Gujarat Board - X Mathematics (Basic)
2022-23

Maximum Marks: **80**
Time allowed: **3 Hours**

Instructions:

1) Write in a clear legible handwriting.
2) This question paper has four Sections A, B, C & D and Question Numbers from 1 to 39.
3) All Sections are compulsory. Internal options are given.
4) The numbers to the right represent the marks of the question.
5) Draw neat diagrams wherever necessary.
6) New sections should be written in a new page. Write the answers in numerical order.
7) Calculator is not allowed.

Section A

- **Answer the following as directed. (1 to 16) (1 mark each)** **[16]**

State whether the following statements are true or false. (1 to 4)

1) HCF $(17, 23) = 1$ **[1]**

Answer: True
Explanation:
The prime factors of $17 = 17\times1$
The prime factors of $23 = 23\times1$

$\therefore HCF(17, 23) = 1$

Hence, the H.C.F. of 17,23 is 1.

Therefore, it is True.

2) If $p(x) = x^2 - 7x + 10$, then the number of zeroes are 3 . **[1]**

Answer: False

Explanation:

Given, $p(x) = x^2 - 7x + 10$

The zeroes of the polynomial are the values of x which satisfies the equation and makes the equation equal to zero as whole.
Let us split the middle term to find the factors of the equation $x^2 - 7x + 10$

$x^2 - 7x + 10 = x^2 - 2x - 5x + 10$
Taking out the common terms, we get

$= x(x - 2) - 5(x - 2)$
$= (x - 2)(x - 5)$
Put both the factors equal to zero

$x - 2 = 0 \; and \; x - 5 = 0$
$x = 2 \; and \; x = 5$
As, $(x - 2)$ and $(x - 5)$ are the factors of the given polynomial, so, Zeroes of the polynomial are 2 and 5.
Therefore, the given statement is False.

3) If $sinA = 1$, then $A = 90^{\circ}$. **[1]**

Answer: True
Explanation:
$\sin sin\, A \; = 1$

$\sin sin\, A = \sin sin\, 90^{\circ}$

$A = 90^{\circ}$

4) The values of $sinA$ and $cosA$ never be greater than 1. **[1]**

Answer: True

Explanation:

That statement is true. The sine (sin) and cosine (cos) functions are trigonometric functions that relate the ratios of the sides of a right-angled triangle to the angles of the triangle.

The values of sin(A) and cos(A) are always between -1 and 1, inclusive. This is because the hypotenuse of a right triangle is always the longest side, so the ratio of the opposite side to the hypotenuse (sin(A)) and the ratio of the adjacent side to the hypotenuse (cos(A)) must always be less than or equal to 1.

Therefore, it is not possible for the values of sin(A) or cos(A) to be greater than 1.

Select the most appropriate answer from the given alternatives: (5 to 10)

5) If p and q are positive integers where $p = ab^2$ and $q = a^2b$ where $a\&b$ are prime numbers then $LCM(p, q) =$ **[1]**

(A) ab

(B) a^2b^2

(C) a^3b^2

(D) a^3b^3

Answer: B

Explanation:

We can find the LCM of p and q by finding their prime factorizations and taking the maximum power of each prime factor.

The prime factorization of p is p = ab^2, which has prime factors of a and b.

The prime factorization of q is q = a^2b, which also has prime factors of a and b.

To find the LCM of p and q, we need to take the maximum power of each prime factor. The maximum power of a is a^2, since it appears as a factor in q. The maximum power of b is b^2, since it appears as a factor in p.

Therefore, LCM(p, q) = a^2b^2.

So, the correct answer is option (B) a^2b^2.

6) Graphically, the pair of linear equations $6x - 3y + 10 = 0, 2x - y + 9 = 0$ represents two lines which are

(A) intersecting at exactly one point
(B) intersecting at exactly two points
(C) coincident
(D) parallel **[1]**

Answer: D
Explanation:
To determine the nature of the lines represented by the pair of linear equations 6x-3y+10=0 and 2x-y+9=0, we can rearrange them into slope-intercept form, y = mx + c, where m is the slope of the line and c is the y-intercept.

For the first equation, we have:

6x - 3y + 10 = 0

-3y = -6x - 10

y = 2x + 10/3

So the slope of the first line is m = 2, and the y-intercept is c = 10/3.

For the second equation, we have:

2x - y + 9 = 0

y = 2x + 9

So the slope of the second line is also m = 2, and the y-intercept is c = 9.

We can see that the two lines have the same slope, but different y-intercepts. Therefore, the lines are parallel and will never intersect. The answer is (D) parallel.

7) If the roots of quadratic equation $ax^2 + bx + c = 0, a\neq 0$ are real and distinct, then **[1]**

(A) $b^2 - 4ac < 0$

(B) $b^2 - 4ac = 0$

(C) $b^2 - 4ac > 0$

(D) $b^2 - 4ac \neq 0$

Answer: C

Explanation:

Given equation : $ax^2 + bx + c = 0, a\neq 0$

It is given that the roots of the equation are real and distinct, i.e. Discriminant > 0

$\Rightarrow b^2 - 4ac > 0$

Hence, the correct option is C.

8) If $A(0, 6)$ and $B(0, -2)$, then the distance between $A \& B$ is **[1]**

(A) 6

(B) 8

(C) 4
(D) 2

Answer: B
Explanation:
Given point are, $A\left(x_1, y_1\right) = (0, 6)$ $B\left(x_2, y_2\right) = (0, -2)$

Use the distance formula to calculate the distance:

$D = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$= \sqrt{(0 - 0)^2 + (-2 - 6)^2}$

$= \sqrt{(0)^2 + (-8)^2}$

$= \sqrt{0 + 64}$

$= \sqrt{64}$

$= 8$

9) Probability of getting 80 marks out of 80 in maths paper is **[1]**
(A) $\frac{79}{80}$
(B) $\frac{1}{80}$
(C) $\frac{1}{81}$
(D) $\frac{79}{81}$

Answer: B
Explanation:
Formula for probability: $P = \frac{Favorable\ outcomes}{Total\ outcomes}$
We need to find the probability of getting 80 marks out of 80 marks in a exam.
Favorable outcome $= 1$

Total outcomes $= 80$
Substitute these values in the above formula.

$P = \frac{1}{80}$

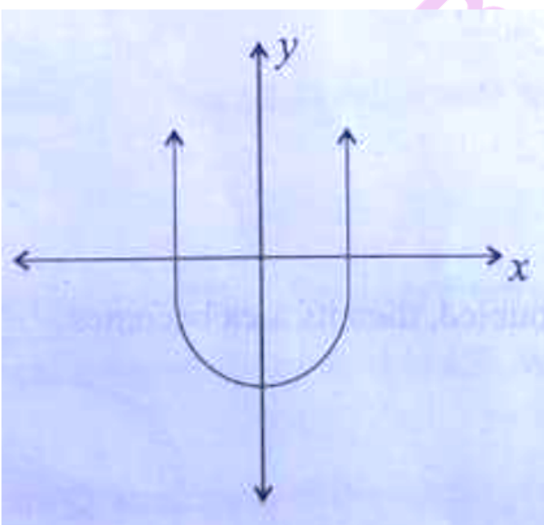
Therefore, the probability of getting 80 marks out of 80 marks in a exam is $\frac{1}{80}$ i.e. equivalent to Option B.

10) For the given figure, if y=p(x), then number of zeroes are **[1]**

A. 1
B. 2
C. 3
D. 0

Answer: B

Explanation:



Because it cuts on X-axis two times that's why the zero's is two.

11) The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its________

[Mean, Median, Mode] **[1]**

Answer: Median

Explanation:

The less than ogive and more than ogive when drawn on the same graph intersect at a point. From this point, if we draw a perpendicular on the x-axis, the point at which it cuts the x-axis gives us the median.
Thus, the abscissa of the point of intersection of less than type and of the more than type cumulative curves of a grouped data gives its median.

12) Probability of a sure event is______ $[0, 1, 2]$ **[1]**

Answer: 1

Explanation:

The probability of a sure event is 1 .
A sure event is an event, which always happens. For example, it's a sure event to obtain a number between 1 and 6 when rolling an ordinary die. The probability of a sure event has the value of 1. The probability of an impossible event has the value of 0 .

13) A tangent to a circle intersect it in______ point. [0,1,2] **[1]**

Answer: 1

Explanation:

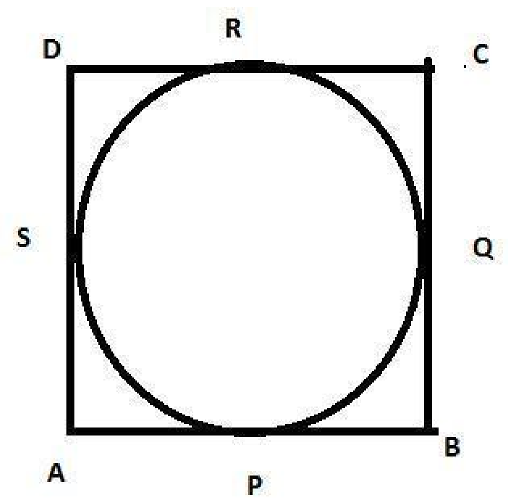
A tangent to a circle is a line that intersects the circle at only one point.

14) A circle which touches all the sides of a quadrilateral $ABCD$, if $AB = 7, BC = 3, CD = 4$, then $AD =$ _____ $[8, 7, 11]$ **[1]**

Answer: 8

Explanation:

In quadrilateral $ABCD$
$AB = 7$ units
$BC = 3$ units
$CD = 4$ units



Suppose circle touch the quadrilateral $ABCD$ and side AB at P, BC at Q, CD at R and AD at S.

We know that tangents drawn from external point to the circle are equal in length.
Therefore, $AP = AS, PB = BQ, CR = QC$ and $DR = DS$
Adding all then we get
$AP + PB + CR + DR = AS + BQ + QC + DS$
$AB + CD = AD + BC$
Substitute the values
$7 + 4 = AD + 3$
$AD = 7 + 4 - 3 = 8$
$AD = 8$ units

15) Length of minor arc is__________ $\left[\frac{\pi r\theta}{180}, \frac{\pi r^2\theta}{360}, \frac{\theta}{360}\right]$ **[1]**

Answer: $\frac{\pi r\theta}{180}$

Explanation:

Formula for length of minor arc $= \frac{\theta}{360^\circ}\times 2\pi r = \frac{\theta\pi r}{180^\circ}$

16) If the radius of a circle is doubled, then its area becomes_______ times.
$[2, 3, 4]$ **[1]**

Answer: 4

Explanation:

Circle is a two dimensional figure, in which all the points lie in a plane and are at the same distance from a fixed point, i.e., the center.
It is given that the radius of the circle is doubled.
Let the radius of the circle be r.

We know that, area of a circle $= \pi r^2$

When radius is doubled, the new area of circle $= \pi(2r)^2 = 4\pi r^2$
∴ Area of new circle $= 4$ (Area of the original circle)
∴ The area of new circle becomes four times the original circle.

SECTION-B

- **Solve the following: (17 to 26) (2 marks each) [20]**

17) Find the zeroes of the quadratic polynomial $x^2 + 2x - 8$. **[2]**

Answer: $(2, -4)$

Explanation:

Given quadratic polynomial is $x^2 + 2x - 8$

$x^2 + 4x - 2x - 8 = 0$

$x(x + 4) - 2(x + 4) = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$

$x = -4$

$x - 2 = 0$

$x = 2$

Hence, the zeroes of Polynomial are $(2, -4)$

OR

17) Find the quotient and remainder:-

$\left(3x^2 - x^3 - 3x + 5\right) \div \left(x - 1 - x^2\right)$ **[2]**

Answer: Quotient = x-2 and Remainder = 3

Explanation:

Quotient : x-2

Remainder: 3

$$
\begin{array}{r|l}
 & \quad x - 2 \\
\hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & + \quad - \quad + \\
\hline
 & \quad 2x^2 - 2x + 5 \\
 & \quad 2x^2 - 2x + 2 \\
 & \quad - \quad + \quad - \\
\hline
 & \qquad 3
\end{array}
$$

18) Find a quadratic polynomial whose sum and product of its zeroes given respectively $\left(\frac{1}{4}, -1\right)$ **[2]**

Answer: $4x^2 - x - 4 = 0$

Explanation:

General formula:

$P(x) = x^2 - (sum\ of\ roots)x + (product\ of\ roots)$

$= x^2 - \left(\frac{1}{4}\right)x - 1 = 0$

$\Rightarrow 4x^2 - x - 4 = 0$

19) How many 3 -digit numbers are divisible by 3 ?

[2]

Answer: 300

Explanation:

Step 1: Condition for divisibility by 3: A number is said to be divisible by 3 when sum of its digits is divisible by 3 . For example, let us consider a random number 423 .

The sum of the digits $= 4 + 2 + 3 = 9$.

We know that $3 \times 3 = 9$, hence 9 is divisible by 3 .

Therefore 423 is divisible by 3 .

Step 2: Estimate the count of natural numbers

Let us enlist such 3 - digit natural numbers

$102, 105, 108, 111, 114 ..... 987, 990, 993, 996, 999$

The first three-digit number which is divisible by 3 is 102 .

The last three-digit number which is divisible by 3 is 999

The number of three-digit numbers divisible by 3

$$= \frac{999-102}{3} + 1 \quad = \frac{897}{3} + 1 \quad = 299 + 1 \quad = 300$$

There are a total of 300 digits in this range.
Hence, the number of 3 -digit natural numbers that are completely divisible by 3 is 300.

20) Evaluate:-

$$2tan^2 45^\circ + cos^2 30^\circ - sin^2 60^\circ$$

[2]

Answer: 2
Explanation:

$\tan tan\, 45^\circ = 1$,

$\cos cos\, 30^\circ = \frac{\sqrt{3}}{2}$

$sin60^\circ = \sqrt{3}/2$

$2tan^2 45 + cos^2 30 - sin^2 60$

$= 2(tan45)^2 + (cos30)^2 - (sin60)^2$

$= 2(1) + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$= 2(1) = 2$

OR

20) If $sin3\,A = cos\left(A - 26^\circ\right)$, where $3\,A$ is an acute angle, find the value of A

[2]

Answer: 29°
Explanation:

Given $sin3A = cos(A - 26)$

$\Rightarrow cos(90 - 3A) = cos(A - 26)$

$\Rightarrow (90 - 3A) = (A - 26)$

$\Rightarrow 4A = 116^{\circ}$

$\therefore A = 29^{\circ}$

21) Find the values of y for which the distance between the points $P\ (2, -\ 3)$ and $Q(10, y)$ is 10 units
[2]

Answer: 3 or -9

Explanation:

We know that the distance between the two points is given by the

Distance Formula $\sqrt{\left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2\right]}$
By substituting the values of points $P(2, -\ 3)$ and $Q(10, y)$ in the distance formula, we get

$PQ = \sqrt{(2 - 10)^2 + (-\ 3 - y)^2} = 10$

$PQ = \sqrt{(-\ 8)^2 + (3 + y)^2} = 10$
Squaring on both sides, we get

$64 + (y + 3)^2 = 100$

$(y + 3)^2 = 36$

$y + 3 = \sqrt{36}$

$y + 3 = \pm 6$

$y + 3 = 6$

$or\ y + 3 =-\ 6$

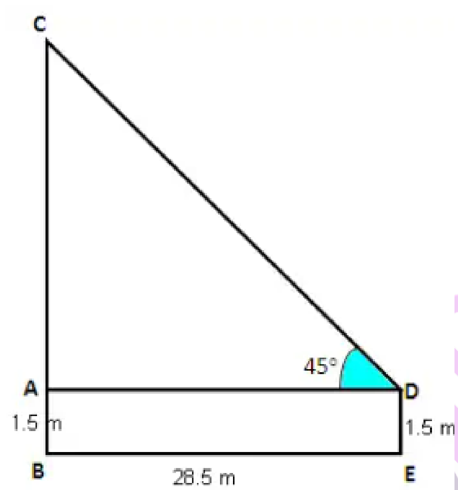
Therefore, $y = 3$ or -9 are the possible values for y.

22) An observer $1.5\ m$ tall is $28.5\ m$ away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?
[2]

Answer: 30m

Explanation:

Consider the diagram shown below.



Height of the observer, $DE = 1.5\ m$
Height of the chimney, $BC = AC + 1.5\ m$
Distance of the observer from the foot of the chimney, $BE = 28.5\ m$
Angle of elevation from the eyes of the observer to the top of the chimney $= 45^{\circ}$
$AD = BE = 28.5\ m$

Therefore,

$tan45^{\circ} = \frac{AC}{AD}\ \ \frac{AC}{28.5} = 1\ \ AC = 28.5\ m$

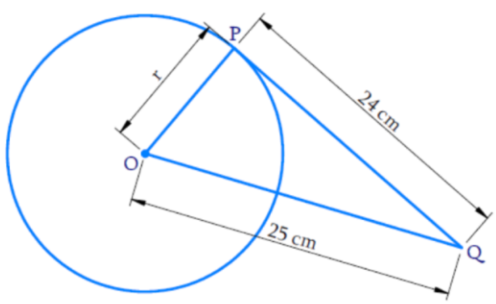
Therefore,
Height of the chimney, $AB = 28.5 + 1.5 = 30\ m$
Hence, the height of the chimney is $30\ m$.

23) From a point Q, the length of the tangent to a circle is $24\ cm$ and the distance of Q from the centre is $25\ cm$, then find the radius of the circle.

[2]

Answer: 7 cm

Explanation:



A tangent at any point of a circle is perpendicular to the radius at the point of contact.
Therefore, OPQ is a right-angled triangle.
By Pythagoras theorem,

$OQ^2 = OP^2 + PQ^2$

$25^2 = r^2 + 24^2$

$r^2 = 25^2 - 24^2$

$r^2 = 625 - 576$

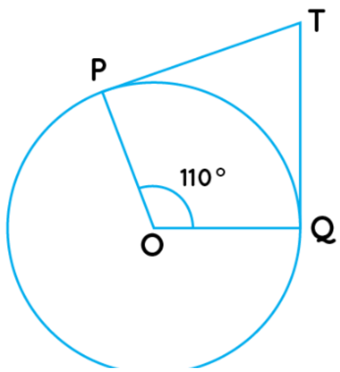
$r^2 = 49$

$r = \pm 7$

Radius cannot be a negative value, hence, $r = 7\ cm$.

OR

23) In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^{\circ}$, then find $\angle PTQ$.



[2]

Answer: 70°

Explanation:

In the above figure, OPTQ is a quadrilateral and $\angle P$ and $\angle Q$ are 90°
The sum of the interior angles of a quadrilateral is 360°.
Therefore, in OPTQ,

$\angle Q + \angle P + \angle POQ + \angle PTQ = 360^{\circ}$

$90^{\circ} + 90^{\circ} + 110^{\circ} + \angle PTQ = 360^{\circ}$

$290^{\circ} + \angle PTQ = 360^{\circ}$

$\angle PTQ = 360^{\circ} - 290^{\circ}$

$\angle PTQ = 70^{\circ}$

24) The edge of a cube is $5\ cm$, then find the total surface area of the cube.

[2]

Answer: $150\ cm^{2}$

Explanation:

Surface area of a cube is $6a^2$ where a is an edge of the cube.
The surface area of the cube with edge $5\ cm$ is given by:

$A = 6a^2$,

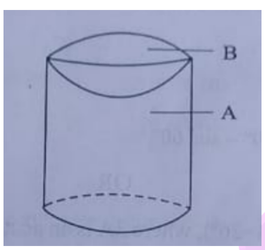
$= 6\times(5)^2$

$= 6\times25$

$= 150\ cm^2$

OR

24) A solid shown in the figure is made up of a cylinder (A) with a hemispherical depression (B). Then write the formula for finding the total surface area of a solid. **[2]**



Answer: πr(2h + 3r)

Explanation:

i) Area of the cylinder (except the top circle) = 2πrh + πr^2

ii) Area of the hemispherical depression = 2πr^2

Total surface area of the solid = 2πrh + πr^2+ 2πr^2

= 2πrh + 3πr^2

= πr(2h + 3r)

25) If $\sum f_i d_1 =- 26, a = 30, \sum f_i = 13$, find $\overline{x}$.

[2]

Answer: $\frac{2}{9}$, $\frac{5}{9}$

Explanation:

Given, $f_i d_1 =- 26, a = 30, \sum f_i = 13$

We know that, Mean $(\overline{x}) = a + \frac{\Sigma fd}{\Sigma f}$

$= 30 + \frac{-26}{13}$

$= 30 - 2 = 28$

Hence, the value of $\overline{x}$ is 28.

26) A box contains 3 blue, 2 white and 4 red marbles. Alok drawn a marble at random from the box, what is the probability that it will be

i) white

ii) not red

[2]

Answer: $\frac{2}{9}$, $\frac{5}{9}$

Explanation:

i) Total number of marbles $= 3 + 2 + 4 = 9$

Number of marbles which are white $= 2$

$P(\, marble\ taken\ out\ is\ white\,) = \frac{Number\ of\ marbles\ which\ are\ white}{Total\ number\ of\ marbles}$

$= \frac{2}{9}$

ii) Total number of marbles $= 3 + 2 + 4 = 9$

Number of marbles which are not red(blue and white) $= 3 + 2 = 5$

$P(\text{not red}) = \frac{\text{Number of marbles which are not red}}{\text{Total number of marbles}}$

$= \frac{5}{9}$

SECTION-C

Answer the following as asked with calculations. (27 to 34) (3 marks each) [24]

27) Solve the linear pair of equations by elimination method:-

$2x + 3y = 46$ & $3x + 5y = 74$
[3]

Answer: $x = 8$ and $y = 10$

Explanation:

$$2x + 3y = 46 \ldots (i)$$

And $3x + 5y = 74 \ldots$ (ii)

On multiplying eq. (i) by 3 and Eq. (ii) by 2 to make the coefficients of x equal, we get the equations as

$$6x + 9y = 138 \ldots\ldots (iii) \quad 6x + 10y = 148 \ldots.. (iv)$$

On subtracting Eq.(iii) from Eq.(iv) we get

$$6x + 10y - 6x - 9y = 148 - 138 \Rightarrow y = 10$$

On putting $y = 10$ in Eq. (ii) we get

$3x + 5y = 74$

$\Rightarrow 3x + 5(10) = 74$

$\Rightarrow 3x + 50 = 74$

$\Rightarrow 3x = 74 - 50$

$\Rightarrow 3x = 24$

$\Rightarrow x = 8$

Hence $x = 8$ and $y = 10$ which is the required solutions

27) The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$, find the numbers. **[3]**

Answer: 10 and 5

Explanation:

Let x and y be two natural numbers such that $x > y$
then Difference of the two natural numbers $= x - y = 5$
Difference of their reciprocals $= \frac{1}{y} - \frac{1}{x} = \frac{1}{10}$ since $\frac{1}{y} > \frac{1}{x}$

$= \frac{x-y}{xy} = \frac{1}{10}$

$= \frac{5}{xy} = \frac{1}{10}$

$= xy = \frac{5}{\frac{1}{10}}$

$= 50 \quad = x = \frac{50}{y}$

Using $x - y = 5 = \frac{50}{y} - y = 5$

$= 50 - y^2 - 5y = 0$

$= y^2 + 5y - 50 = 0$

$= y^2 + 10y - 5y - 50 = 0$

$= y(y + 10) - 5(y + 10) = 0$

$= (y + 10)(y - 5) = 0$

$= y = -10, 5$

$\therefore y = 5$ since $y = -10$ is not a natural number.

$= x - y = 5$

$= x - 5 = 5\ or\ x = 10$

$\therefore$ the required natural numbers are $x = 10, y = 5$

28) Find the nature of the roots of the given quadratic equation. If the real root exist find them.

$2x^2 - 6x + 3 = 0$ **[3]**

Answer: $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$

Explanation:

We have $2x^2 - 6x + 3 = 0$
Here, $a = 2, b = -6, c = 3$
Therefore, $D = b^2 - 4ac = (-6)^2 - 4\times2\times3 = 36 - 24 = 12 > 0$
Hence, given quadratic equation has real and distinct roots
Thus, $x = \frac{-b\pm\sqrt{D}}{2a} = \frac{-(-6)\pm\sqrt{12}}{2\times2} = \frac{3\pm\sqrt{3}}{2}$
Hence roots of the given equation are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$

OR

28) A train travels $360\ km$ at a uniform speed. If the speed had been $5\ km/h$ more, it would have taken 1 hour less for the same journey. Find the speed of the train. **[3]**

Answer: $40\ km/hr$

Explanation:

Given distance $= 360\ km$
Let the speed of the train be xkm/hr.
Speed when increased by $5\ km/hr = (x + 5)km/hr$

$\frac{360}{x} - \frac{360}{(x+5)} = 1$

$\frac{[360x+1800-360x]}{x(x+5)} = 1$

$x^2 + 5x - 1800 = 0$

$x^2 + 45x - 40x - 1800 = 0$

$x(x + 45) - 40(x + 45) = 0$

$(x - 40)(x + 45) = 0$

$x = 40, - 45$

The speed of the train is $40\ km/hr$.

29) Find the sum of the first 40 positive integers divisible by 6.

[3]

Answer: 4920

Explanation:

The first 40 positive integers that are divisible by 6 are $6, 12, 18, 24$...
$a = 6$ and $d = 6$.
We need to find S_{40}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{40} = \frac{40}{2}[2(6) + (40 - 1)6]$

$= 20[12 + (39)6]$

$= 20(12 + 234)$

$= 20\times246$

$= 4920$

30) Find the 20^{th} term from the last term of the A.P. : $3, 8, 13, \ldots\ldots\ldots, 253.$

[3]

Answer: 158

Explanation:

For the given A.P.,
First term(a) $= 3$
Last term $\left(t_n\right) = 253$
Common Difference $(d) = 8 - 3 = 5$

$\because t_n = a + (n - 1)d$

$\Rightarrow 253 = 3 + (n - 1)\times 5$

$\Rightarrow 253 - 3 = (n - 1)\times 5$

$\Rightarrow 250\div 5 = n - 1$

$\Rightarrow 50 + 1 = n$

$\therefore n = 51$

In 51 terms, 20^{th} term from the last term will be $51 - 20 + 1 = 32^{th}$

$$\therefore t_{32} = a + (32 - 1)d = 3 + 31\times 5 = 3 + 155 = 158$$

31) Find the value of " k " for which the points are collinear.
$(8, 1), (k, -4), (2, -5)$ **[3]**

Answer: k = 3

Explanation:

Since the given points are collinear, they do not form a triangle, which means area of the triangle is Zero.

Area of a triangle with vertices (x_1, y_1); (x_2, y_2) and (x_3, y_3) is

$$\left|\frac{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)}{2}\right|$$

Hence, substituting the points $(x_1, y_1) = (8, 1)$; $(x_2, y_2) = (k, -4)$ and $(x_3, y_3) = (2, -5)$
the area formula, we get

$$\left|\frac{8(-4+5)+k(-5-1)+2(1+4)}{2}\right| = 0$$

$=> 8 - 6k + 10 = 0$

$=> 6k = 18$

$=> k = 3$

OR

31) In what ratio does the point $(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$? **[3]**

Answer: 2 : 7

Explanation:

Step 1: Find (x_1, y_1) and (x_2, y_2) and then applying the section formula.
Given: $(x_1, y_1) = (-6, 10)$

$(x_2, y_2) = (3, -8)$ $(x, y) = (-4, 6)$

Using the section formula, we get,

$A(-6, 10), B(3, -8), C(-4, 6)$

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

$\Rightarrow -4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$

$\Rightarrow -4(m_1 + m_2) = 3m_1 - 6m_2$

$\Rightarrow -4m_1 - 4m_2 = 3m_1 - 6m_2$

$\Rightarrow -7m_1 = -2m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{2}{7}$

$\Rightarrow$ *Ratio is* $2:7$.

Hence, the ratio is 2: 7.

32) The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in ₹)	$100 - 150$	$150 - 200$	$200 - 250$	$250 - 300$	$300 - 350$
Number of households	4	5	12	2	2

[3]

Answer: 211

Explanation:

Daily Expense	Households (fi)	Mid Point (xi)	fi xi

$100 - 150$	4	125	500
$150 - 200$	5	175	875
$200 - 250$	12	225	2700
$250 - 300$	2	275	550
$300 - 350$	2	325	650
	25		5275

$$Mean = \frac{\sum fixi}{\sum fi} = \frac{5275}{25} = 211$$

33) A die is thrown once. Find the probability of getting
i) a prime number.
ii) a number lying between 2 and 6.
iii) an odd number. **[3]**

Explanation:

Sample space $= \{1, 2, 3, 4, 5, 6\}$
i) $P($ Prime number $) = \frac{Number\ of\ prime\ numbers}{Sample\ space}$

P (Prime number) $= \frac{3}{6} = \frac{1}{2}$
ii) P (Number between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

iii) P (odd number) = $\frac{3}{6} = \frac{1}{2}$

OR

34) One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting
i) a king of red colour.
ii) Not a spade.
iii) The queen of hearts **[3]**

Explanation:

i) King of red colour can be hearts King or a diamond king. So, the possibility is 2.

$P(Red\ king) = \frac{2}{52} = \frac{1}{26}$

ii) There are 13 spades in total.
So, $P(\text{Spade}) = \frac{13}{52} = \frac{1}{4}$

$P(\text{not a spade}) = 1 - P(\text{Spade})$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

iii) There is only one queen of heart.
Hence, $P(Queen\text{ of heart}) = \frac{1}{52}$

SECTION-D

- **Solve the following. (35 to 39)**

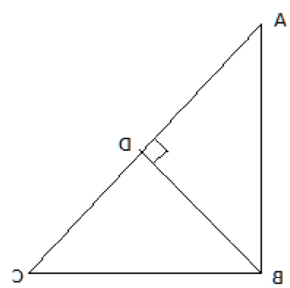
  (4 marks each) **[20]**

35) State and prove Pythagoras theorem. **[4]**

Explanation:

Statement: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: ABC is a triangle in which $\angle ABC = 90^{\circ}$
Construction: Draw BD $\perp AC$.
Proof:
In $\triangle ADB$ and $\triangle ABC$
$\angle A = \angle A$ [[Commonangle]



$\angle ADB = \angle ABC$ [Each 90°]
$\triangle ADB \sim \triangle ABC$ [$A - A$ Criteria]
So, $\frac{AD}{AB} = \frac{AB}{AC}$
Now, $AB^2 = AD \times AC$
Similarly,

$BC^2 = CD \times AC$

Adding equations (1) and (2) we get,

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$= AC(AD + CD)$

$= AC \times AC$

$\therefore AB^2 + BC^2 = AC^2$ [*hence proved*]

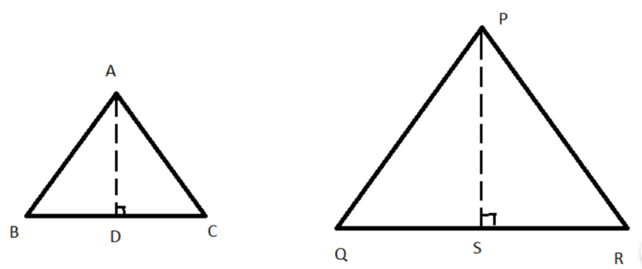
OR

35) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. **[4]**

Explanation:

Let the two triangles be:
$\triangle ABC$ and $\triangle PQR$.
Area of $\triangle ABC = \frac{1}{2} \times BC \times AM$.
Area of $\triangle PQR = \frac{1}{2} \times QR \times PN$.
Dividing (1) by (2)

$\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$



$\frac{ar(ABC)}{ar(PQR)} = \frac{BC \times AM}{QR \times PN}$

In $\triangle ABM$ and $\triangle PQN$
$\angle B = \angle Q$ (Angles of similar triangles)
$\angle M = \angle N$(Both 90°)
Therefore, $\triangle ABM \sim \triangle PQN$
So, $\frac{AB}{AM} = \frac{PQ}{PN}$

From 1 and 2

$\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AMN}{PN}$

$= \frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ}$

.

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$............$(\triangle ABC \sim \triangle PQR)$

Putting in (3) $\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ} = \left(\frac{AB}{PQ}\right)^2$

$= \frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

36) If the median of the distribution given below is 28.5, find the values of P and Q. **[4]**

Weight in kg.	$0 - 10$	$10 - 20$	$20 - 30$	$30 - 40$	$40 - 50$	$50 - 60$	Total
Number of students	5	P	20	15	Q	5	60

Answer: 8 and 7

Explanation:

Here, it is given that Median $= 28.5$ and $n = \sum f_i = 60$

Cumulative frequency table for the following data is given.

Here $n = 60 \Rightarrow \frac{n}{2} = 30$

Class - interval	Frequency	Cumulative frequency
$0-10$	5	5
$10-20$	x	$5+x$
$20-30$	20	$25+x$
$30-40$	15	$40+x$
$40-50$	y	$40+x+y$
$50-60$	5	$45+x+y$
Total	$n=60$	

Since, median is 28.5 , median class is $20-30$

Hence, $1=20, h=10, f=20$, c.f. $=5+P$

Therefore, Median $=1+\left(\frac{\frac{n}{2}-cf}{f}\right)h$

$28.5=20+\left(\frac{30-5-P}{20}\right)10$

$\Rightarrow 28.5=20+\frac{25-P}{2}$

$\Rightarrow 8.5\times 2=25-P$

$\Rightarrow P=8$

Also, $45+P+Q=60$

$\Rightarrow Q=60-45-P=15-8=7$

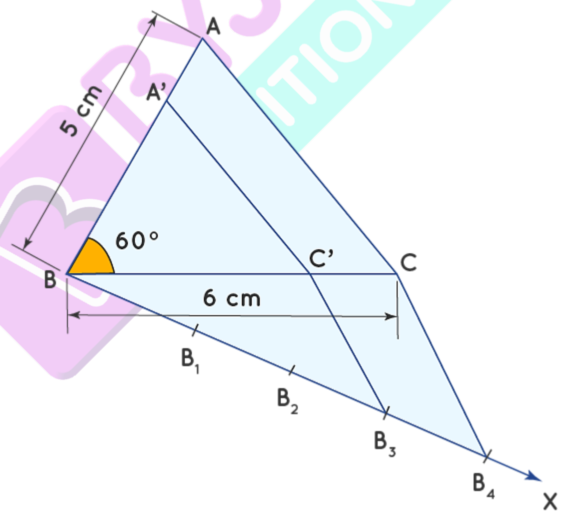
Hence, $P=8, Q=7$

37) Draw a triangle ABC with side $BC = 6\ cm, AB = 5\ cm$ and $\angle ABC = 60^{\circ}$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. **[4]**

Explanation:

Draw the triangle with the given conditions.

- Then draw another line that makes an acute angle with the baseline Divide the line into $m + n$ parts where m and n are the ratios given.
- Two triangles are said to be similar if their corresponding angles are equal, are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- The basic proportionality theorem states that "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".



Steps of constructions:

- Draw a line $BC = 6\ cm$.
- At B, make $\angle C = 60^{\circ}$ and cut an arc at A on the same line so that $BA = 5\ cm$. Join AC, $\triangle ABC$ is obtained.

- Draw the ray BX such that $\angle CBX$ is acute.
- Mark 4 (since $4 >$ in $3/4$) points B_1, B_2, B_3, B_4 on BX such that $B_1 = B_1B_2 = B_2B_3 = B_3B_4$
- Join B_4 to C and draw $B_3C^{'}$ parallel to B_4C to intersect BC at $C^{'}$.
- Draw $C^{'}A^{'}$ parallel to CA to intersect BA at $A^{'}$.

  Now, $\triangle A^{'}BC^{'}$ is the required triangle similar to $\triangle ABC$ where $BA^{'}/BA = B^{'}/BC = C^{'}A^{'}/CA = 3/4$

Proof:

In $\triangle BB_4C, B_3C^{'} \parallel B_4C$

Hence by Basic proportionality theorem,

$B_3B_4/BB_3 = C^{'}C/BC^{'} = 1/3$

$C^{'}C/BC^{'} + 1 = 1/3 + 1$

$\left(C^{'}C + BC^{'}\right)/BC^{'} = 4/3$

$BC/BC^{'} = 4/3\ or\ BC^{'}/BC = 3/4$

Consider $\triangle BA^{'}C^{'}$ and $\triangle BAC$

$\angle A^{'}B^{'} = \angle ABC = 60^{\circ}$

$\angle BCA^{'} = \angle BCA$ (Corresponding angles $\because C^{'}A^{'} \| ICA$)

$\angle BA^{'}C^{'} = \angle BAC$ (Corresponding angles)

By AAA axiom, $\triangle BA^{'}C^{'} \sim \triangle BAC$

Therefore, corresponding sides are proportional,

$BC^{'}/BC = BA^{'}/BA = C^{'}A^{'}/CA = 3/4$

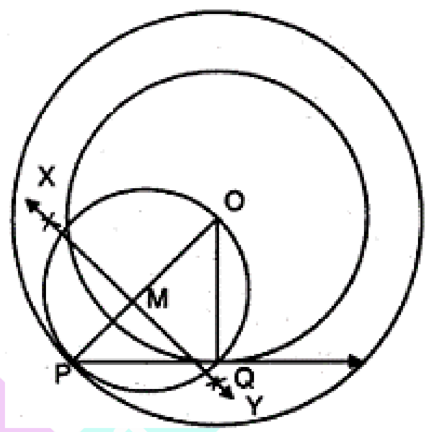
OR

37) Construct a tangent to a circle of radius $4\ cm$ from a point on the concentric circle of radius $6\ cm$ and measure its length.

[4]

Answer:

Explanation:



Steps of construction:

1 Draw two concentric circles with centre ' O ' and radii $4\ cm$ and $6\ cm$.

2 Take a point ' P ' on larger circle and join O, P.

3 Draw the perpendicular bisector of OP which intersects it at M.

4 Taking M as centre and PM or MO as radius draw a circle which intersects smaller circle at Q.

5 Join PQ, which is a tangent to the smaller circle.

38) A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	$1-3$	$3-5$	$5-7$	$7-9$	$9-11$
Number of families	7	8	2	2	1

Find the mode of this data.

[4]

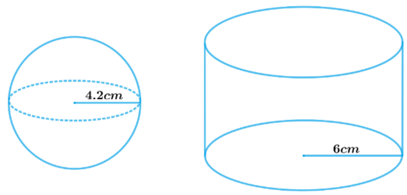
Explanation:

⇒ Here, modal class $= 3-5$
⇒$l = 3, f_0 = 7, f_1 = 8, f_2 = 2$ and $h = 2$
⇒ Mode $= 1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$
$= 3 + \frac{8-7}{2\times8-7-2} \times 2 = 3 + \frac{1}{7} \times 2 = 3 + 0.286 \therefore$
Mode $= 3.286$
∴ Given mode value in question is correct

39) A metallic sphere of radius $4.2\ cm$ is melted and recast into the shape of cylinder of radius $6\ cm$. Find the height of the cylinder. **[4]**

Explanation:

Given:
Radius $\left(r_1\right)$ of hemisphere $= 4.2\ cm$
Radius $\left(r_2\right)$ of cylinder $= 6\ cm$
Height (h) = ?
The object fromed by recasting the hemisphere will be same in volume.



So, Volume of sphere = Volume of cylinder

$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\Rightarrow \frac{4}{3}\pi \times (4.2)^3 = \pi (6)^2 h$

$\Rightarrow \frac{4}{3} \times \frac{4.2 \times 4.2 \times 4.2}{36} = h$

$h = (1.4)^3 = 2.74\ cm$

Therefore, the height of cylinder so formed will be $2.74\ cm$.

OR

39) A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in figure. If the height of the cylinder is $10\ cm$ and its base is of radius $3.5\ cm$, find the total surface area.

[4]

Explanation:

We have
Height of cylinder = $10\ cm$
radius of base = $3.5\ cm$
Total surface area of the article = Curved surface area of the cylinder +2 surface area of a hemisphere

$= 2\pi rh + 2\left(2\pi r^2\right)$

$= 2\pi r(h + 2r)$

$= 2\times\frac{22}{7}\times3.5(10 + 2\times3.5)cm^2$

$= 374\ cm^2$