Gujarat Board - X Mathematics (Standard)
2022-23

Maximum Marks: **80**
Time allowed: **3 Hours**

Instructions:

1) Write in a clear legible handwriting.
2) This question paper has four Sections A, B, C & D and Question Numbers from 1 to 39.
3) All Sections are compulsory. General options are given.
4) The numbers to the right represent the marks of the question.
5) Draw neat diagrams wherever necessary.
6) New sections should be written in a new page. Write the answers in numerical order.
7) Calculator is not allowed.

Section A

- **Answer the following as directed. (1 to 16) (1 mark each)** **[16]**
  State whether the following statements are true or false.

1) $8\,sec^2\theta - 8tan^2\theta = 8$ **[1]**
Answer: True

Explanation:
$8\,sec^2\theta - 8tan^2\theta$
$= 8(sec^2\theta - tan^2\theta)$
$= 8(1)$
$= 8$

2) $7\times11\times13 + 13$ is a prime number. **[1]**

Answer: False

Explanation:

$(7\times11\times13 + 13) = 13\{(7\times11) + 1\} = 13\times78 = 1014$
Since, 1014 is divided by 13,78 which is other than 1 and the number itself, so it is a composite number.

3) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, ... is an arithmetic progression. **[1]**
Answer: False
Explanation:
$a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, a_3 = \frac{1}{4}$
Common difference:
$d_1 = a_2 - a_1 = \frac{1}{3} - \frac{1}{2} = \frac{-1}{6}$
$d_2 = a_3 - a_2 = \frac{1}{4} - \frac{1}{3} = \frac{-1}{12}$
$d_2 \neq d_1$
Therefore, the given sequence is not an arithmetic progression.

4) If one of the root of quadratic $x^2 - 4x + m = 0$ is 3 then m = 3. **[1]**

Answer: True
Explanation:
If x = 3,
Then, $3^2 - 4(3) + m = 0$
$9 - 12 + m = 0$
$m = 3$

5) If H.C.F (12, k) = 6 and L.C.M. (12, k) = 36 then k = _____. **[1]**

Answer: 18
Explanation:

H.C.F x L.C.M = Product of two numbers

$6\times36 = 12\times k$

$k = \frac{6\times36}{12}$

$k = 18$

6) If α and β are zeros of quadratic polynomial $3x - x^2 + 8,$ $then$ $\alpha\beta =$ ___.
[1]

Answer: -8
Explanation:

$3x - x^2 + 8$ can be written as $- x^2 + 3x + 8$

a = -1, b = 3 and c = 8

Product of the roots $\alpha\beta$ = $\frac{c}{a}$ =$\frac{8}{-1}$ -8

7) $27x + 63y = 45$ and $63x + 27y = 135$, then $x + y =$ ____
[1]

Answer: x + y = 2
Explanation:

$27x + 63y = 45$ .... (i) ÷ 9

$63x + 27y = 135$ .... (ii) ÷ 9

$3x + 7y = 5$ .... (iii) ×7

$7x + 3y = 15$ .... (iv) ×3

$21x + 49y = 35$ ....

$21x + 9y = 45$ .... (-)

$40y =- 10$

$y =- \frac{1}{4}$

Sub y in (iii),

$3x + 7(-\frac{1}{4}) = 5$

$3x = 5 + \frac{7}{4}$

$3x = \frac{27}{4}$

$x = \frac{9}{4}$

$x + y = \frac{9}{4} - \frac{1}{4}$

$= \frac{8}{4}$

$= 2$

8) Co-ordinates of midpoint M of line segment AB joining the points A (2a-b,b) and B(b, 2a-b) is ______. **[1]**

Answer: (a, a)
Explanation:
Midpoint of AB = $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$= (\frac{2a-b+b}{2}, \frac{b+2a-b}{2}) = (a, a)$

9) Which is the median class for the following frequency distribution? **[1]**

Class	$0 - 10$	$10 - 20$	$20 - 30$	$30 - 40$	$40 - 50$
Frequency	7	15	13	17	10

Answer: 20-30
Explanation:

To find the median class of a frequency distribution, we first need to calculate the cumulative frequency of each class. We can do this by adding up the frequencies of each class, starting from the first class.

Class	Frequency	Cumulative Frequency
0-10	7	7
10-20	15	22
20-30	13	35
30-40	17	52
40-50	10	62

The total frequency is 62, which is an even number. To find the median class, we need to find the class that contains the middle value. The middle value is the average of the two middle values, which in this case are the 31st and 32nd values.

To find the class that contains the 31st value, we look at the cumulative frequency column and find the first class that has a cumulative frequency greater than or equal to 31. In this case, it is the class 20-30, which has a cumulative frequency of 35.

To find the class that contains the 32nd value, we look at the cumulative frequency column and find the first class that has a cumulative frequency greater than or equal to 32. In this case, it is also the class 20-30.

Therefore, the median class is the class 20-30.

10) If $P(A) - P(\overline{A}) = 0.8$ then find the value of $P(A)$. **[1]**

Answer: 0.9

Explanation:

We know that the complement of event A is denoted as A', $\overline{A}$, or A-bar. The probability of an event and its complement always sum up to 1. Therefore:

P(A) + P($\overline{A}$) = 1

We are given that:

P(A) - P($\overline{A}$)= 0.8

Substituting the value of P($\overline{A}$) from the first equation into the second equation, we get:

P(A) - (1 - P(A)) = 0.8

Simplifying the equation, we get:

2P(A) - 1 = 0.8

Adding 1 to both sides, we get:

2P(A) = 1.8

Dividing both sides by 2, we get:

P(A) = 0.9

Therefore, the value of P(A) is 0.9.

11) Find the diameter of a circle whose circumference and area are equal in number. **[1]**

Answer: Diameter = 4 units
Explanation:
Given that,
Area of circle = Circumference of circle
$\pi r^2 = 2\pi r$

$r = 2\,units$
Diameter = 2 × Radius
$\therefore Diameter = 4\,units$

12) For what value of an acute angle $\theta, cot2\theta \cdot cot7\theta = 1$? **[1]**

Answer: 10°

Explanation.

$cot2\theta \times cot7\theta = 1$
Rearranging the above equation
$\Rightarrow cot7\theta = \frac{1}{cot\,2\theta}$
We know that $tan2\theta = \frac{1}{cot\,2\theta}$
$\Rightarrow cot7\theta = tan2\theta$
Since $tan\theta = cot(90 - \theta)$

$\Rightarrow cot7\theta = cot\,(90 - 2\theta)$

$\therefore 7\theta = 90^{\circ} - 2\theta \Rightarrow 9\theta = 90^{\circ} \Rightarrow \theta = 10^{\circ}$

13) If a sphere of radius r is divided in four equal parts then the total surface area of each part is? **[1]**

a) πr^{2}

b) $2\pi r^{2}$

c) 3 πr^{2}

d) $\frac{1}{2}\pi r^{2}$

Answer: (a) πr^{2}
Explanation:
Total surface area of sphere = $4\pi r^{2}$

So, one fourth of it will be πr^2.

14) If the pair of equations 2x + 2y + 2 = 0 and 4x + ky + 8 = 0 has unique solution then k≠___. **[1]**

a) 4
b) 2
c) -4
d) 8

Answer: (a) 4
Explanation:

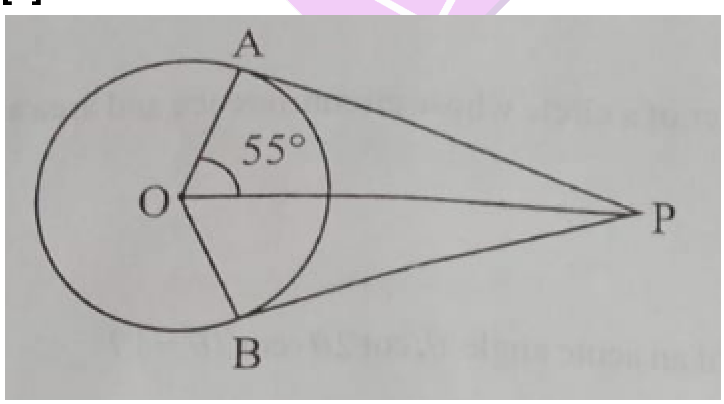
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{4} \neq \frac{2}{k}$

$\frac{1}{2} \neq \frac{2}{k}$

$k \neq 4$

15) In a figure below, PA and PB are tangents to the circle with centre O, ∠AOP = 55° then ∠APB = _____.
[1]



a) 35°
b) 70°
c) 125°
d) 110°

Answer: (b) 70°

Explanation:

In ΔOAP, ∠A = 90°,

So, ∠AOP+ ∠OAP+ ∠APO=180°

55°+90° $+ \angle APO$ =180°

$\angle APO = 35°$

So, ∠APB=2 $\angle APO$

∠APB = 70°

16) An unbiased coin is tossed thrice. What is the probability of getting at least two heads? **[1]**

a) 3/8

b) 1/8

c) ½

d) 2/3

Answer: (c) 1/2

Explanation:

S = {HHH, HHT, HTH, HTT, TTT, THH, THT, TTH}

At Least 2 heads A = {HHH, HHT, HTH, THH}

P(A) = n(S)/n(A) = 4/8 = 1/2

SECTION-B

- **Solve the following questions showing calculation. (17 to 26) (2 marks each) [20]**

17) Prove that $5 + 2\sqrt{7}$ is irrational. **[2]**

Explanation:

Let, $5 + 2\sqrt{7}$ be rational.

So $5 + 2\sqrt{7} = \frac{a}{b}$,

where ' a ' and b ' are integers and $b \neq 0$

$2\sqrt{7} = [\frac{a}{b}] - 5$

$2\sqrt{7} = \frac{[a-(5b)]}{2b}$

Since ' a ' and ' b ' are integers $a - 5b$ is also an integer. [$\frac{[a-(5b)]}{2b}$ is rational. So RHS is rational. LHS should be rational. but it is given that $2\sqrt{7}$ is irrational. Our assumption is wrong.

18) Find the zeroes of the quadratic polynomial $6x^2 - 13x + 6$. **[2]**

Answer: $\frac{2}{3}, \frac{3}{2}$

Explanation:

$6x^2 - 13x + 6 = 0$

$6x^2 - 9x - 4x + 6 = 0$

$3x(2x - 3) - 2(2x - 3) = 0$

$(3x - 2)(2x - 3) = 0$

$x = \frac{2}{3}, \frac{3}{2}$

19) Verify whether the pair of linear equations: $\frac{4}{3}x + 2y = 8$ and $2x + 3y = 12$ is consistent or not. **[2]**

Explanation:

$\frac{4}{3}x + 2y = 8, 2x + 3y = 12$

$\Rightarrow 4x + 6y = 24, 2x + 3y = 12$

$\Rightarrow 2x + 3y = 12, 2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{4}{2} = 2 \quad \frac{b_1}{b_2} = \frac{6}{3} = 2$

$\frac{c_1}{c_2} = \frac{24}{12} = 2 \qquad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ the linear equations are consistent

$\therefore$ They have infinite solutions.

OR

19) Solve the pair of equations by substitution method:

$x + y = 4 \; and \; 2x = 8 + 3y.$

Answer:

Explanation:

$x + y = 4 ... (i) \quad 2x = 8 + 3y ... - (ii)$

Put $x = 4 - y$ in equation (ii)

$2\{4 - y\} = 8 + 3y$

$8 - 2y = 8 + 3y$

$5y = 0$

$y = 0, \; x = 4$

20) If P, Q and R are interior angles of a triangle PQR then show that $sec\left(\frac{P+Q}{2}\right) = cosec\frac{R}{2}$ **[2]**

Explanation:

Given P, Q, R are the interior angles of $\triangle PQR$

$\Rightarrow P + Q + R = 180^{\circ}$

$\Rightarrow P + Q = 180^{\circ} - R$

Since, we know that, $P + Q + R = 180^{\circ}$

$\Rightarrow P + Q = 180^{\circ} - R$

Now,

$$LHS = sec(\frac{P+Q}{2}) = sec\left(\frac{180^{\circ}-R}{2}\right) = sec\left(90^{\circ} - \frac{R}{2}\right) = cosec\frac{R}{2} = RHS\ proved.$$

21) If $2sin\theta + cos\theta = 2$, find $tan\theta \cdot (cos\theta \neq 0)$ **[2]**

Answer: ∞

Explanation:

$cos\theta = 2 - 2sin\theta$

$1 - sin^{2}\theta = 4 + 4sin^{2}\theta - 8sin\theta$

$5sin^{2}\theta - 8sin\theta + 3 = 0$

$5sin^{2}\theta - 5sin\theta - 3sin\theta + 3 = 0$

$5sin\theta(sin\theta - 1) - 3(sin\theta - 1) = 0$

$(5sin\theta - 3)(sin\theta - 1) = 0$

$sin\theta = \frac{3}{5}\ or\ sin\theta = 1$

$tan\theta = \frac{3}{4}\ or\ tan\theta = \infty$

OR

21) If $2tan^{2}45^{\circ} + x - sin^{2}60^{\circ} = 2$, find the value of x. **[2]**

Answer: $\frac{3}{4}$

Explanation:

$2tan^{2}45^{\circ} + x - sin^{2}60$

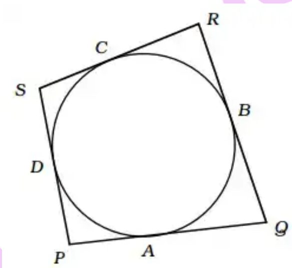
$= 2\,2 * 1^{2} + x - \left(\frac{\sqrt{3}}{2}\right)^{2} = 2$

$2 + x - \frac{3}{4} = 2$

$x = 2 - 2 + \frac{3}{4}$

$x = \frac{3}{4}$

22) As shown in the figure, a quadrilateral $PQRS$ is drawn to circumscribe a circle. Prove that $PQ + RS = QR + PS$. **[2]**



Explanation:

Given:
Quadrilateral $PQRS$ circumscribes a circle
To prove that:

$PQ + RS = PS + QR$
Proof:
Lengths of tangents drawn to a circle from an external point are equal.
Hence, $PA = PD$...........(i)

$QA = QB$......... (ii) $RB = RC$.......... (iii) $SC = SD$......... (iv)
LHS

$PQ + RS = PA + AQ + RC + CS$

RHS

$PS + QR \;\; = PD + DS + QB + BR$
$= PA + CS + AQ + RC$ $using\ equations\ (i),\ (ii),\ (iii)\ and\ (iv)$
From above, it can be seen LHS = RHS
Hence proved

OR

22) Two concentric circles are of radii $29\ cm$ and $21\ cm$. Find the length of the chord of the larger circle which touches the smaller circle. **[2]**

Answer: 40 cm

Explanation:

The length of the chord of the larger circle which touches the smaller circle is $40\ cm$.
Step-by-step explanation:
Let PQ be the chord of the larger circle which touches the smaller circle at the point L. Since PQ is tangent at the point L. Since PQ is tangent at the point L to the smaller circle with center O.Thus
$OL = 21\ cm$ and $OP = 29\ cm$
Therefore, OL is perpendicular to PQ
Since PQ is a chord of the bigger circle and OL is perpendicular PQ.
Therefore, $PQ = 2PL$
In the right angled triangle OPL.

$PL = \sqrt{OP^2 - OL^2} \;\; = \sqrt{29^2 - 21^2} \;\; = \sqrt{400} \;\; = 20\ cm$
Therefore, chord $PQ = 2PL = 2\times20\ cm = 40\ cm$
So, length of the chord PQ is $40\ cm$.

23) For the following grouped frequency distribution, find the mode. **[2]**

Class	$10 - 25$	$25 - 40$	$40 - 55$	$55 - 70$	$70 - 85$

Frequency	2	3	7	6	6

Answer: 52

Explanation:

The class having highest frequency is $40 - 55$, therefore, it is the modal class
Now lower limit of modal class $(l) = 40$
Frequency $\left(f_1\right)$ of modal class $= 7$
frequency $\left(f_0\right)$ of preceding the modal class $= 3$
frequency $\left(f_2\right)$ of the class succeeding the modal class $= 6$
Class size $= 15$

Mode $= L + \left(\frac{f_1 - f_0}{2f_i - f_0 - f_2} \times h\right)$

$= 40 + \left(\frac{7-3}{14-3-6} \times 15\right)$

$= 40 + \frac{60}{5}$

$= 40 + 12$

= 52

24) Salma and Mona are friends. What is the probability that both will have
i) different birthdays?
ii) the same birthday in the year 2019? **[2]**

Answer: $\frac{364}{365}$ & $\frac{1}{365}$

Explanation:

(i) Let E be the event of both having different birthdays, and (not E) be the event same birthdays

If Salma's b'day is different from Mona's b'day, the number of favorable outcomes for her bday is $365 - 1 = 364$

$P(E)$ = (Number of outcomes favorable to E)/(Total number of outcomes) = $\frac{364}{365}$

(ii)Probability of Salma and Mona sharing the same b'day in 2019 (which is not a leap year)will be:

$$P(\,not\ E) = 1 - P(E) = 1 - \frac{364}{365} = \frac{1}{365}$$

25) Find the roots of the quadratic equation $5x = 6 + \frac{2}{x}$ by the method of completing the square. **[2]**

Answer: $\frac{3}{5} \pm \frac{\sqrt{19}}{5}$

Explanation:

We have, after rearranging the equation,

$$5x^2 - 6x - 2 = 0$$
$$x^2 - \frac{6}{5}x - \frac{2}{5} = 0$$
$$x^2 - \frac{6}{5}x = \frac{2}{5}$$
$$x^2 - 2\left(\frac{3}{5}\right)x + \left(\frac{3}{5}\right)^2 = \frac{2}{5} + \left(\frac{3}{5}\right)^2$$
$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$
$$x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$
$$x = \frac{3}{5} \pm \frac{\sqrt{19}}{5}$$

OR

25) Find the roots of quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ by the method of factorisation. **[2]**

Solution.

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$Splitting\ the\ middle\ term\ method \quad We\ will\ factorize\ by$

$We\ need\ to\ find\ two\ numbers\ whose \quad splitting\ the\ middle\ term\ method$d

$Sum\ = 7\ \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$Product\ = 5\sqrt{2} \times \sqrt{2} \quad = 5\times2$

$= 10\ \sqrt{2}x^2 + (\sqrt{2} \times \sqrt{2})x + 5x + 5\sqrt{2}$

$= 0\ \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2})$

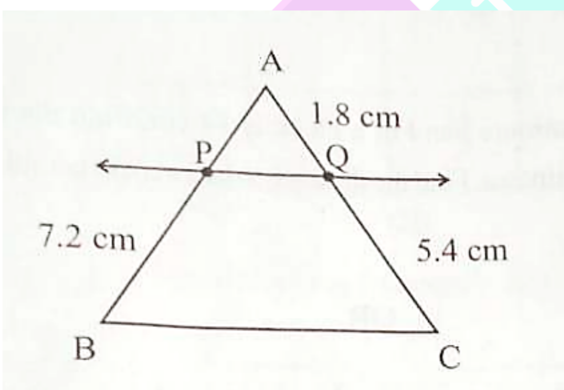
$= 0\ \ (\sqrt{2}x + 5)(x + \sqrt{2})$

$= 0\ \sqrt{2}x + 5$

$= 0|\ x + \sqrt{2} = 0$

$\sqrt{2}x =- 5\ x =- \sqrt{2}\ \ x = \frac{-5}{\sqrt{2}}$

26) In the given figure if $PQ \parallel BC$ then find AB. **[2]**



Answer: 10.6 cm

Explanation:

We can apply Basic proportionality theorem here, which will give -

$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC} \quad \Rightarrow \frac{AP}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow AP\ = 3.4\ cm$

$AB = AP + PB$
$AB = 3.4 + 7.2$
$AB = 10.6\, cm$

SECTION-C

Answer the following questions showing calculations. (27 to 34) (3 marks each) [24]

27) On dividing $3x^3 + x^2 + 2x + 5$ by a polynomial $g(x)$, the quotient and remainder were $3x - 5$ and $9x + 10$ respectively. Find $g(x)$. **[3]**

Answer: $x^2 + 2x + 1$

Explanation:

If we divide $f(x) = 3x^3 + x^2 + 2x + 5$ by $g(x)$ we get $q(x) = (3x - 5)$ as quotient and $r(x) = (9x + 10)$ as remainder.
By using division algorithm,

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
3x-5 & 3x^3 + x^2 - 7x - 5 \\
 & 3x^3 - 5x^2 \\
 & (-)\ \ (+) \\
\hline
 & 6x^2 - 7x \\
 & 6x^2 - 10x \\
 & (-)\ \ (+) \\
\hline
 & 3x - 5 \\
 & 3x - 5 \\
 & (-)\ \ (+) \\
\hline
 & 0 \\
\hline
\end{array}
$$

$f(x) = g(x)q(x) + r(x) \Rightarrow 3x^3 + x^2 + 2x + 5$

$= g(x)(3x - 5) + (9x + 10)$

$\Rightarrow g(x) = \frac{3x^3+x^2+2x+5-9x-10}{(3x-5)}$

$\Rightarrow g(x) = \frac{3x^3+x^2-7x-5}{(3x-5)}$

Hence, $g(x) = x^2 + 2x + 1$

28) Sum of the areas of two squares is $468\, m^2$. If the difference of their perimeters is $24\, m$, find the sides of the two squares. **[3]**

Answer: 12 m and 18 m

Explanation:

Let the side of the first square be 'a'm and that of the second be 'A' m. Area of the first square $= a^2$ sq m .

Area of the second square $= A^2 sqm$.
Their perimeters would be $4a$ and $4\, A$ respectively.
Given $4\, A - 4a = 24$

$A - a = 6 - (1) A^2 + a^2 = 468$___$(2)$
From (1), $A = a + 6$
Substituting for A in (2), we get

$(a + 6)^2 + a^2 = 468$

$a^2 + 12a + 36 + a^2 = 468$

$2a^2 + 12a + 36 = 468$

$a^2 + 6a + 18 = 234\ \ 6$

$a^2 + 6a - 21 = 0$

$a^2 + 18a - 12a - 216 = 0$

$a(a + 18) - 12(a + 18) = 0$

$(a - 12)(a + 18) = 0$

$a = 12, -18$

So, the side of the first square is $12\ m$. and the side of the second square is 18 m

29) For what value of n, are the n^{th} terms of two APs : $65, 67, 69,$ .... and $10, 17, 24,$ equal? **[3]**

Answer: n = 12

Explanation:

for the first A.P.

$a = 65\ \ d = 2$

Its n^{th} term will be -

$t_n = a + (n - 1)d \quad t_n = 65 + (n - 1)2$

for the second A.P.

$A = 10\ \ d = 7$

$t_n = 10 + (n - 1) - 7$

$A.T.Q\ \ 65 + (n - 1)^2 = 10 + (n - 1)7$

$55 = 7n - 7 - 2n + 2$

$55 = 5n - 5$

$11 = n - 1$

$n = 12$

So, for $n = 12$ the n^{th} term of both the APs will be same.

OR

29) Find the sum of all the terms of the AP: $-2, -5, -8,$ ......,. $-227.$
[3]

Answer: -8702

Explanation:

Given -

$a = \ - 2\ d = \ - 3\ l = \ - 227$
Now sum of all forms will be given by -

$S_n = \frac{n}{2}[a + l]$

We know n^{th} form of an A.P. will be-

$t_n \ = a + (n - 1)d$
$\Rightarrow \ - 227 \ = - 2 + (n - 1)(- 3)$
$\Rightarrow \ - 225 \ = (n - 1)(- 3)$
$\Rightarrow 75 \ = n - 1$
$\Rightarrow n \ = 76$
Now -

$s_n = \frac{n}{2}[a + l]$
$s_n = \frac{76}{2}[- 2 - 227]$
$s_n = 38[- 229]$
$s_n = - 8702$

30) If $P(2, 3), Q(3, - 2), R(- 3, - 5)$ and $S(- 4, - 2)$ are the vertices of a quadrilateral, find the area of the quadrilateral $PQRS$. **[3]**

Answer: $28\ sq.\ units$

Explanation:

Area of quadrilateral $PQRS = ar\triangle RSQ + ar\triangle PRS$.

Area $\triangle RSQ$

$= \frac{1}{2}\left[x_1\left(y_2 - y_1\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}[-4(-2-(-5)) + 3(-5-(-2)) + -3(-2-(-2))]$

$= \frac{1}{2}[-4\times3 + 3\times -3 + 3\times0]$

$= \frac{1}{2}\times(12 + 9)$
$= \frac{21}{2}\ sq.\ units$

Area of triangle PRS

$= \frac{1}{2}[-4(-2-3) + 3(3+2) + 2(-2+2)]$
$= \frac{1}{2}[-4\times -5 + 3\times5 + 0]$
$= \frac{1}{2}\times(20 + 15)$
$= \frac{35}{2}\ sq.\ units$
$\therefore area\ \square PQRS = \frac{21}{2} + \frac{35}{2} = 28\ sq.\ units$

31) The length of the minute hand of a clock is $14\ cm$. Find the area swept by the minute hand in 15 minutes. Find the distance to be swept to complete one revolution. **[3]**

Explanation:

Minute hand completes full circle degree in 60 minutes.
Angle swept by minute in 60 minutes $= 360^\circ$
Angle swept by the minute hand in 15 minutes $= \frac{360^\circ}{60}\times15 = 90^\circ$
Therefore,

$$\theta = 90^\circ$$

Length of minute hand = $r = 14\ cm$
Area swept by minute hand in 15 minutes = Area of sector
As we know that area of sector is given as-

$A = \frac{\theta}{360^\circ} \times \pi r^2$

Therefore,

Area swept by the minute hand in 15 minutes = $\frac{90^\circ}{360^\circ} \times \left(\frac{22}{7} \times (14)^2\right) = \frac{1}{4} \times 616 = 154\ cm^2$

Hence the area swept by the minute hand in 15 minutes is $154\ cm^2$.

Hence the correct answer is $154\ cm^2$.

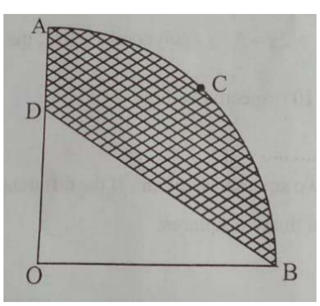
(ii) Distance swept in complete one complete revolution

$= 2\pi r\ \frac{2 \times 22}{7} \times 14 = 88\ cm$

OR

31) In the given figure $OACB$ is quadrant of a circle with centre O and diameter $7\ cm$. If $OD = 2\ cm$, find the area of the

i) quadrant $OACB$
ii) shaded region **[3]**



Explanation:

(i) Area of Quadrant $OACB = \frac{1}{4}\times \pi r^2$ where r is the radius of circle.

$$= \frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{8}\,cm^2$$

(ii) Area of Shaded region .

Area of Quadrant OACB – Area OF $\triangle ODB = .\frac{77}{8}\,cm^2 - \frac{1}{2}\times$ Base x height

$$\frac{77}{8}\,cm^2 - \frac{1}{2}\times\frac{7}{2}\times 2\,cm^2 = \frac{49}{8}\,cm^2$$

32) A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is $28\,cm$ and the total height of the vessel is $26\,cm$. Find the inner surface area of the vessel. **[3]**

Answer: $2288\,cm^2$.

Explanation:

As per the parameters given in the question itself, we have
Diameter of the hemisphere, $d = 28\,cm$
Radius of the hemisphere, $r = 14\,cm\left[\because r = \frac{d}{2}\right]$
Height of cylindrical portion, $h = 26 - 14 = 12\,cm$
Curved surface area of cylindrical portion can be calculated as follows:
$= 2\pi rh = 2\times\frac{22}{7}\times14\times12 = 1056\,cm^2$
Curved surface area of hemispherical portion can be calculated as follows:
$= 2\pi r^2 = 2\times\frac{22}{7}\times 14^2 = 1232\,cm^2$

Total inner surface area $= 1056\,cm^2 + 1232\,cm^2 = 2288\,cm^2$

Hence, the inner surface area of the vessel is $2288\,cm^2$.

33) The following frequency distribution shows the ages of 100 persons. Find the median of the data.

Age (years)	$20-3$	$30-4$	$40-50$	$50-60$	$60-70$	$70-80$
No. of persons	15	16	38	15	9	7

Answer: 45

Explanation:

$Class\ (1)$	$Frequency\ (f)\ (2)$	$cf\ (7)$
$20-30$	15	15
$30-40$	16	31
$40-50$	38	69
$50-60$	15	84
$60-70$	9	93
$70-80$	7	100
	$n = 100$	

To find Median Class

$=$ value of $\left(\frac{n}{2}\right)^{th}$ observation

$=$ value of $\left(\frac{100}{2}\right)^{th}$ observation

$=$ value of 50^{th} observation

From the column of cumulative frequency cf, we find that the 50^{th} observation lies in the class $40-50$.

$\therefore$ The median class is $40-50$.

Now

$\therefore L =$ lower boundary point of median class $= 40$

$\therefore n$ = Total frequency = 100
$\therefore cf$ = Cumulative frequency of the class preceding the median class = 31

$\therefore f = Frequency\ of\ the\ median\ class = 38$

$\therefore c = class\ length\ of\ median\ class = 10$

$Median\ M = L + \frac{\frac{n}{2} - cf}{f} \cdot c$

$= 40 + \frac{50-31}{38} \cdot 10$

$= 40 + \frac{19}{38} \cdot 10$

$= 40 + 5 = 45$

Therefore, median = 45

OR

33) The mean of the following frequency distribution is 18. Find the missing frequency f. **[3]**

Class	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 -25
Frequency	7	6	f	13	20	5	4

Answer: 9

Explanation:

Class interval	Mid value x_i	Frequency f_i	$f_i x_i$
11 - 13	12	7	84
13 -15	14	6	84

15 - 17	16	f	16f
17 - 19	18	13	234
19 - 21	20	20	400
21 - 23	22	5	110
23 - 25	24	4	96
		$\sum f = 55 + f$	$\sum f_i x_i = 1008 + 16f$

Given : Mean $= 18$

We know, $Mean(\overline{x}) = \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow 18 = \frac{1008+16f}{55+f}$

$\Rightarrow 18(55 + f) = 1008 + 16f$

$\Rightarrow 990 + 18f = 1008 + 16f$

$\Rightarrow 2f = 1008 - 990 = 18$

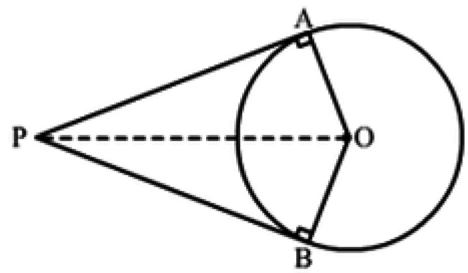
$\Rightarrow f = \frac{18}{2} = 9$

OR

34) Prove that the lengths of tangents drawn from an external point to a circle are equal. **[3]**

Explanation:

Let AP and BP be the two tangents to the circle with centre O.



To Prove: AP = BP

Proof:

In ΔAOP and ΔBOP

OA = OB (radii of the same circle)

$\angle OAP = \angle OBP = 90^\circ$ (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

OP = OP (common)

$\therefore \Delta AOP \cong \Delta BOP$ (by R.H.S. congruence criterion)

$\therefore$ AP = BP (corresponding parts of congruent triangles)

Hence the length of the tangents drawn from an external point to a circle are equal.

SECTION-D

- **Answer the following questions as required by showing calculations. (35 to 39)**
  (4 marks each) **[20]**

35) A boat goes $40\ km$ upstream and $49\ km$ downstream in 15 hours. In the same river it can go $25\ km$ upstream and $35\ km$ downstream in 10 hours. Determine the speed of the stream and that of the boat in still water.
[4]

Answer: 1 km/h and 6 km/h

Explanation:

Let speed of boat in still water be $x\ km/h$ and speed of stream be $y\ km/h$.
Speed Upstream = $(x - y)km/h$
Speed downstream = $(x + y)km/h$
According to the question,

$\frac{40}{x-y} + \frac{49}{x+y} = 15$
And $\frac{25}{x-y} + \frac{35}{x+y} = 10$

Let $\frac{1}{x-y} = p$ and $\frac{1}{x+y} = q$ ... (1)
Therefore, the equation becomes

$40p + 49q = 15 ...(2)$
$25p + 35q = 10 ...(3)$
$\Rightarrow 25p = 10 - 35q$
$\Rightarrow p = \frac{10-35q}{25} ...(4)$
Substituting the value of p in (2), we get

$40\left(\frac{10-35q}{25}\right) + 49q = 15$
$\Rightarrow 8\left(\frac{10-35q}{5}\right) + 49q = 15$
$\Rightarrow \frac{8(10-35q)+5(49q)}{5} = 15$
$\Rightarrow 80 - 280q + 245q = 75$
$\Rightarrow -35q = 75 - 80 = -5$
$\Rightarrow q = \frac{-5}{-35} = \frac{1}{7} ...(5)$
Substituting the value of q in (4), we get

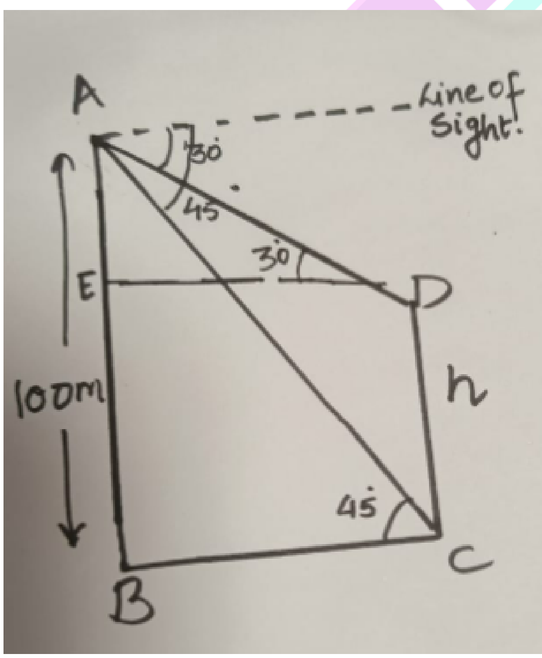
$\Rightarrow p = \frac{10-35(\frac{1}{7})}{25}$

$\Rightarrow p = \frac{10-5}{25} = \frac{5}{25} = \frac{1}{5} \ldots(6)$

From (1), (5) and (6), we get $x - y = 5$ and $x + y = 7$
Solving both, we get $x = 6$ and $y = 1$
Hence, the speed of the stream and that of the boat in still water is 1 km/h and 6 km/h, respectively.

36) As observed from the top of a $100\ m$ high hill the angle of depressions of the top of a tower is 30° and the angle of depressions of the bottom of a tower is 45°. Find the height of the tower and the distance between base of a tower and base of a hill. **[4]**
(Take $\frac{1}{\sqrt{3}} = 0.58$)

Answer: $74.24\ m$
Explanation:



In $\triangle ABC$ right angled at B

$tan45^{\circ} = \frac{AB}{BC}$ $1 = \frac{AB}{BC}$ $\Rightarrow AB = BC = 100\, m$

Since DE and BC are equal $SO\ DE = 100$ metre

In right-angled triangle AED,

$tan30^{\circ} = \frac{AE}{ED}$ $\frac{1}{\sqrt{3}} = \frac{100-h}{45}$

By cross multiplication

$100\sqrt{3} - \sqrt{3}h = 45$

$\sqrt{3}h = 100\sqrt{3} - 45$

$\sqrt{3}h = 100\times1.73 - 45$

$\sqrt{3}h = 173 - 45$

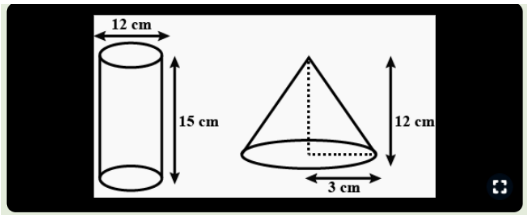
$h = \frac{173-45}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}\times128$

$= 120\times0.58$

$= 74.24\, m$

37) A container shaped like a right circular cylinder having radius $6\ cm$ and height $15\ cm$ is full of ice cream. The ice cream is to be filled into cones of height $12\ cm$ and radius $3\ cm$, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream. **[4]**



Answer: 10

Explanation:

Let n ice-cream cones be filled with ice cream of the container.
Volume of ice cream in the cylinder $=$ n (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$
$$\Rightarrow \pi \times 6^2 \times 15 = n\left(\frac{1}{3}\pi 3^2 \times 12 + \frac{2}{3}\pi 3^3\right)$$
$$\Rightarrow n = \frac{30\times15}{\frac{1}{3}\times9\times12+\frac{2}{3}\times27}$$
$$\Rightarrow n = \frac{36\times15\times3}{108+54}$$
$$n = 10$$

38) In $\triangle XYZ$ if $XY^2 + XZ^2 = YZ^2$, prove that $\angle X = 90^\circ$. **[4]**

Explanation:

We are given that in triangle XYZ, $XY^2 + XZ^2 = YZ^2$.

We need to prove that angle X is 90 degrees.
To begin the proof, we will use the Pythagorean theorem, which states that in a right triangle, the sum of the squares of the two shorter sides is equal to the square of the longest side (the hypotenuse).

Let's assume that angle X is not 90 degrees. Then, either angle Y or angle Z must be acute, because the sum of angles in a triangle is always 180 degrees. Without loss of generality, let's assume that angle Y is acute.
Now, we can apply the Law of Cosines, which states that for any triangle with sides a, b, and C and angles A, B, and C opposite those sides:

$c^2 = a^2 + b^2 - 2ab\, cos(C)$
In triangle XYZ, let's label the sides as follows:

$a = XY, \; b = XZ, \; c = YZ$
Then, we can rewrite the given equation as:

$XY^2 + XZ^2 = YZ^2$

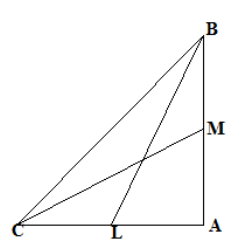
Using the Law of Cosines, we can write:

$$YZ^2 = XY^2 + XZ^2 - 2X \times XZ\, cos(Y)$$
Since angle Y is acute, $cos(Y)$ is positive. Therefore, we can rewrite the above equation as:

$$YZ^2 < XY^2 + XZ^2$$
This contradicts the given equation, so our assumption that angle X is not 90 degrees must be false. Therefore, angle X is 90 degrees, as required.

OR

38) BL and CM are medians of a triangle ABC right angled at A. Prove that $4\left(BL^2 + CM^2\right) = 5BC^2$ **[4]**



Explanation:

BL is median

$= AL = CL = \frac{1}{2}AC \rightarrow (1)$

CM is median

$= AM = NB = \frac{1}{2}AB \rightarrow (2)$

In $\triangle BAC$

$(BC)^2 = (AB)^2 + (AC)^2$

In $\triangle BAC$

$(BL)^2 = (AB)^2 + \left(\frac{AC}{2}\right)^2 \; 4BL^2 = 4 \cdot AB^2 + (AC)^2$

In $\triangle MAC$,

$(CM)^2 = (AM)^2 + (AC)^2$

$(CM)^2 = \left(\frac{AB}{2}\right)^2 + (AC)^2 \; 4CM^2 = (AB)^2 + (AC)^2$

$Now,\ (BC)^2 = (AB)^2 + (AC)^2 \rightarrow (1)$

$4BC^2 = 4(AB)^2 + (AC)^2 \rightarrow (2) \; 4M^2 = AB^2 + 4AC^2 \rightarrow (3)$

ADD (2) & (3)

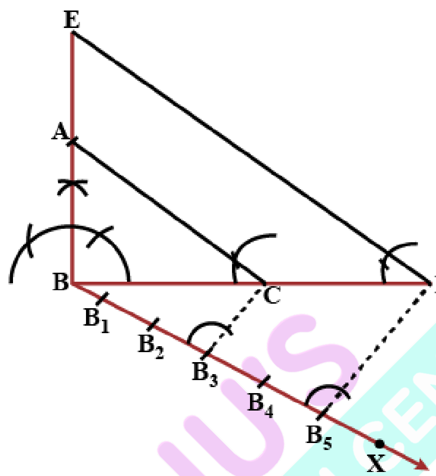
$4BC^2 + 4CM^2 = 5AB^2 + 5AC^2$

$4\left(BL^2 + CM^2\right) = 5$

39) Draw a right triangle in which the sides (other than hypotenuse) are of lengths $4\ cm$ and $3\ cm$. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle. Write steps of construction. **[4]**

Explanation:

Construct $\triangle ABC$, the given right angled triangle, as follows:
Step 1: Draw base BC of length $4\ cm$.
Step 2: Draw a right angle at point B.



Step 3: Draw an arc of radius 3 cm on the right angle drawn from B and label that point as A
Step 4: Join $A - C$.
$\triangle ABC$ is the given triangle.
To construct $\triangle EBD \sim \triangle ABC$:
Step 1: Draw a ray BX at an acute angle to line BC on the opposite side of A.
Step 2: Mark 5 equidistant points B_1, B_2, B_3, B_4, B_5 on ray BX.
Step 3: Join B_3 and C.
Step 4: Draw B_5D parallel to B_3C and label the intersection on the extension of BC as D.

Step 5: Draw DE parallel to BA and label the intersection with the extension of BA as E

$\triangle EBD$ *is the required triangle.*

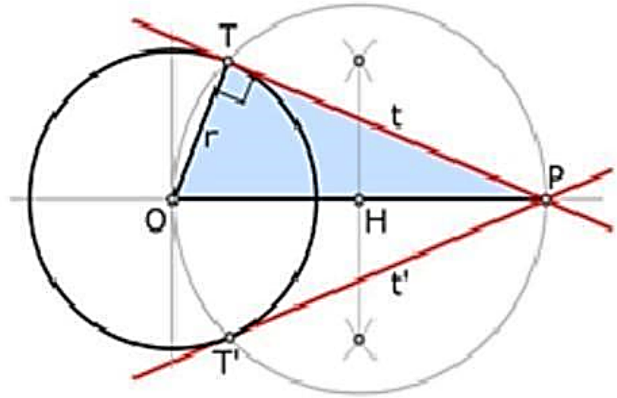
OR

39) Draw a circle of radius $4.5\ cm$. From a point $7.5\ cm$ away from its centre, construct the pair of tangents to the circle and measure their lengths. Write steps of construction. **[4]**

Explanation:

Let's say that we are given a circle with centre O and a point P outside it, and we have to construct two tangents from the point P to the circle without the help of a ruler. Follow the steps of construction as below:

1. Join OP and bisect it. To bisect OP, take a compass, and open it slightly more than half of the length of the line segment.



2. From point P, mark a minimal arc above and below the line segment. Repeat the similar step from point O keeping the opening of the compass as same as it was from point P. Two points will be created where the two arcs, produced from point O and point P, meet.
3. Join these two points with a line segment using a scale. This line segment bisects the OP. Let's consider H as the mid-point of PO.
4. Taking the point H as a centre and HO as a radius, draw a circle. Let it intersect the given circle at the points, T and T '.
5. Join PT and PT'. Then PT and PT' are the required two tangents and after measure the tangents PT and PT' using scale will be PT = PT' = 6cm