1. Find the derivative of $x^2 - 2$ at x = 10.

Solution:

Let f (x) = $x^2 - 2$

From first principle

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Put x = 10, we get

$$f'(10) = \lim_{h\to 0}\frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h\to 0}\frac{[(10+h)^2 - 2] - (10^2 - 2)}{h}$$

$$= \lim_{h\to 0}\frac{10^2 + 2\times 10\times h + h^2 - 2 - 10^2 + 2}{h}$$

$$= \lim_{h\to 0}\frac{20h + h^2}{h}$$

$$= \lim_{h\to 0}(20 + h)$$

$$= 20 + 0$$

$$= 20$$

2. Find the derivative of x at x = 1.

Solution:

Let f (x) = x

Then,

From first principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Let f (x) = x

From first principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(10)}{h}$$

Put x = 1, we get

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h) - 1}{h}$$

$$= \lim_{h \to 0} \frac{1+h-1}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1$$

= 1

3. Find the derivative of 99x at x = l00.

Solution:

Let f (x) = 99x,

From the first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Put x = 100, we get

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$

$$= \lim_{h \to 0} \frac{99(100+h) - 99 \times 100}{h}$$

$$= \lim_{h \to 0} \frac{99 \times 100 + 99h - 99 \times 100}{h}$$

$$= \lim_{h \to 0} \frac{99 \times h}{h}$$

$$= \lim_{h \to 0} 99$$

= 99

4. Find the derivative of the following functions from the first principle.

(i) $x^3 - 27$

(ii) (x – 1) (x – 2)

(iii) 1 / x^2

(iv) x + 1 / x – 1

Solution:

(i) Let f (x) = $x^3 - 27$

From the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^3 - 27] - (x^3 - 27)}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{h^3 + 3x^2h + 3xh^2}{h}$$

$$= \lim_{h\to 0}(h^2 + 3x^2 + 3xh)$$

$$= 0 + 3x^2$$

$$= 3x^2$$

(ii) Let f (x) = (x – 1) (x – 2)

From the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h-1)(x+h-2) - (x-1)(x-2)}{h}$$

$$= \lim_{h\to 0} \frac{(x^2 + hx - 2x + hx + h^2 - 2h - x - h + 2) - (x^2 - 2x - x + 2)}{h}$$

$$= \lim_{h\to 0} \frac{hx + hx + h^2 - 2h - h}{h}$$

$$= \lim_{h\to 0}(h + 2x - 3)$$

$= 0 + 2x – 3$

$= 2x – 3$

(iii) Let f (x) = 1 / x^2

From the first principle, we get

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0}\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{x^2-x^2-h^2-2hx}{x^2(x+h)^2}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{-h^2-2hx}{x^2(x+h)^2}\right]$$

$$= \lim_{h\to 0}\left[\frac{-h-2x}{x^2(x+h)^2}\right]$$

$= (0 – 2x) / [x^2 (x + 0)^2]$

$= (- 2 / x^3)$

(iv) Let f (x) = x + 1 / x – 1

From the first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{(x-1)(x+h+1) - (x+1)(x+h-1)}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{(x^2 + hx + x - x - h - 1) - (x^2 + hx + x - x + h - 1)}{(x-1)(x+h-1)} \right]$$

$$= \lim_{h \to 0} \frac{-2h}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(x-1)(x+h-1)}$$

$$= -\frac{2}{(x-1)(x-1)}$$

$$= -\frac{2}{(x-1)^2}$$

5. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots \frac{x^2}{2} + x + 1$ **, prove that f’ (1) =100 f’ (0).**

Solution:

Given function is:

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots \frac{x^2}{2} + x + 1$$

By differentiating both sides, we get

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left[\frac{x^{100}}{100} + \frac{x^{99}}{99} + \cdots + \frac{x^2}{2} + x + 1\right]$$

$$= \frac{d}{dx}\left(\frac{x^{100}}{100}\right) + \frac{d}{dx}\left(\frac{x^{99}}{99}\right) + \cdots + \frac{d}{dx}\left(\frac{x^2}{2}\right) + \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\therefore \frac{d}{dx} f(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \cdots + \frac{2x}{2} + 1 + 0$$

$$f'(x) = x^{99} + x^{98} + \cdots + x + 1$$

At $x = 0$, we get

$$f'(0) = 0 + 0 + \ldots + 0 + 1$$

$$f'(0) = 1$$

At $x = 1$, we get

$$f'(1) = 1^{99} + 1^{98} + \ldots + 1 + 1 = [1 + 1 \ldots. + 1]\ 100 \text{ times} = 1 \times 100 = 100$$

Hence, $f'(1) = 100\, f'(0)$

6. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n$ **for some fixed real number a.**

Solution:

Given function is:

$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}\left(x^n + ax^{n-1} + a^2x^{n-2} + \ldots + a^{n-1}x + a^n\right)$$

$$= \frac{d}{dx}(x^n) + a\frac{d}{dx}(x^{n-1}) + a^2\frac{d}{dx}(x^{n-2}) + \cdots + a^{n-1}\frac{d}{dx}(x) + a^n\frac{d}{dx}(1)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n - 2)x^{n-3} + \ldots + a^{n-1} + a^n(0)$

$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n - 2)x^{n-3} + \ldots + a^{n-1}$

7. For some constants a and b, find the derivative of
(i) (x − a) (x − b)

(ii) $(ax^2 + b)^2$

(iii) x – a / x – b

Solution:

(i) (x – a) (x – b)

Let $f(x) = (x - a)(x - b)$

$f(x) = x^2 - (a + b)x + ab$

Now, by differentiating both sides, we get

$$f'(x) = \frac{d}{dx}(x^2 - (a+b)x + ab)$$

$$= \frac{d}{dx}(x^2) - (a+b)\frac{d}{dx}(x) + \frac{d}{dx}(ab)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f'(x) = 2x - (a + b) + 0$$

$$= 2x - a - b$$

(ii) $(ax^2 + b)^2$

Let $f(x) = (ax^2 + b)^2$

$f(x) = a^2x^4 + 2abx^2 + b^2$

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}(a^2x^4 + 2abx^2 + b^2)$$

$$f'(x) = \frac{d}{dx}(x^4) + (2ab)\frac{d}{dx}(x^2) + \frac{d}{dx}(b^2)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f'(x) = a^2 \times 4x^3 + 2ab \times 2x + 0$$

$$= 4a^2x^3 + 4abx$$

$= 4ax\ (ax^2 + b)$

(iii) $x - a / x - b$

$$\text{Let } f(x) = \frac{(x-a)}{(x-b)}$$

By differentiating both sides and using quotient rule, we get

$$f'(x) = \frac{d}{dx}\left(\frac{x-a}{x-b}\right)$$

$$f'(x) = \frac{(x-b)\frac{d}{dx}(x-a) - (x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$$

$$= \frac{(x-b)(1) - (x-a)(1)}{(x-b)^2}$$

By further calculation, we get

$$= \frac{x - b - x + a}{(x - b)^2}$$

$$= \frac{a - b}{(x - b)^2}$$

8. Find the derivative of $\frac{x^n - a^n}{x - a}$ **for some constant a.**

Solution:

Let $f(x) = \frac{x^n - a^n}{x - a}$

By differentiating both sides and using quotient rule, we get

$$f'(x) = \frac{d}{dx}\left(\frac{x^n - a^n}{x - a}\right)$$

$$f'(x) = \frac{(x - a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x - a)}{(x - a)^2}$$

By further calculation, we get

$$= \frac{(x - a)(nx^{n-1} - 0) - (x^n - a^n)}{(x - a)^2}$$

$$= \frac{nx^n - anx^{n-1} - x^n + a^n}{(x - a)^2}$$

9. Find the derivative of

(i) 2x – 3 / 4

(ii) $(5x^3 + 3x - 1)(x - 1)$

(iii) $x^{-3}(5 + 3x)$

(iv) $x^5(3 - 6x^{-9})$

(v) $x^{-4}(3 - 4x^{-5})$

(vi) (2 / x + 1) – x^2 / 3x – 1

Solution:

(i)

Let f (x) = 2x – 3 / 4

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}\left(2x - \frac{3}{4}\right)$$

$$= 2\frac{d}{dx}(x) - \frac{d}{dx}\left(\frac{3}{4}\right)$$

= 2 – 0

= 2

(ii)

Let f (x) = $(5x^3 + 3x – 1) (x – 1)$

By differentiating both sides and using the product rule, we get

$$f'(x) = (5x^3 + 3x - 1)\frac{d}{dx}(x - 1) + (x - 1)\frac{d}{dx}(5x^3 + 3x + 1)$$

$$= (5x^3 + 3x - 1) \times 1 + (x - 1) \times (15x^2 + 3)$$

$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$

$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4$

(iii)

Let f (x) = x^{-3} (5 + 3x)

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^{-3}\frac{d}{dx}(5+3x) + (5+3x)\frac{d}{dx}(x^{-3})$$

$$= x^{-3}(0+3) + (5+3x)(-3x^{-3-1})$$

By further calculation, we get

$$= x^{-3}(3) + (5+3x)(-3x^{-4})$$

$$= 3x^{-3} - 15x^{-4} - 9x^{-3}$$

$$= -6x^{-3} - 15x^{-4}$$

$$= -3x^{-3}\left(2+\frac{5}{x}\right)$$

$$= \frac{-3x^{-3}}{x}(2x+5)$$

$$= \frac{-3}{x^4}(5+2x)$$

(iv)

Let $f(x) = x^5 (3 - 6x^{-9})$

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^5 \frac{d}{dx}\left(3-6x^{-9}\right) + \left(3-6x^{-9}\right)\frac{d}{dx}\left(x^5\right)$$

$$= x^5\left\{0-6(-9)x^{-9-1}\right\} + \left(3-6x^{-9}\right)\left(5x^4\right)$$

By further calculation, we get

$$= x^5\left(54x^{-10}\right) + 15x^4 - 30x^{-5}$$

$$= 54x^{-5} + 15x^4 - 30x^{-5}$$

$$= 24x^{-5} + 15x^4$$

$$= 15x^4 + \frac{24}{x^5}$$

(v)

Let $f(x) = x^{-4} (3 - 4x^{-5})$

By differentiating both sides and using Leibnitz product rule, we get

$$f'(x) = x^{-4} \frac{d}{dx}\left(3-4x^{-5}\right) + \left(3-4x^{-5}\right)\frac{d}{dx}\left(x^{-4}\right)$$

$$= x^{-4}\left\{0-4(-5)x^{-5-1}\right\} + \left(3-4x^{-5}\right)(-4)x^{-4-1}$$

By further calculation, we get

$$= x^{-4}\left(20x^{-6}\right) + \left(3-4x^{-5}\right)\left(-4x^{-5}\right)$$

$= 20x^{-10} - 12x^{-5} + 16x^{-10}$

$= 36x^{-10} - 12x^{-5}$

$= -\frac{12}{x^5} + \frac{36}{x^{10}}$

(vi)

Let $f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$

By differentiating both sides we get,

$$f'(x) = \frac{d}{dx}\left(\frac{2}{x+1} - \frac{x^2}{3x-1}\right)$$

Using quotient rule we get,

$$f'(x) = \left[\frac{(x+1)\frac{d}{dx}(2) - 2\frac{d}{dx}(x+1)}{(x+1)^2}\right] - \left[\frac{(3x-1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}\right]$$

$$= \left[\frac{(x+1)(0) - 2(1)}{(x+1)^2}\right] - \left[\frac{(3x-1)(2x) - (x^2)\times 3}{(3x-1)^2}\right]$$

$$= -\frac{2}{(x+1)^2} - \left[\frac{6x^2 - 2x - 3x^2}{(3x-1)^2}\right]$$

$$= -\frac{2}{(x+1)^2} - \frac{x(3x-2)}{(3x-1)^2}$$

10. Find the derivative of cos x from the first principle.

Solution:

Let f (x) = cos x

Accordingly, f (x + h) = cos (x + h)

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

So, we get

$$= \lim_{h\to 0} \frac{1}{h}[\cos(x+h) - \cos(x)]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]$$

By further calculation, we get

$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$= \lim_{h\to 0} -\sin\left(\frac{2x+h}{2}\right) \times \lim_{h\to 0} \frac{\sin(\frac{h}{2})}{\frac{h}{2}}$$

$$= -\sin\left(\frac{2x+0}{2}\right) \times 1$$

= - sin (2x / 2)

= - sin (x)

11. Find the derivative of the following functions.

(i) sin x cos x

(ii) sec x

(iii) 5 sec x + 4 cos x

(iv) cosec x

(v) 3 cot x + 5 cosec x

(vi) 5 sin x – 6 cos x + 7

(vii) 2 tan x – 7 sec x

Solution:

(i) sin x cos x

Let f (x) = sin x cos x

Accordingly, from the first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h)\cos(x+h) - \sin x \cos x}{h}$$

$$= \lim_{h \to 0} \frac{1}{2h}\left[2\sin(x+h)\cos(x+h) - 2\sin x \cos x\right]$$

$$= \lim_{h \to 0} \frac{1}{2h}\left[\sin 2(x+h) - \sin 2x\right]$$

$$= \lim_{h \to 0} \frac{1}{2h}\left[2\cos\frac{2x+2h+2x}{2} \cdot \sin\frac{2x+2h-2x}{2}\right]$$

By further calculation, we get

$$= \lim_{h \to 0} \frac{1}{h}\left[\cos\frac{4x+2h}{2}\sin\frac{2h}{2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\cos(2x+h)\sin h\right]$$

$$= \lim_{h \to 0} \cos(2x+h) . \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \cos(2x+0).1$$

$$= \cos 2x$$

(ii) sec x

Let f (x) = sec x

= 1 / cos x

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

Using quotient rule, we get

$$f'(x) = \frac{\cos x \frac{d}{dx}(1) - 1\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \times 0 - (-\sin x)}{\cos^2 x}$$

We get

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$$

= tan x sec x

(iii) 5 sec x + 4 cos x

Let f (x) = 5 sec x + 4 cos x

By differentiating both sides, we get

$$f'(x) = \frac{d}{dx}(5\sec x + 4\cos x)$$

By further calculation, we get

$$= 5\frac{d}{dx}(\sec x) + 4\frac{d}{dx}(\cos x)$$

$$= 5\sec x\tan x + 4 \times (-\sin x)$$

= 5 sec x tan x – 4 sin x

(iv) cosec x

Let f (x) = cosec x

Accordingly f (x + h) = cosec (x + h)

By first principle, we get

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\operatorname{cosec}(x+h) - \operatorname{cosec} x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sin(x+h)} - \frac{1}{\sin x}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\sin(x+h)}\right]$$

By further calculation, we get

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{-\sin\left(\frac{h}{2}\right)\cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right)\sin(x+h)}\right]$$

$$=-\frac{1}{\sin x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$$=-\frac{1}{\sin x}\times 1\times\frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)}$$

$$=-\frac{1}{\sin x}\times\frac{\cos x}{\sin x}$$

= -cosec x cot x

(v) 3 cot x + 5 cosec x

Let f (x) = 3 cot x + 5 cosec x

$f'(x) = 3 (\cot x)' + 5 (\text{cosec } x)'$

Let $f_1(x) = \cot x$,

Accordingly $f_1(x + h) = \cot(x + h)$

By using first principle, we get

$$f_1'(x) = \lim_{x \to 0} \frac{f_1(x+h) - f_1(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cot(x+h) - \cot x}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}\right)$$

By further calculation, we get

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\sin x \cos(x+h) - \cos x \sin(x+h)}{\sin x \sin(x+h)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\sin(x-x-h)}{\sin x \sin(x+h)}\right)$$

$$= 1 / \sin x \lim_{h \to 0} \frac{1}{h}\left[\frac{\sin(-h)}{\sin(x+h)}\right]$$

$$= -\frac{1}{\sin x}\left(\lim_{h \to 0} \frac{\sin h}{h}\right)\left(\lim_{h \to 0} \frac{1}{\sin(x+h)}\right)$$

$$= -\frac{1}{\sin x} \times 1 \times \frac{1}{\sin(x+0)}$$

$$= -\frac{1}{\sin^2 x}$$

$$= -\text{cosec}^2 x$$

Let $f_2(x) = \operatorname{cosec} x$,

Accordingly $f_2(x+h) = \operatorname{cosec}(x+h)$

By using first principle, we get

$$f_2'(x) = \lim_{h\to 0}\frac{f_2(x+h)-f_2(x)}{h}$$

$$=\lim_{h\to 0}\frac{\operatorname{cosec}(x+h)-\operatorname{cosec} x}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sin(x+h)}-\frac{1}{\sin x}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)}\right]$$

By further calculation, we get

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\left[\frac{-\sin\left(\frac{h}{2}\right)\cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right)\sin(x+h)}\right]$$

$$=-\frac{1}{\sin x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$$=-\frac{1}{\sin x}\times 1\times\frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)}$$

$$= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$$

= -cosec x cot x

Now, substitute the value of (cot x)' and (cosec x)' in f'(x), we get

f'(x) = 3 (cot x)' + 5 (cosec x)'

f'(x) = 3 × (-cosec^2 x) + 5 × (-cosec x cot x)

f'(x) = -3cosec^2 x – 5cosec x cot x

(vi)5 sin x – 6 cos x + 7

Let f (x) = 5 sin x – 6 cos x + 7

Accordingly, from the first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[5\sin(x+h) - 6\cos(x+h) + 7 - 5\sin x + 6\cos x - 7\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[5\{\sin(x+h) - \sin x\} - 6\{\cos(x+h) - \cos x\}\right]$$

$$= 5\lim_{h \to 0} \frac{1}{h}\left[\sin(x+h) - \sin x\right] - 6\lim_{h \to 0} \frac{1}{h}\left[\cos(x+h) - \cos x\right]$$

By further calculation, we get

$$= 5\lim_{h \to 0} \frac{1}{h}\left[2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right] - 6\lim_{h \to 0} \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= 5\lim_{h \to 0} \frac{1}{h}\left[2\cos\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}\right] - 6\lim_{h \to 0}\left[\frac{-\cos x(1-\cos h) - \sin x\sin h}{h}\right]$$

Now, we get

$$=5\lim_{h\to 0}\left(\cos\left(\frac{2x+h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)-6\lim_{h\to 0}\left[\frac{-\cos x(1-\cos h)}{h}-\frac{\sin x\sin h}{h}\right]$$

$$=5\left[\lim_{h\to 0}\cos\left(\frac{2x+h}{2}\right)\right]\left[\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right]-6\left[(-\cos x)\left(\lim_{h\to 0}\frac{1-\cos h}{h}\right)-\sin x\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\right]$$

$$=5\cos x.1-6[(-\cos x).(0)-\sin x.1]$$
$$=5\cos x+6\sin x$$

(vii) 2 tan x – 7 sec x

Let f (x) = 2 tan x – 7 sec x

Accordingly, from the first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}[2\tan(x+h)-7\sec(x+h)-2\tan x+7\sec x]$$

$$=\lim_{h\to 0}\frac{1}{h}[2\{\tan(x+h)-\tan x\}-7\{\sec(x+h)-\sec x\}]$$

$$=2\lim_{h\to 0}\frac{1}{h}[\tan(x+h)-\tan x]-7\lim_{h\to 0}\frac{1}{h}[\sec(x+h)-\sec x]$$

By further calculation, we get

$$=2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{1}{\cos(x+h)}-\frac{1}{\cos x}\right]$$

$$= 2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)\cos x-\sin x\cos(x+h)}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{\cos x-\cos(x+h)}{\cos x\cos(x+h)}\right]$$

$$= 2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos x\cos(x+h)}\right]$$

Now, we get

$$= 2\lim_{h\to 0}\left[\left(\frac{\sin h}{h}\right)\frac{1}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos x\cos(x+h)}\right]$$

$$= 2\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{1}{\cos x\cos(x+h)}\right)-7\left(\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{2x+h}{2}\right)}{\cos x\cos(x+h)}\right)$$

$$= 2.1.\frac{1}{\cos x\cos x}-7.1\left(\frac{\sin x}{\cos x\cos x}\right)$$

$$= 2\sec^2 x-7\sec x\tan x$$