) $\sqrt{72}$(d) $\sqrt{216}$

Answer: (b)
Explanation:

Given the series

$\sqrt{6}, \sqrt{24}, \sqrt{54}$

We can write this series.

$\sqrt{6}, 2\sqrt{6}, 3\sqrt{6}$ which is A.P.

and common difference is $\sqrt{6}$

Next term can be obtained by adding common difference $\sqrt{6}$ than

$\sqrt{6}, 2\sqrt{6}, 3\sqrt{6}, 4\sqrt{6}$

We can write it as

$\sqrt{6}, \sqrt{24}, \sqrt{54}, \sqrt{96}$

[The next term is $\sqrt{96}$]

4. The distance of the point $(-\ 1, 7)$ from x-axis is :

(a) -1

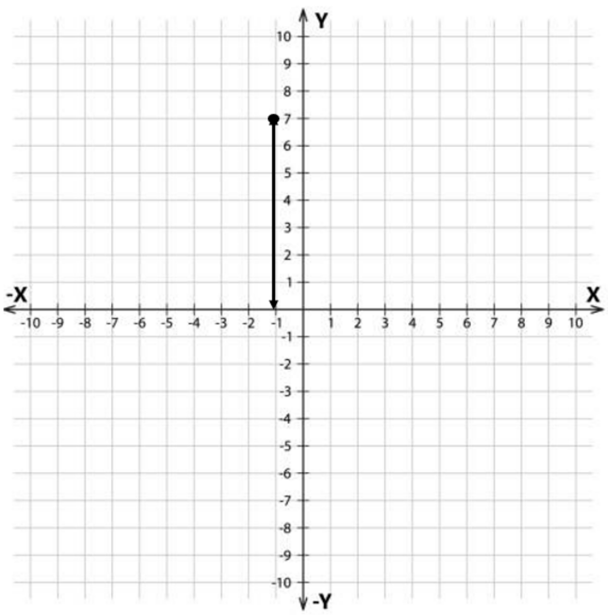
(b) 7

(c) 6

(d) $\sqrt{50}$

Answer: (b)
Explanation:

The x-axis is the horizontal line with an equation of y = 0. Therefore, the distance between the point (-1,7) and the x-axis is simply the vertical distance from the point to the x-axis, which is equal to the absolute value of the y-coordinate of the point.



Therefore, the distance of the point (-1,7) from the x-axis is = 7

So, the answer is (b) 7.

5. What is the area of a semi-circle of diameter ' d ' ?

(a) $\frac{1}{16}\pi d^2$

(b) $\frac{1}{4}\pi d^2$

(c) $\frac{1}{8}\pi d^2$

(d) $\frac{1}{2}\pi d^2$

Answer: (c)
Explanation:
The area of a semi-circle of diameter 'd' can be found using the formula for the area of a circle, which is A = πr^2, where r is the radius of the circle.

In a semi-circle, the diameter 'd' is equal to twice the radius 'r'. Therefore, we can write:
d = 2r
Solving for 'r', we get:
r = $\frac{d}{2}$
Now, substituting this value of 'r' in the formula for the area of a circle, we get:
A = $\pi(\frac{d}{2})^2$
Simplifying this expression, we get:
A = $\pi\frac{d^2}{4}$
But this is only the area of half the circle. Therefore, the area of the semi-circle is:
$\frac{A}{2} = \pi\frac{d^2}{4}/2 = \pi\frac{d^2}{8}$
So, the correct answer is (c) $\frac{1}{8}\pi d^2$.

6. The empirical relation between the mode, median and mean of a distribution is:
(a) Mode $=$ 3 Median $-$ 2 Mean
(b) Mode $=$ 3 Mean $-$ 2 Median
(c) Mode $=$ 2 Median $-$ 3 Mean
(d) Mode $=$ 2 Mean $-$ 3 Median
Answer: (a)
Explanation:
Empirical relationship between mean, median and mode is:
Mode = 3 Median - 2 Mean

7. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:
(a) intersecting
(b) parallel
(c) coincident
(d) either intersecting or parallel
Answer: (c)
Explanation:
The equation can be written as

$2x - 5y - 6 = 0 \text{ and } 6x - 15y - 18 = 0$

Here $a_1 = 2,\ b_1 = -5, c_1 = -6$ and $a_2 = 6,\ b_2 = -15, c_2 = -18$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{-5}{-15} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-6}{-18} = \frac{1}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence the given pair of linear equation represents coincident line.

8. If α, β are zeroes of the polynomial $x^2 - 1$, then value of $(\alpha + \beta)$ is :

(a) 2
(b) 1
(c) -1
(d) 0

Answer: (d)
Explanation:

$f(x) = x^2 - 1 \quad a = 1 \quad b = 0 \quad c = -1$

$\because \alpha$ and β are the zeroes of above polynomial.

$\therefore Sum\ of\ roots = \frac{-b}{a} \quad \Rightarrow \alpha + \beta = \frac{0}{1} \quad \Rightarrow \alpha + \beta = 0$

Hence, 0 is the correct answer.

9. If a pole $6\ m$ high casts a shadow $2\sqrt{3}\ m$ long on the ground, then sun's elevation is :

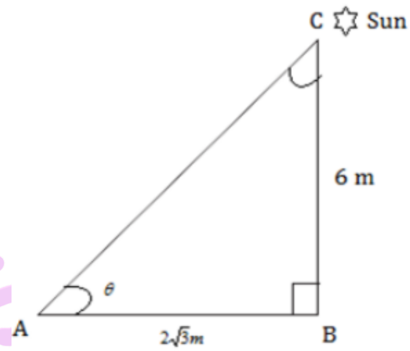
(a) 60°
(b) 45°
(c) 30°
(d) 90°

Answer: (a)
Explanation:

The answer is (a).

Let $BC = 6\,m$ be the height of the pole and $AB = 2\sqrt{3}\,m$ be the length of the shadow on the ground.

Let the Sun makes an angle θ on the ground.



Now, in $\triangle BAC$,

$tan\theta = \frac{BC}{AB}$

$\Rightarrow tan\theta = \frac{6}{2\sqrt{3}}$

$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow tan\theta = \frac{3\sqrt{3}}{3}$

$= \sqrt{3} = tan60^{\circ}$

$\therefore \theta = 60^{\circ}$

Hence, the Sun's elevation is 60°

10. $sec\theta$ when expressed in terms of $cot\theta$, is equal to :

(a) $\frac{1+cot^2\theta}{cot\theta}$

(b) $\sqrt{1 + cot^2\theta}$

(c) $\frac{\sqrt{1+cot^2\theta}}{cot\theta}$

(d) $\frac{\sqrt{1-cot^2\theta}}{cot\theta}$

Answer: (c)
Explanation:

$sec^2\theta = 1 + tan^2\theta$

$= 1 + \frac{1}{cot^2\theta}$

$= \frac{cot^2\theta+1}{cot^2\theta}$

$sec\theta = \sqrt{\frac{cot^2\theta+1}{cot^2\theta}}$

$= \frac{\sqrt{cot^2\theta+1}}{cot\theta}$

11. Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is :

(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

Answer: (c)

Explanation:

To find the probability of getting the difference of numbers on two dice equals to 3, we need to count the number of ways we can get a difference of 3, and then divide by the total number of possible outcomes.

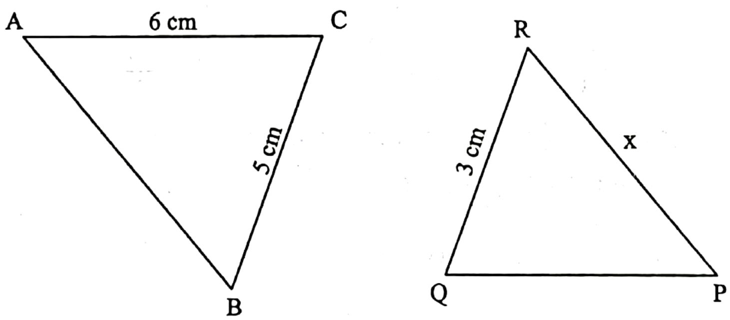
Let A be the event that the difference of numbers on the two dice is 3. There are two ways this can happen: either the first die shows a number 3 higher than the second die, or the second die shows a number 3 higher than the first die.

Let's consider each case separately:

The first die shows a number 3 higher than the second die: there are 3 possible outcomes for the second die (1, 2, or 3), and for each of these outcomes, there is exactly 1 outcome for the first die that gives a difference of 3. So there are a total of 3 outcomes in this case.
The second die shows a number 3 higher than the first die: this is the same as the first case, but with the dice swapped. So there are also 3 outcomes in this case.
Therefore, the total number of outcomes in event A is 3 + 3 = 6.

Now let's consider the total number of possible outcomes when two dice are thrown together. There are 6 possible outcomes for the first die and 6 possible outcomes for the second die, giving a total of 6 x 6 = 36 possible outcomes.
Therefore, the probability of getting a difference of 3 is
P(A) = number of outcomes in A / total number of possible outcomes $= \frac{6}{36} = \frac{1}{6}$

12. In the given figure, $\triangle ABC \sim \triangle QPR$. If $AC = 6\ cm, BC = 5\ cm, QR = 3\ cm$ and $PR = x$; then the value of x is :



(a) $3.6\ cm$
(b) $2.5 cm$
(c) $10\ cm$
(d) $3.2\ cm$
Answer: (b)
Explanation:
So, by proportionality theorem

$\frac{AC}{BC} = \frac{QR}{PR}$

$\frac{6}{5} = \frac{3}{x}$

$\therefore x = 2.5\ cm$

13. The distance of the point $(-\ 6, 8)$ from origin is :
(a) 6
(b) -6
(c) 8
(d) 10
Answer: (d)

Explanation:
We know that the formula to find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

It is given that, we have to find the distance of the point $P(-6, 8)$ from origin i.e. $(0, 0)$
Substituting these values in the equation we get

Distance $= \sqrt{(0 - (-6))^2 + (0 - 8)^2}$

So we get, Distance of point $P = \sqrt{(36 + 64)} = \sqrt{100} = 10$
Distance of point $P = 10$ units
Therefore, the distance of the point $P(-6, 8)$ from the origin is 10 units.

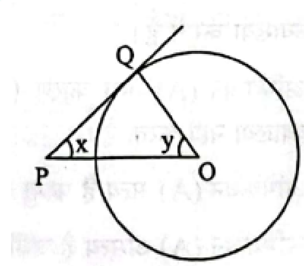
14. In the given figure, PQ is a tangent to the circle with centre O. If $\angle OPQ = x, \angle POQ = y$,
then $x + y$ is:

(a) 45°

(b) 90°

(c) 60°

(d) 180°



Answer: (b)
Explanation:
In $\triangle OPQ$,
We know that

$$\angle POQ + \angle PQO + \angle OPQ = 180$$

$$y + x + \angle OPQ = 180^\circ$$

OP is a radius through P, the point of contact of the tangent PQ with the given circle

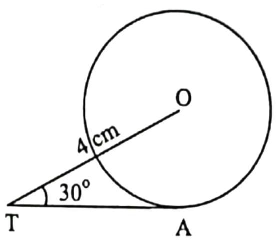
$\angle OPQ = 90^\circ$ since the radius through the point of contact of a tangent to a circle is perpendicular to the tangent.

$$\angle OPQ = 90^\circ$$

Thus, $y + x + 90^\circ = 180^\circ$

Or, $x + y = 90^\circ$

15. In the given figure, TA is a tangent to the circle with centre O such that $OT = 4\ cm, \angle OTA = 30^{\circ}$, then length of TA is :



(a) $2\sqrt{3}\ cm$
(b) $2\ cm$
(c) $2\sqrt{2}\ cm$
(d) $\sqrt{3}\ cm$

Answer: (a)

Explanation:

We know tangent makes 90° with the centre.

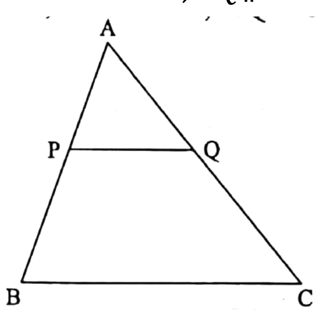
Given $\angle ATO = 30^{\circ}$ & $OT = 4\ cm$

$cos30^{\circ} = \frac{AT}{OT} = \frac{AT}{4}$

$\frac{\sqrt{3}}{2} = \frac{AT}{4}$

$\Rightarrow AT = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm

16. In $\triangle ABC, PQ \| BC$. If $PB = 6\ cm, AP = 4\ cm, AQ = 8\ cm$, find the length of AC.



(a) $12\ cm$
(b) $20 cm$

(c) $6\ cm$
(d) $14\ cm$
Answer: (b)
Explanation:
In $\triangle ABC, PQ \parallel BC$
So, by proportionality theorem

$$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{PQ}{BC}$$

$$\frac{4}{6} = \frac{8}{QC}$$

$$\therefore QC = 12\ cm$$

$$AC = AQ + QC$$

$$AC = 8 + 12$$

$$AC = 20cm$$

17. If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to :
(a) $\frac{7}{3}$
(b) $\frac{-7}{3}$
(c) $\frac{3}{7}$
(d) $\frac{-3}{7}$
Answer: (d)
Explanation:
Given: α, β are zeroes of the polynomial $4x^2 - 3x - 7$
To find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$
We know that equation $a^2 + bx + c = 0$
Then sum of roots $= \frac{-b}{a}$ and product of roots $= \frac{c}{a}$
From the given quadratic equation, $\alpha + \beta =- \frac{(-3)}{4} = \frac{3}{4}$ and $\alpha\beta = \frac{-7}{4}$
Therefore,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

$= \frac{\frac{3}{4}}{\frac{-7}{4}} = -\frac{3}{7}$

$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{7}$

18. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is :

(a) $\frac{1}{13}$

(b) $\frac{9}{13}$

(c) $\frac{4}{13}$

(d) $\frac{12}{13}$

Answer: (d)

Explanation:

Total number of cards $= 52$.

So the number of possible outcomes $= 52$.

Let E be the event of getting a non-ace card.

There will be 48 non-ace cards.

Number of favorable outcomes $= 48$

$P(E) = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes}$

$P(E) = \frac{48}{52} = \frac{24}{26} = \frac{12}{13}$

Hence the probability of getting a non-ace card is $\frac{12}{13}$.

19. Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R) : The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Answer: (c)

Explanation:

Assertion: In a leap year (366) days, $\frac{364}{7} = 52$ are complete weeks, rest two days can be:-

Sun M, M T, T We, We Th, Th Fr, Fr Sa, Sa Su.

Out of which, 2 combinations would given 53 Sundays.

$\Rightarrow Probability = \frac{2}{7}$, It is true

Reason: A non-leap year has 365 days

A year has 52 weeks. Hence there will be 52 Sundays for sure.

52 weeks $= 52\times7 = 364$ days .

$365 - 364 = 1$ day extra.

In a non-leap year there will be 52 Sundays and 1 day will be left.

This 1 day can be Sunday, Monday, Tuesday, Wednesday, Thursday, friday, Saturday, Sunday.

Of these total 7 outcomes, the favourable outcomes are 1.

Hence the probability of getting 53 sundays $= \frac{1}{7}$ but given $\frac{5}{7}$ (False)

So, Assertion (A) is true but Reason (R) is false.

20. Assertion (A): a, b, c are in A.P. if and only if $2b = a + c$.

Reason (R) **:** The sum of first n odd natural numbers is n^2.

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, and Reason (R) is not the correct explanation of Assertion (A).

(e) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Answer: (b)

Explanation:

Assertion (A) is the definition of an arithmetic progression. If a, b, and c are in AP, then the common difference is d = b − a = c − b. Thus, we have b = (a + c)/2, which is equivalent to 2b = a + c.

Reason (R) is also true.
The first n odd natural numbers are provided by 1,3,5,7,9,….,(2n-1). This is formed through Arithmetic Progression.
Here,
a=1
d=2
t_n= (2n-1)
The formula of an A.P series –
S = (n/2)[2a + (n − 1) × d]
S = (n /2)[2 + 2n – 2]
S = (n /2) [2n]
S = n^2
Hence, Both Assertion (A) and Reason (R) are true, and Reason (R) is not the correct explanation of Assertion (A).

SECTION B

Section - B consists of Very Short Answer (VSA) type questions of 2 marks each.

21. Two numbers are in the ratio $2:3$ and their LCM is 180 . What is the HCF of these numbers?

Answer: 30

Explanation:

Given: Ratio of the numbers $= 2:3$, LCM of numbers $= 180$, $HCF\ =?$
We know that, product of LCM and HCF of two numbers is equal to the product of the numbers
$LCM\times HCF\ = a\times b$ (a, b are the two numbers)
Let, numbers $= 2x$ and $3x$
$\therefore LCM\ = 2\times 3\times x\ = 6x$
$\therefore 6x\ = 180$
$\therefore x\ = 180/6\ = 30$
⇒ Numbers are $30\times 2\ = 60$ and $30\times 3\ = 90$
Now, LCM $\times HCF\ = ab(a, b$ are the numbers)
$\therefore 180\times HCF\ = 60\times 90$
$\therefore HCF\ = 5400/180\ = 30$

Therefore, $HCF = 30$

22. If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k.

Answer: -4

Explanation:

Given: One zero of $p(x) = 6x^2 + 37x - (k - 2)$is reciprocal to the other.

let α be the one root of the quadratic equation.

So, the other root will be $\frac{1}{\alpha}$.

We know that for the quadratic equation $ax^2 + bx + c$ product of the roots is given by $\frac{c}{a}$.

Thus, the product of the roots of given quadratic equation $= \frac{-(k-2)}{6}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{-(k-2)}{6}$

$\Rightarrow 6(1) =- k + 2$

$\Rightarrow k = 2 - 6 =- 4$

Hence, the value of k is -4 .

23 (A). Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$

Answer: $\frac{9}{2}$ *and* 2

Explanation:

We know that for a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $-\frac{b}{a}$ and the product of the roots is $\frac{c}{a}$.

Here, the given quadratic equation $2x^2 - 9x + 4 = 0$ is in the form $ax^2 + bx + c = 0$ where $a = 2,\ b =- 9$ and $c = 4$.

The sum of the roots is $-\frac{b}{a}$ that is:

$-\frac{b}{a} =- \frac{(-9)}{2} = \frac{9}{2}$

The product of the roots is $\frac{c}{a}$ that is:

$\frac{c}{a} = \frac{4}{2} = 2$

Hence, sum of the roots is $\frac{9}{2}$ and the product of the roots is 2.

OR

23 (B). Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation.

Answer: 80 and Roots are real and distinct

Explanation:

Given equation: $4x^2 - 5 = 0$

Comparing with $ax^2 + bx + c = 0$, we get, $a = 4, b = 0, c = - 5$

Therefore, $D = b^2 - 4ac$

$D = 0^2 - 4(4)(- 5) = 80$

Thus $D > 0$

∴ Roots are real and distinct.

24. If a fair coin is tossed twice, find the probability of getting 'atmost one head'.

Answer: $\frac{3}{4}$

Explanation:

Given, a coin is tossed two times.

We have to find the probability of getting at most one head.

When a coin is tossed two times, The possible outcomes are $\{TT, HH, TH, HT\}$

Number of possible outcomes $= 4$

Favourable outcomes $= \{HH, HT, TH\}$

Number of favourable outcomes $= 3$

Probability $=$ number of favourable outcomes / number of possible outcomes

Probability of getting at most one head $= \frac{3}{4}$

Therefore, the probability of getting at most one head is $\frac{3}{4}$.

25 (A). Evaluate $\frac{5cos^2 60^\circ + 4sec^2 30^\circ - tan^2 45^\circ}{sin^2 30^\circ + cos^2 30^\circ}$

Answer: $\frac{67}{12}$

Explanation:

Given, $\frac{5cos^2 60+4sec^2 30-tan^2 45}{sin^2 30+cos^2 30}$

$$\Rightarrow \frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}}$$

$$\Rightarrow \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{4}{4}} \Rightarrow \frac{3+64}{12} = \frac{67}{12}$$

Hence, $\frac{5cos^2 60^\circ+4sec^2 30^\circ-tan^2 45^\circ}{sin^2 30^\circ+cos^2 30^\circ} = \frac{67}{12}$

OR

25 (B). If A and B are acute angles such that $sin(A - B) = 0$ and $2cos(A + B) - 1 = 0$, then find angles A and B.

Answer: $A = 30 \; and \; B = 30$

Explanation:

Given, $sin\,(A - B) = 0$ and $2cos(A + B) - 1 = 0$

$sin\,(A - B) = 0 = sin\,0$

$A - B = 0$..... (1)

$cos(A + B) = 1/2 = cos60$

$A + B = 60$..... (2)

By adding eq 1 and 2 , we get

$A - B + A + B = 0 + 60$

$2A = 60$

$A = 60/2 = 30$

according to eq 1, $A - B = 0$

Then, $B = A - 0 = 30 - 0 = 30$

Hence, $A = 30, B = 30$

SECTION - C

Section - C consists of Short Answer (SA) type questions of 3 marks each.

26(A). How many terms are there in an A.P. whose first and fifth terms are -14 and 2, respectively and the last term is 62.

Explanation: Given:-

first term of the A.P. $a = -14$

Let the common difference be d.

ATQ

$a + 4d = 2$

$-14 + 4d = 2$

$4d = 16$

$d = 4$

Now the last form will be given by

$l = a + (n - 1)d$

$62 = -14 + (n - 1)4$

$76 = (n - 1)4$

$(n - 1) = 19$

$n = 20$

OR

26(B). Which term of the A.P. : $65, 61, 57, 53,$ is the first negative term?

Explanation:

Here first term (a) $= 65$

second term $= 61$

Common difference $(d) = 61 - 65$

$d = -4$

Let the n^{th} term of the given AP be the first negative term.

$\therefore a_n < 0 \Rightarrow a + (n - 1)d < 0$

$\Rightarrow (n - 1)(-4) < -65$

$\Rightarrow 65 + (n - 1)(-4) < 0$

$\Rightarrow n - 1 > \frac{65}{4}$

$\Rightarrow n > \frac{65}{4} + 1$

$\Rightarrow n > \frac{69}{4}$

$\Rightarrow n > 17\frac{1}{4}$

Since 18 is the natural number just greater than $17\frac{1}{4}$

So $n = 18$

$\therefore 18^{th}$ term is first negative term.

27. Prove that $\sqrt{5}$ is an irrational number

Explanation: Let us prove that $\sqrt{5}$ is an irrational number.
This question can be proved with the help of the contradiction method.
Let's assume that $\sqrt{5}$ is a rational number. If $\sqrt{5}$ is rational, that means it can be written in the form of a/b, where a and b integers that have no common factor other than 1 and $b \neq 0$.

$\frac{\sqrt{5}}{1} = \frac{a}{b}$

$\sqrt{5}b = a$

Squaring both sides,

$5b^2 = a^2$

$b^2 = a^2/5$

This means 5 divides a^2.
That means it also divides a.

$\frac{a}{5} = c$

$a = 5c$

On squaring, we get

$a^2 = 25c^2$

Put the value of a^2 in equation (1).

$5b^2 = 25c^2$

$b^2 = 5c^2$

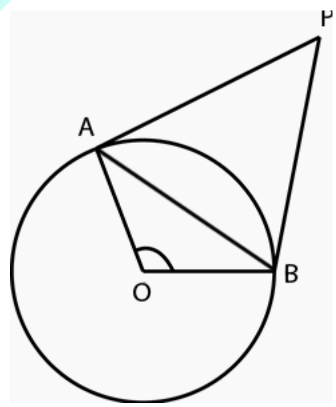
$b^2/5 = c^2$

This means b^2 is divisible by 5 and so b is also divisible by 5 . Therefore, a and b have 5 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number. So, we conclude that $\sqrt{5}$ is irrational.

28. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Explanation:

Draw a circle with center O and take a external point P. PA and PB are the tangents. As radius of the circle is perpendicular to the tangent.



$OA \perp PA$

Similarly $OB \perp PB$

$\angle OBP = 90^\circ$

$\angle OAP = 90^\circ$

In quadrilateral OAPB, the sum of all interior angles $= 360^\circ$

$\Rightarrow \angle OAP + \angle OBP + \angle BOA + \angle APB = 360^\circ$

$\Rightarrow 90^\circ + 90^\circ + \angle BOA + \angle APB = 360^\circ$

$\angle BOA + \angle APB = 180^\circ$

It proves the angle between the two tangents drawn from an external point to a circle supplementary to the angle subtented by the line segment.

29(A). Prove that $\frac{sinA - 2sin^3A}{2cos^3A - cosA} = tanA$

Explanation:

LHS $= \frac{sinA - 2sin^3A}{2cos^3A - cosA} = \frac{sinA}{cosA}\left(\frac{1-2sin^2A}{2cos^2A-1}\right)$

$= \frac{sinA}{cosA}\left(\frac{sin^2A+cos^2A-2sin^2A}{2cos^2A-(sin^2A+cos^2A)}\right)$

$= \frac{sinA}{cosA}\left(\frac{cos^2A-sin^2A}{cos^2A-sin^2A}\right)$

$= tanA$

OR

29(B). Prove that $secA(1 - sinA)(secA + tanA) = 1$

Explanation:

Solving L.H.S

$secA(1 - sinA)(secA + tanA)$

We write everything in terms of $sinA$ and $cosA$

$= \frac{1}{cosA}(1 - sinA)\left(\frac{1}{cosA} + \frac{sinA}{cosA}\right)$

$= \frac{(1-sinA)}{cosA}\left(\frac{1+sinA}{cosA}\right)$

$= \frac{(1-sinA)(1+sinA)}{cosA \times cosA}$

We know that $(a - b)(a + b) = a^2 - b^2$

$= \frac{\left(1^2-sin^2A\right)}{cos^2A} \quad = \frac{\left(1-sin^2A\right)}{cos^2A}$

$= \frac{cos^2A}{cos^2A}$

$= 1 \quad = R.H.S$

Thus, L.H.S = R.H.S

Hence proved

$Formula\ used$

$cos^2A + sin^2A = 1$

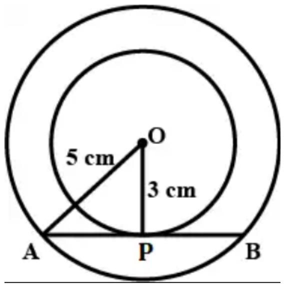
$\Rightarrow cos^2A = 1 - sin^2A$

$\Rightarrow 1 - sin^2A = cos^2A$

30. Two concentric circles are of radii $5\ cm$ and $3\ cm$. Find the length of the chord of the larger circle which touches the smaller circle.

Explanation:

Let O be the centre of the concentric circle of radii $5\ cm$ and $3\ cm$ respectively. Let AB be a chord of the larger circle touching the smaller circle at P.



Then

$AP = PB$ and $OP \perp AB$

Applying Pythagoras theorem in $\triangle OPA$, we have

$$OA^2 = OP^2 + AP^2$$
$$\Rightarrow 25 = 9 + AP^2$$
$$\Rightarrow AP^2 = 16$$
$$\Rightarrow AP = 4\,cm$$
$$\therefore AB = 2AP = 8\,cm$$

31. Find the value of ' p ' for which the quadratic equation $px(x - 2) + 6 = 0$ has two equal real roots.

Explanation:

Given that the roots of the quadratic equation $px(x - 2) + 6 = 0$ are equal

$$px^2 - 2px + 6 = 0$$

Comparing with general equation $ax^2 + bx + c = 0$, for the given equation

$$a = p,\ b = - 2p,\ c = 6$$

Hence $D = b^2 - 4ac = 0$

$$(-2p)^2 - 4 \cdot p \cdot 6 = 0$$
$$4p^2 - 24p = 0$$
$$4p(p - 6) = 0$$

$4p = 0$ or $(p - 6) = 0$

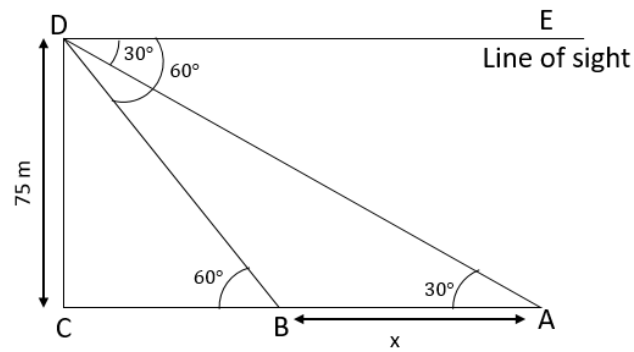
$p = 0$ or $p = 6$

Putting $p = 0$ in equation given we get $6 = 0$ that is not possible

Hence value of $p = 6$ for which the equation has equal roots.

32. (A) A straight highway leads to the foot of a tower. A man standing on the top of the $75\ m$ high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (use $\sqrt{3} = 1.73$)

Explanation:



Let x be the distance between two cars

$In\ \triangle CBD$

$tan60^{\circ} = \frac{75}{BC}$

$BC \ = \frac{75}{\sqrt{3}}$

$= \ 25\sqrt{3}\ meters \ \ ---------- \ (i)$

$In\triangle ACD$

$tan30^{\circ} = \frac{75}{AC}$

$\frac{1}{13} = \frac{75}{25\sqrt{3}+x}$

By cross multiplication

$(x \ + \ 25\sqrt{3}) \ = \ 75\sqrt{3}$

$x = 50\sqrt{3}$

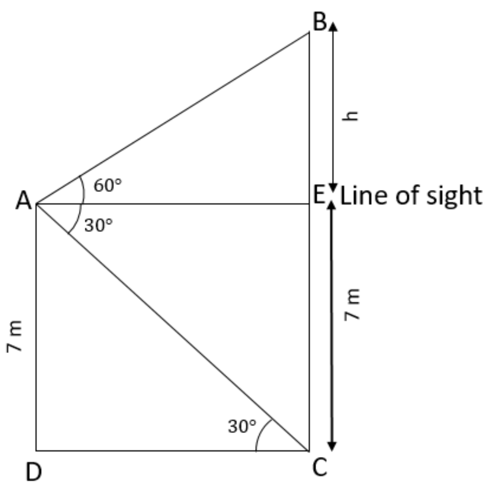
$= 50 \times 1.73\, metre$

$= 86.5\, m$

OR

(B) From the top of a $7\ m$ high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Explanation:



$In\ \triangle ABE(\ upper\ \triangle)$:

$tan60^\circ = \frac{AB}{AE}$

$tan60^\circ = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}, ------ (i)$

$In\ \triangle ADC,$

$tan30^\circ = \frac{AD}{DC}$

$\frac{1}{\sqrt{3}} = \frac{7}{x}$

$\frac{1}{\sqrt{3}} = \frac{7}{\frac{h}{\sqrt{3}}}$

$h = 21\, metres$

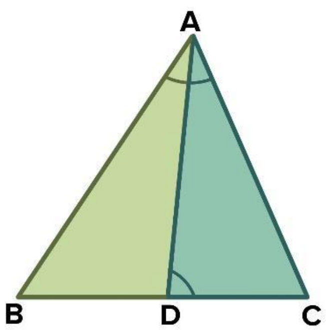
The tower = 21 + 7 = 28 meters high.

SECTION - D

Section - D consists of Long Answer (LA) type questions of 5 marks each.

33(A). D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that $CA^2 = CB.CD$

Explanation:



In $\triangle ADC$ and $\triangle BAC$

$\angle ADC = \angle BAC\ (Given)$

$\angle C$ is Common

$\therefore$ by AA Criterion of Similarity, $\triangle ADC \sim \triangle BAC$

$\Rightarrow \frac{AD}{BA} = \frac{DC}{AC} = \frac{AC}{BC}$

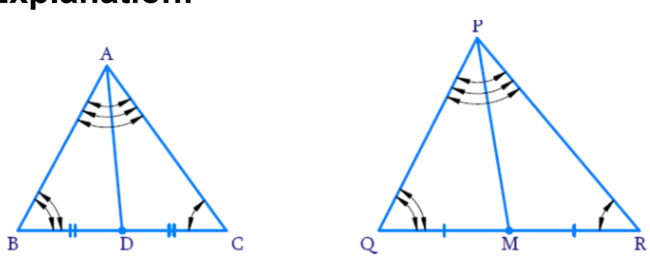
$\Rightarrow \frac{DC}{AC} = \frac{AC}{BC}$

$\therefore CA^2 = CB \cdot CD$

OR

33(B). If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Explanation:



Consider the triangles $\triangle ABC$ and $\triangle PQR$

AD and PM being the mediums from vertex A and P respectively.

Given: $\triangle ABC \sim \triangle PQR$

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

It is given that $\triangle ABC \sim \triangle PQR$

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

[from the side-ratio property of similar $\triangle s$]

$$\Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R ...... (A)$$

$BC = 2BD; QR = 2QM$ [P, M being the mid points of $BCqQR$ respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AC}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AC}{PR}$ .... (1)

Now in $\triangle ABD$ & $\triangle PQM$

$\frac{AB}{PQ} = \frac{BP}{QM}$ ......[from (1)]

$\angle B = \angle Q$........[from (A)]

$\Rightarrow \triangle ABD \sim \triangle PQM$ [By SAS property of similar $\triangle s$] from the side property of similar $\triangle s$

$\frac{AB}{PQ} = \frac{AD}{PM}$ Hence proved

34. A student was asked to make a model shaped like a cylinder with two cones attached to its ends by using a thin aluminum sheet. The diameter of the model is $3\ cm$ and its total length is $12\ cm$. If each cone has a height of $2\ cm$, find the volume of air contained in the model.

Answer: $66\ cm^3$

Explanation:

For the given statement first draw a diagram, In this diagram, we can observe that

Height $\left(h_1\right)$ of each conical part $= 2\ cm$

Height $\left(h_2\right)$ of cylindrical part = $12 - 2 - 2 = 8\ cm$

Radius (r) of cylindrical part = Radius of conical part = $\frac{3}{2}\ cm = 1.5\ cm$

Volume of air present in the model = Volume of cylinder + $2\times$ Volume of a cone

Volume of cylinder $= \pi r^2 h$

$= \pi \times 1.5^2 \times 8 = 18\pi cm^3$

Volume of a cone= $\frac{1}{3}\pi r^2 h$

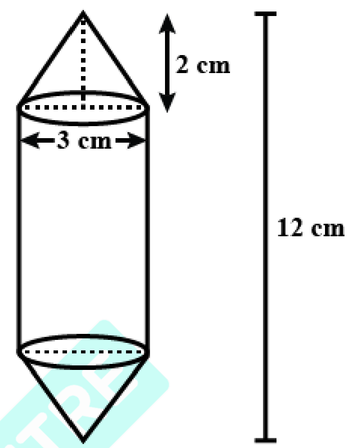
$= \frac{1}{3}\pi \times 1.5^2 \times 2$

$= 1.5\pi cm^3$

Volume of air present in the model= $18\pi + 2\times 1.5\pi$

$= 18\pi + 3\pi = 21\pi$

$= 21 \times \frac{22}{7} = 66\ cm^3$



Hence, Volume of air present in the model is $66\ cm^3$.

35. The monthly expenditure on milk in 200 families of a Housing Society is given below :

Monthly Expenditure (in ₹)	1000 – 1500	1500 – 2000	2000 –2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Number of families	24	40	33	x	30	22	16	7

Find the value of x and also, find the median and mean expenditure on milk.

Explanation:

To find the value of x, we need to use the information that the total number of families is 200.

Therefore,

200 = 24 + 40 + 33 + x + 30 + 22 + 16 + 7

Simplifying the equation, we get:

x = 28

So, there are 28 families who spend between ₹2000 and ₹2500 per month on milk.

$Class\ \ (1)$	$Frequency\ (f)\ (2)$	$Mid\ value\ (x)$	$d = \frac{x-A}{h} = \frac{x-3250}{500}\ A$	$f \cdot d\ (5) = (2)\times(4)$	$cf\ \ (7)$
$1000 - 1500$	24	1250	-4	-96	24
$1500 - 2000$	40	1750	-3	-120	64
$2000 - 2500$	33	2250	-2	-66	97
$2500 - 3000$	28	2750	-1	-28	125
$3000 - 3500$	30	$3250 = A$	0	0	155
$3500 - 4000$	22	3750	1	22	177
$4000 - 4500$	16	4250	2	32	193
$4500 - 5000$	7	4750	3	21	200
...	...	...	...	---	---
	$n = 200$	...	...	$\sum f \cdot d = -235$	----

$Mean x^{-} = A + \frac{\sum fd}{n} \times h$

$= 3250 + \frac{-235}{200} \times 500\ = 3250 + (-1.175) \times 500$

$= 3250 - 587.5\ = 2662.5$

To find Median Class

= value of $\left(\frac{n}{2}\right)^{th}$ observation

= value of $\left(\frac{200}{2}\right)^{th}$ observation

= value of 100^{th} observation

From the column of cumulative frequency cf, we find that the 100^{th} observation lies in the class $2500 - 3000$.

∴ The median class is $2500 - 3000$.

Now,

$\therefore L = \ lower\ boundary\ point\ of\ median\ class\ = 2500$

$\therefore n = \text{Total frequency} = 200$
$\therefore cf = \text{Cumulative frequency of the class preceding the median class} = 97$
$\therefore f = \text{Frequency of the median class} = 28$
$\therefore c = \text{class length of median class} = 500$

$\text{Median } M = L + \frac{\frac{n}{2} - cf}{f} \times c$

$= 2500 + \frac{100-97}{28} \times 500$

$= 2500 + \frac{3}{28} \times 500$

$= 2500 + 53.5714$

$= 2553.5$

SECTION - E

Section - E consists of three Case Study Based questions of 4 marks each.

36. Two schools ' P ' and ' Q ' decided to award prizes to their students for two games of Hockey ₹x per student and Cricket ₹ y per student. School ' P ' decided to award a total of ₹$9,500$ for the two games to 5 and 4 students respectively; while school ' Q ' decided to award ₹$7,370$ for the two games to 4 and 3 students respectively.

Based on the above information, answer the following questions :
(i) Represent the following information algebraically (in terms of x and y).
(ii) (a) What is the price amount of Hockey?

OR

(b) Prize amount on which game is more and by how much ?
(iii) What will be the total prize amount if there are 2 students each for two games?

Explanation:

(i) For school P, 5x + 4y = 9500

For School Q, 4x + 3y = 7370

(ii) (a) Let's take

5x + 4y = 9500 …..eq(i)

4x + 3y = 7370 …..eq(ii)

After solving both equations we get

x = 980 and y = 1150

So, Price amount of Hockey is Rs 980

OR

(b) Price amount of Hockey ₹980 per student and Cricket ₹ 1150

So, Cricket price money is more by 1150 – 980 = ₹170

(iii) Total prize amount if there are 2 students each for two games

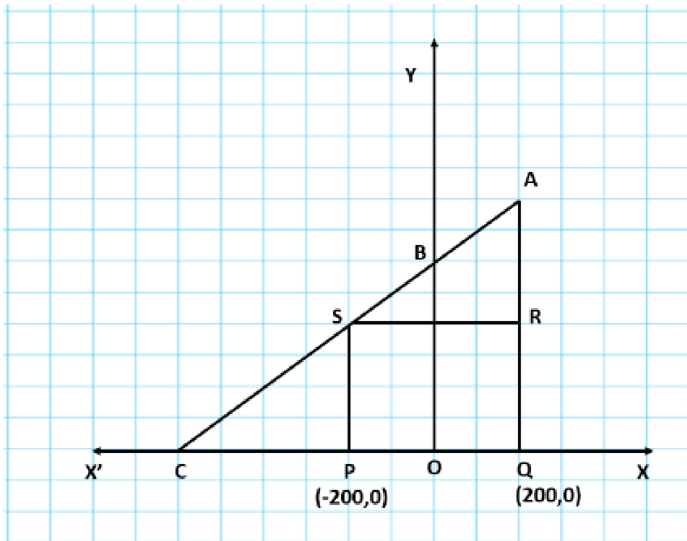
2x + 2y

Let keep the values of x and y

= 2 x 980 + 2 x 1150

= ₹3860

37. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are $(-\ 200, 0)$ and of Q are $(200, 0)$. PQRS being a square, what are the coordinates of R and S ?

(ii) (a) What is the area of square PQRS?

OR

(b) What is the length of diagonal PR in square PQRS ?

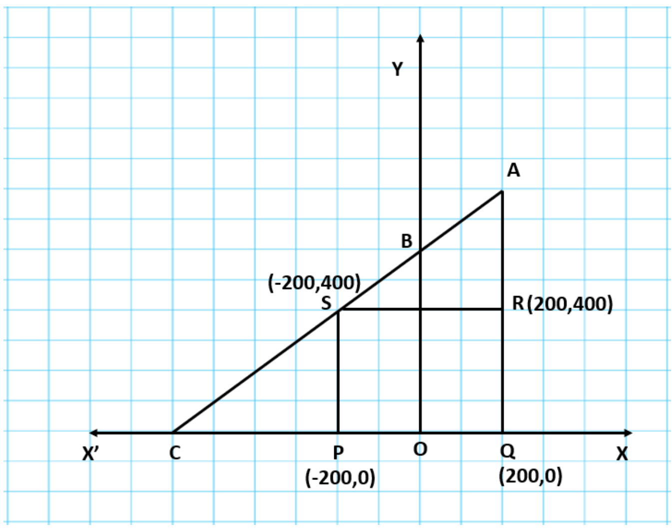
(iii) If S divides CA in the ratio $K: 1$, what is the value of K, where point A is $(200, 800)$?

Explanation:

(i) Taking O as origin, coordinates of P are $(-\ 200, 0)$ and of Q are $(200, 0)$. PQRS being a square,

Co-ordinates of R = (200, 400)

Co-ordinates of S = (-200, 400)



(ii) (a) What is the area of square PQRS?

Area of square PQRS = $Side^2$

Side = PQ = PO + OQ

= 200 + 200

= 400 units

Area of square PQRS = 400^2 = 160000 sq units

OR

(b) Length of diagonal PR in square PQRS = $\sqrt{2} \times$ Side = 400$\sqrt{2}$ $Units$

(iii) To find the value of K, we can use the section formula, which states that the coordinates of a point dividing a line segment in a given ratio can be found using the following formula:

Let P(x,y) be the point dividing the line segment AB in the ratio K:1, where A(x_1,y_1) and B(x_2,y_2) are the given points, then the coordinates of P are given by:

x = (Kx_2 + x_1)/(K+1) and y = (Ky_2 + y_1)/(K+1)

Here, C is (-600,0) and A is (200,800). Therefore, the coordinates of CA can be found as:

x_2 - x_1 = 200 - (-600) = 800
y_2 - y_1 = 800 - 0 = 800

Thus, the slope of the line segment CA is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{800}{800} = 1$$

Now, let S be the point dividing CA in the ratio K:1. Therefore, the coordinates of S can be found using the section formula as:

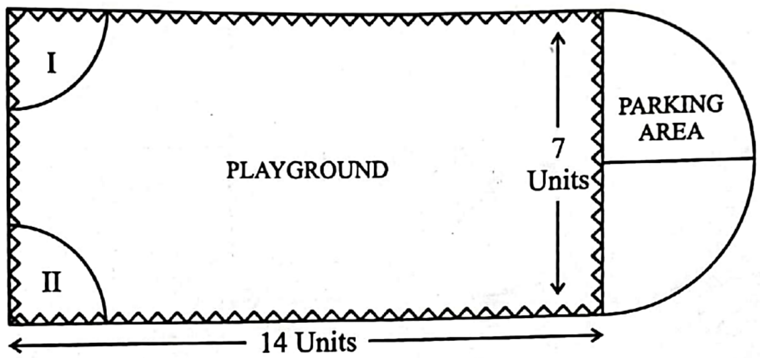
$$x = \frac{K200 + (-600)}{K+1} = \frac{200K - 600}{K+1}$$
$$y = \frac{K800 + 0}{K+1} = \frac{800K}{K+1}$$

Also, we know that S has coordinates (-200,400). Therefore, equating the values of x and y, we get:

$$\frac{200K - 600}{K+1} = -200$$
200K - 600 = -200K - 200
400K = 400
K = 1

Therefore, the value of K is 1.

38. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, answer the following questions :

(i) What is the total perimeter of the parking area?

(ii) (a) What is the total area of parking and the.two quadrants?

OR

(b) What is the ratio of area of playground to the area of parking area?

(iii) Find the cost of fencing the playground and parking area at the rate of ₹2 per unit.

Explanation:

(i) To find the perimeter of the parking area, we need to find the circumference of the semi-circle and add it to the sum of the lengths of the straight sides of the rectangular playground.

The radius of the semi-circle is 2 units, so its circumference is $2\pi r = 2\pi(2) = 4\pi$ units. The length of the rectangular playground is 14 units, so the length of the straight side adjacent to the semi-circle is also 14 units. The breadth of the rectangular playground is 7 units, so the length of the straight side opposite to the semi-circle is also 7 units. Therefore, the total perimeter of the parking area is:

Perimeter of parking area = Circumference of semi-circle + Length of straight side adjacent to semi-circle + Length of straight side opposite to semi-circle

Perimeter of parking area = $4\pi + 14 + 7$

Perimeter of parking area = $21 + 4\pi$

Perimeter of parking area ≈ 33.85 units

(ii) (a) To find the total area of the parking and the two quadrants, we need to find the area of the semi-circle and the area of the two quadrants, and add them together.

The area of the semi-circle is $1/2\ \pi r^2 = 1/2\ \pi(2)^2 = 2\pi$ square units. Since there are two quadrants, their combined area is 2 times the area of one quadrant, which is $1/4\ \pi r^2 = 1/4\ \pi(2)^2 = \pi/2$ square units. Therefore, the total area of the parking and the two quadrants is:

Total area = Area of semi-circle + Area of two quadrants

Total area = $2\pi + \pi/2$

Total area = $5\pi/2$

Total area ≈ 7.85 square units

OR

(b) The area of the playground is the area of the rectangle, which is length x breadth = 14 x 7 = 98 square units. The area of the parking and the two quadrants is $5\pi/2$ square units, as we found in part (ii)(a). Therefore, the ratio of the area of the playground to the area of the parking area is:

Area of playground : Area of parking area = 98 : ($5\pi/2$)

Area of playground : Area of parking area ≈ 98 : 7.85

Area of playground : Area of parking area ≈ 12.46 : 1

Therefore, the area of the playground is about 12.46 times greater than the area of the parking area.

(iii) To find the cost of fencing the playground and parking area at the rate of ₹2 per unit, we need to find the perimeter of the playground and parking area and multiply it by ₹2 per unit.

The perimeter of the playground is 2(length + breadth) = 2(14 + 7) = 42 units. The perimeter of the parking area is the same as we found in part (i), which is approximately 33.85 units. Therefore, the total perimeter of the playground and parking area is:

Total perimeter = Perimeter of playground + Perimeter of parking area

Total perimeter = 42 + 33.85

Total perimeter ≈ 75.85 units

Multiplying this by ₹2 per unit, we get the cost of fencing the playground and parking area:

Cost of fencing = Total perimeter x ₹2 per unit

Cost of fencing ≈ 75.85 x 2

Cost of fencing ≈ ₹151.70

Therefore, the cost of fencing the playground and parking area at the rate of ₹2 per unit is approximately ₹151.70.