Maharashtra Board - II (X) Mathematics
2022-23

Maximum Marks: **40**
Time allowed: **2 Hours**

Note:-
(i) All questions are compulsory.
(ii) Use of calculator is not allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case of MCQs [Q. No. 1(A)] only the first attempt will be evaluated and will be given credit.
(v) For every MCQ, the correct alternative (A), (B), (C) or (D) with: sub-question number is to be written as an answer.
(vi) Draw the proper figures for answers wherever necessary.
(vii) The marks of construction should be clear and distinct. Do not erase them.
(viii) Diagram is essential for writing the proof of the theorem,

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Question 1

A. **Four alternative answers are given for every subquestion, Select the correct alternative and write the alphabet of that answer : [4]**

(1) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle :
(A) Obtuse angled triangle
(B) Acute angled triangle
(C) Right angled triangle
(D) Equilateral triangle

Answer: (C) Right angled triangle

Explanation:
In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is right angled triangle.

Hence, the correct option is the Right angled triangle.

(2) Chords AB and CD of a circle intersect inside the circle at point E. If $AE = 4, EB = 10, CE = 8$, then find ED:
(A) 7
(B) 5
(C) 8
(D) 9

Answer: B
Explanation:

$$AE \times EB = CE \times ED$$

Plugging in the given values, we get:

$$4 \times 10 = 8 \times ED$$

Simplifying, we get:

$$40 = 8 \times ED$$

ED = 5

(3) Co-ordinates of origin are ___________
(A) $(0,0)$
(B) $(0,1)$
(C) $(1,0)$
(D) $(1,1)$

Answer: A

Explanation:

The coordinates of the origin are (0, 0).

(4) If radius of the base of cone is $7\ cm$ and height is $24\ cm$, then find its slant height :
(A) $23\ cm$

(B) $26\ cm$
(C) $31\ cm$
(D) $25\ cm$

Answer: D

Explanation: The slant height of a cone can be calculated using the Pythagorean theorem, which states that the square of the slant height is equal to the sum of the square of the radius and the square of the height.
So, slant height $= \sqrt{radius^2 + height^2}$
Substituting the given values, we get:
slant height $= \sqrt{7^2 + 24^2}$
slant height $= \sqrt{49 + 576}$
slant height $= \sqrt{625}$
slant height $= 25\ cm$
Therefore, the answer is option (D) $25\ cm$.

B. Solve the following sub-questions: [4]

1. If $\Delta ABC \sim \Delta PQR$ and $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{16}{25}$, then find AB : PQ.
Answer: $4 : 5$

Explanation:
$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$
$\frac{16}{25} = \frac{AB^2}{PQ^2}$
$\frac{AB}{PQ} = \sqrt{\frac{16}{25}}$
$\frac{AB}{PQ} = \frac{4}{5}$
$\therefore AB : PQ = 4 : 5$

2. In $\Delta RST, \angle S = 90°, \angle T = 30°, RT = 12\ cm,$ then find RS.
Answer: $6\ cm$

Explanation:

$sin\ 30° = \frac{RS}{RT}$

$\frac{1}{2} = \frac{RS}{12}$

$\therefore RS = 6\ cm$

3. If radius of a circle is 5 cm, then find the length of longest chord of a circle.

Answer: $10\ cm$

Explanation:

The longest chord of a circle is the diameter of the circle.

$\therefore\ Diameter = 2 \times Radius$

$= 2 \times 5$

$= 10\ cm$

$\therefore$ Diameter = 10 cm

4. Find the distance between the points O(0, 0) and P(3, 4).
Answer: 5 units

Explanation:

Distance between a point and origin = $\sqrt{x^2 + y^2}$

= $\sqrt{3^2 + 4^2}$

= $\sqrt{9 + 16}$

= $\sqrt{25}$

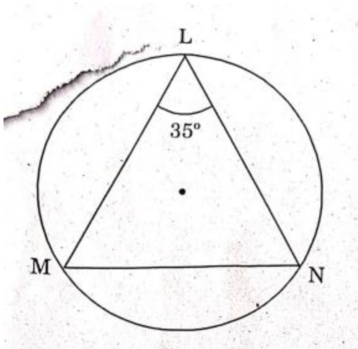
= 5 units

$\therefore$ The distance b/w 2 points is 5 units.

Question 2

A. Complete the following activities (any two): [4]

1.



In the above figure, $\angle L = 35°$, find:
(i) $m(arc\ MN)$
(ii) $m(arc\ MLN)$

Explanation:
(i) $\angle L = \frac{1}{2} m(arc\ MN)$ ……..… (By inscribed angle theorem)

$$\therefore\ 35 = \frac{1}{2} m(arc\ MN) \quad \therefore\ 2 \times 35 = m(arc\ MN) \quad \therefore\ m(arc\ MN) = 70^{\circ}$$

(ii) $m(arc\ MLN) = 360° - m(arc\ MN)$
…………….[Definition of measure of arc]

$$= 360° - 70°$$

$\therefore\ m(arc\ MLN) = 290^{\circ}$

2. Show that, $cot\ \theta + tan\ \theta = cosec\ \theta \times sec\ \theta$

Explanation:

$$L.H.S. \quad = cot\ \theta + tan\ \theta \quad = \frac{cos\ \theta}{sin\ \theta} + \frac{sin\ \theta}{cos\ \theta}$$

$$= \frac{Sin^2\theta + \theta}{sin\ \theta \times cos\ \theta}$$

$= \frac{1}{sin\ \theta \times cos\ \theta}$……………$(Sin^2\theta + \theta\ = 1)$

$$= \frac{1}{sin\ \theta} \times \frac{1}{cos\ \theta}$$
$$= cosec\ \theta \times sec\ \theta\ \ L.H.S.\ \ = \ R.H.S.$$
$$\therefore\ cot\ \theta\ + tan\ \theta = cosec\ \theta \times sec\ \theta$$

3. Find the surface area of a sphere of radius $7\ cm$.

Answer: 616 sq.cm.

Explanation:
Surface area of sphere $= 4\pi r^2$

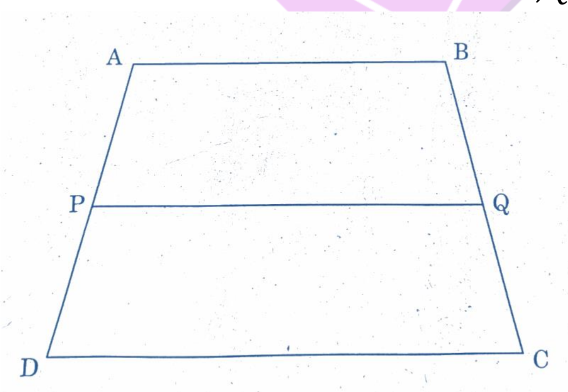
$$= 4 \times \frac{22}{7} \times 7^2 \quad = 4 \times \frac{22}{7} \times 49 \quad = 88 \times 7$$

$\therefore$ Surface area of sphere $= 616$ sq.cm.

B. Solve the following sub-questions (any four) **[8]**

1. In trapezium $ABCD$ side AB ∥ side PQ ∥ side $DC. AP = 15$

$$PD = 12, QC = 14,\ find\ BQ$$



Answer: 17.5

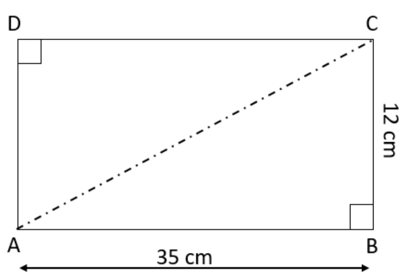
Explanation: According to property of three parallel lines and their transversals if $AB \parallel PQ \parallel DC$ then -

$$\frac{AP}{PD} = \frac{BQ}{QC}\ \frac{15}{12} = \frac{BQ}{14}\ BQ = 17.5$$

2. Find the length of the diagonal of a rectangle whose length is $35\ cm$ and breadth is $12\ cm$.

Answer: 37 cm

Explanation:



According to Pythagoras theorem, in $\triangle ABC$

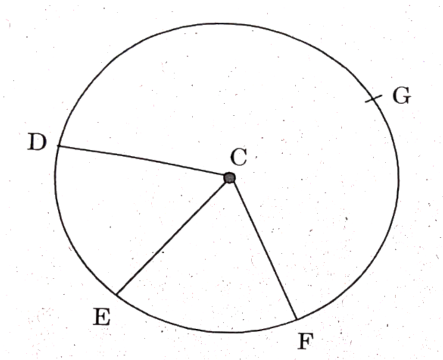
$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow 35^2 + 12^2 = AC^2$$
$$\Rightarrow 1225 + 144 = AC^2$$
$$\Rightarrow AC^2 = 1369$$
$$\Rightarrow AC = 37\ cm$$

Hence, the length of the diagonal is $37\ cm$.

3. In the given figure points G, D, E, F are points of a circle with centre $C, \angle ECF = 70°, m(arc$ DGF $) = 200°$.
Find :

(i) $m(arc\ DE)$
(ii) m (arc DEF).



Answer: $90^{\circ}\ and\ 160^{\circ}$

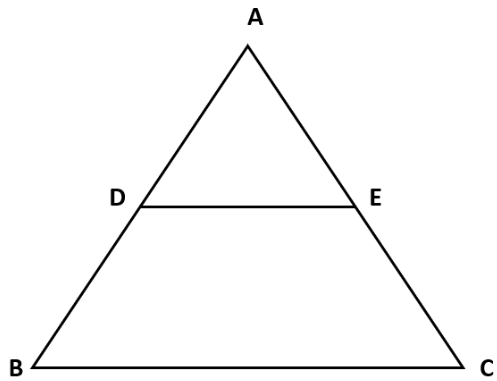
Explanation:

$$(i)\ \ m(arc\ DE) = 360^{\circ} - [m(arc\ DGF) + m(arcECF)]$$
$$m(arc\ DE) = 360^{\circ} - [200^{\circ} + 70^{\circ}]$$
$$m(arc\ DE) = 90^{\circ}$$

(ii) $m(arc\ DEF) = m(arc\ DE) + m(arc\ EF)$
$$m(arc\ DEF) = 90^{\circ} + 70^{\circ}$$
$$m(arc\ DEF) = 160^{\circ}$$

4. Show that points A (-1, -1), B (0, 1), C (1, 3) are collinear.

Explanation:



$A(-1,-1), B(0,1), C(1,3)$

Slope of $AB = \frac{1-(-1)}{0-(-1)} = \frac{2}{1} = 2$

Slope of $BC = \frac{3-1}{1-0} = \frac{2}{1} = 2$

Slope of $AB =$ Slope of $BC = 2$

Thus, the given points are collinear.

5. A person is standing at a distance of 50 m from a temple looking at its top. The angle of elevation is of 45°, Find the height of the temple.

Answer: $50\sqrt{2}\,m$

Explanation:

Let ABC be the triangle right angled at B and angle C be 45°.
Given that, BC = 50 m
To find, AB (height).
$cos\ cos\ 45° = \frac{BC}{AB}$
$\frac{1}{\sqrt{2}} = \frac{50}{AB}$

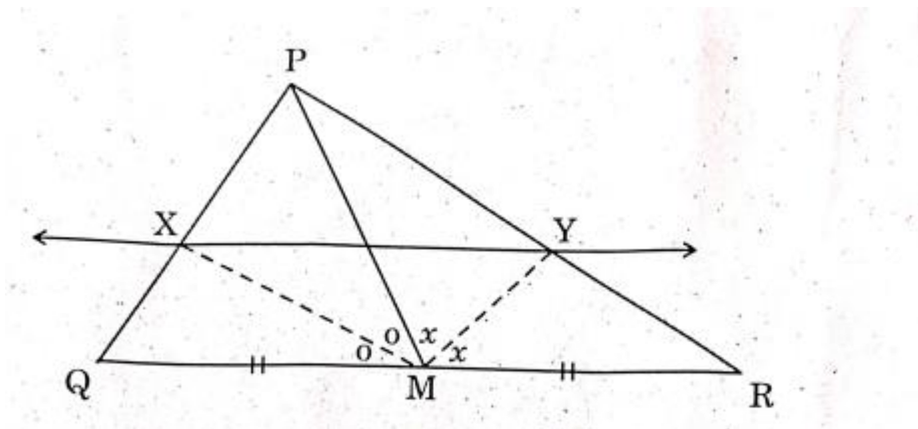
$\therefore AB = 50\sqrt{2}\, m$

$\therefore Height\ of\ the\ temple = 50\sqrt{2}\, m$

Question 3.

A. Complete the following activities (any one): [3]

1.



In $\triangle PQR$, seg PM is a median. Angle bisectors of $\angle PMQ$ and $\angle PMR$ intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR. Complete the proof by filling in the boxes.

Explanation:

In $\triangle PMQ$,

Ray MX is the bisector of $\angle PMQ$

$\therefore \frac{MP}{MQ} = \frac{XQ}{PX}$ ………………………….. (I) [Theorem of angle bisector]

Similarly, in △ PMR, Ray MY is bisector of $\angle PMR$

$\therefore \frac{MP}{MR} = \frac{YR}{PY}$ …………………………………. (II) [Theorem of angle bisector]

But $\frac{MP}{MQ} = \frac{MP}{MR}$ ……………………………….(III) [As M is the midpoint of QR]

Hence $MQ = MR$

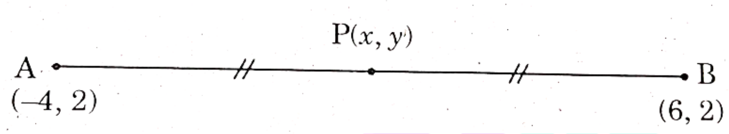
$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$ [From (I), (II) and (III)]

$\therefore XY \parallel QR$ … … …. [Converse of basic proportionality theorem]

2. Find the coordinates of point P where P is the midpoint of a line segment AB with $A(-4,2)$ and $B(6,2)$.

Answer: (1, 2)

Explanation:



Suppose, $(-4,2) = (x_1, y_1)$ and $(6,2) = (x_2, y_2)$ and co-ordinates of P are (x, y)

$\therefore$ According to midpoint theorem,

$$x = \frac{x_1 + x_2}{2} = \frac{-4+6}{2} = \frac{2}{2} = 1$$
$$y = \frac{y_1 + y_2}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

$\therefore$ Co-ordinates of midpoint P are (1, 2).

B. Solve the following sub-questions (any two) : [6]

1. In $\triangle ABC$, seg AP is a median. If $BC = 18, AB^2 + AC^2 = 260$, find AP.

Answer: 7 cm

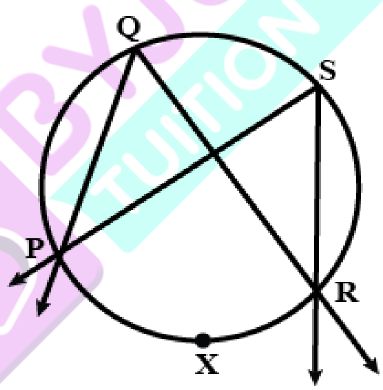
Explanation:

$$AB^2 + AC^2 = 2AP^2 + 2BP^2\ (by\ Apollonius\ theorem)$$
$$\Rightarrow 260 = 2AP^2 + 2(9^2)$$
$$\Rightarrow 260 = 2AP^2 + 2(81)$$
$$\Rightarrow 260 = 2AP^2 + 162$$
$$\Rightarrow 2AP^2 = 260 - 162$$
$$\Rightarrow 2AP^2 = 98$$
$$\Rightarrow AP^2 = 49$$
$$\Rightarrow AP = 7$$

2. Prove that, "Angles inscribed in the same arc are congruent."

Explanation:

Proof: -



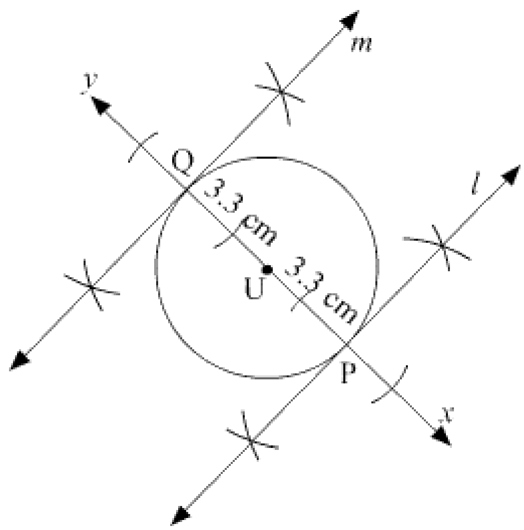
$$m\angle PQR = \frac{1}{2}m(\ arc\ P \times R) - \ (i)\ \ (inscribed\ angle\ \ theorem)$$
$$m\angle PSR = \frac{1}{2}m(arc\ \ PXR) - \ (i)\ \ (\ Inscribed\ angle\ \ theorem)$$
$$\therefore\ m\angle PQR = m\angle PSR - [\ form\ (i)\ \&\ (ii)\ ]$$

$\therefore \angle PQR \cong \angle PSR$. (Angles equal in measure are congruent).

3. Draw a circle of radius $3.3\ cm$. Draw a chord PQ of length $6.6\ cm$. Draw tangents to the circle at points P and Q.

Explanation:

Steps of Construction:
Step 1: Draw a circle with radius $3.3\ cm$. Mark any point P on it.
Step 2: Draw chord $PQ = 6.6\ cm$ (PQ is the diameter of the circle).



Step 3: Draw rays CX and CY.
Step 4: Draw line I perpendicular to ray CX through point P.
Step 4: Draw line m perpendicular to ray CY through point Q.

Here, line I and line m are the required tangents to the circle at points P and Q, respectively. It can be observed that the tangents I and m are parallel to each other.

4. The radii of circular ends of a frustum are $14\ cm$ and $6\ cm$ respectively and its height is $6\ cm$. Find its curved surface area. $(\pi = 3.14)$

Answer: 628 sq. cm

Explanation:

Here, $r_1 = 14\ cm, r_2 = 6\ cm$ and $h = 6\ cm$.
Slant height of the frustum, l= $[\sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{6^2 + (14-6)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}] = 10\ cm$
Curved surface area of frustum = $\pi(r_1 + r_2)l$

$$= 3.14 \times (14 + 6) \times 10$$
$$= 3.14 \times 20 \times 10$$
$$= 628\ cm^2$$

∴ The curved surface area of frustum is 628 sq. cm

Question 4. Solve the following sub-questions (any two): **[8]**

1. In $\triangle ABC$, seg $DE \parallel$ side BC. If $2\ A(\triangle ADE) = A(\square\ DBCE)$, find $AB{:}AD$ and show that $BC = \sqrt{3}DE$.

Explanation:

Given:
In $\triangle ABC$, segment $DE \parallel$ side BC

$$2\ A(\triangle ADE) = A(DBCE)$$

To Find:

$$BC = \sqrt{3} \times DE$$

Solution:
Since DE is parallel to BC
In $\triangle ADE$ and $\triangle ABC$,
$\angle A$ is common
The corresponding angles of a triangle are equal

$$\therefore \angle ADE = \angle ABC$$

So, $\triangle ADE$ is similar to $\triangle ABC$ by AA similarity criteria.

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \ .(1)$$

Then,

$$\frac{A(\text{ triangle } ADE)}{A(\text{ triangle } ABC)} = \frac{AD^2}{AB^2} = \frac{DE^2}{BC^2} = \frac{AE^2}{AC^2} .. (II)$$

Also, 2 $A(\triangle ADE) = A(DBCE)$..(III)   [given]
Since, $A(\triangle ABC) = A(\triangle ADE) + A(DBCE)$

$$A(\triangle ABC) = A(\triangle ADE) + 2\,A(\triangle ADE)$$
$$\therefore A(\triangle ABC) = 3\,A(\triangle ADE)$$

$\frac{A(\text{ triangle } ADE)}{A(\text{ triangle } ABC)} = \frac{1}{3}$

Using (II) and (III), we get

$$\frac{AD^2}{AB^2} = \frac{DE^2}{BC^2} = \frac{1}{3}$$

$\therefore \frac{AD^2}{BD^2} = \frac{1}{3}$ and $\frac{DE^2}{BC^2} = \frac{1}{3}$

$\frac{AD}{BD} = \frac{1}{\sqrt{3}}$. (v) and $\frac{DE}{BC} = \frac{1}{\sqrt{3}}$.(v)

Now, by applying Inverted In (v)

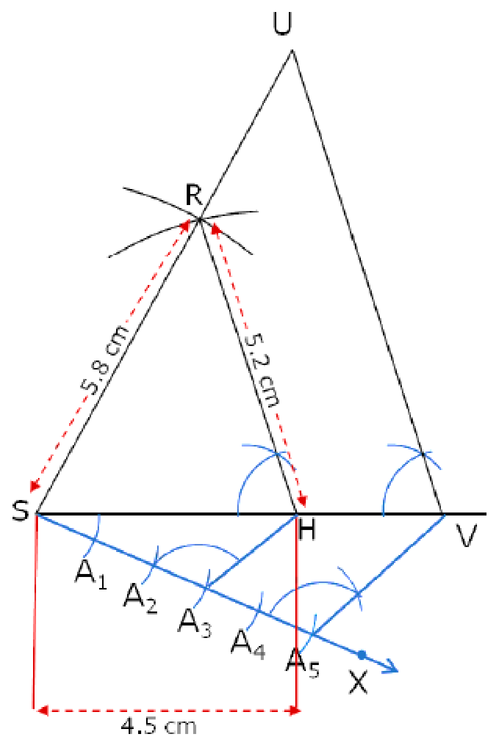
We get,
$\frac{AB}{BD} = \sqrt{3}$ and
Using equation (v), we get

$$BC = \sqrt{3} \times DE$$

Hence proved.

2. $\Delta SHR \sim \Delta SVU$. In $\Delta SHR, SH = 4.5\ cm, HR = 5.2\ cm, SR = 5.8\ cm$ and $\frac{SH}{SV} = \frac{3}{5}$, construct ΔSVU.

Explanation:



Steps of Construction:

1 Construct the △ SHR with the given measurements. For this draw SH of length $4.5\ cm$.

2 Taking S as the center and radius equal to $5.8\ cm$ draw an arc above SH.

3 Taking H as the center and radius equal to $5.2\ cm$ draw an arc to intersect the previous arc. Name the point of intersection as R.

4 Join SR and HR. △ SHR with the given measurements is constructed. Extend SH and SR further on the right side.

5 Draw any ray SX making an acute angle (i.e; 45°) with SH on the side opposite to the vertex R.

6 Locate 5 points. (the ratio of old triangle to new triangle is $3/5$ and $5>3$). Locate A_1, A_2, A_3, A_4 and A_5 on AX so that $SA_1=A_1A_2=A_2A_3=A_3A_4=A_4A_5$

7 Join A_3H and draw a line through A_5 parallel to A_3H_1, intersecting the extended part of SH at V.

8 Draw a line VU through V parallel to HR.
$\triangle SVU$ is the required triangle.

3. An ice cream pot has a right circular cylindrical shape. The radius of the base is $12\ cm$ and height is $7\ cm$. This pot is completely filled with ice-cream. The entire ice cream is given to the students in the form of right circular ice-cream cones, having diameter $4\ cm$ and height is $3.5\ cm$. If each student is given one cone, how many students can be served?

Answer: 647 students

Explanation:

Volume of ice cream in the cylindrical pot = $\pi \times (radius)^2$ ×(height)

$$= \pi(12\ cm^2)(7\ cm) \quad = 3{,}024\pi cm^3$$

Volume of one ice-cream cone = $1/3\pi\ (radius)^2$ (height)

$$= 1/3\pi(2\ cm)^2(3.5\ cm)$$
$$= 4.67\pi cm^3 \text{ (Rounded to two decimal places)}$$

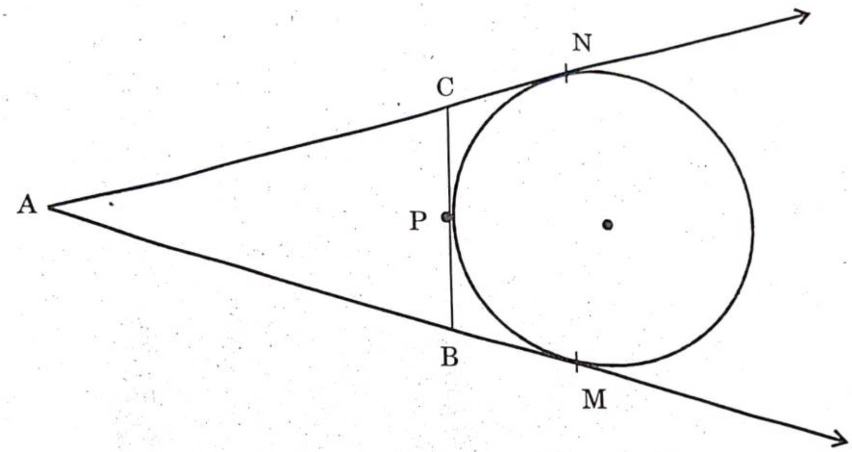
Number of cones that can be filled with the ice-cream in the pot = (Volume of ice-cream in the cylindrical pot) ÷ (Volume of one ice-cream cone)

$$= (3{,}024\pi cm^3) \div (4.67\pi cm^3)$$
$$= 647.46 \text{ (Rounded to two decimal places)}$$

Therefore, 647 students can be served with one cone each. Note that this is an approximation and assumes that no ice cream is lost or wasted in the process of filling the cones.

Question 5. Solve the following sub-questions (anyone): [3]

1.



A circle touches side BC at point P of the $\triangle ABC$, from out-side of the triangle. Further extended lines AC and AB are tangents to the circle at N and M respectively. Prove that :

$$AM = \frac{1}{2}(\,Perimeter\ of\ \triangle ABC)$$

Explanation:

Lengths of Δ drawn from an external pt to a circle are equal.

$$\Rightarrow AM = AN, BM = BP, CP = CN\ + (AN - CN)$$
$$Perimeter\ of\ \triangle ABC = AB + BC + CA$$
$$= AB + (BP + PC)$$
$$= (AB + BM) + PC + (AM - PC)$$
$$= AM + AM = 2AM$$
$$AM = \frac{1}{2}\ (perimeter\ of\ \triangle ABC)$$

2. Eliminate θ if $x = r\cos\ \theta$ and $y = r\sin\ \theta$.

Explanation:

$$x = r\cos\ \theta, y = r\sin\ \theta$$

Squaring on both term

$$x^2 = r^2 cos^2\ \theta\ ,\_\_\_\_\_\_(1)$$
$$y^2 = r^2 sin^2\ \theta\_\_\_\_\_\_\_(2)$$

Add (1) + (2)

$$x^2 + y^2 = r^2 sin^2\ \theta + r^2 cos^2\ \theta$$
$$x^2 + y^2 = r^2 \left(sin^2\ \theta + cos^2\ \theta\right)$$

But we know $(sin^2\ \theta + cos^2\ \theta) = 1$

$$\therefore\ x^2 + y^2 = r^2$$