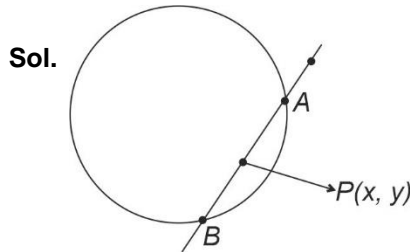# MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let a circle $x^2 + y^2 = 16$ and line passing through (1, 2) cuts the curve at A and B then the locus of the mid-point of AB is

   (1) $x^2 + y^2 + x + y = 0$ (2) $x^2 + y^2 - x + 2y = 0$

   (3) $x^2 + y^2 - x - 2y = 0$ (4) $x^2 + y^2 + x + 2y = 0$

Answer (3)

Sol.

Let $P(x_1 y_1)$ be the mid-point of AB

Then $T = S_1$

$x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16$

$\Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$ …(i)

$\because$ (i) passes through (1, 2)

$\therefore x_1 + 2y_1 = x_1^2 + y_1^2$

$\therefore$ required locus

$x^2 + y^2 - x - 2y = 0$

2. Consider $f(x) = \sec^{-1}\left(\dfrac{2x}{5x+3}\right)$, domain of $f(x)$ is

   $[\alpha, \beta) \cup (\gamma, \delta]$, then the value of $|3\alpha + 10\beta + 5\gamma + 21\delta|$ is

   (1) 22   (2) 23

   (3) 21   (4) 19

Answer (3)

Sol. $\dfrac{2x}{5x+3} \geq 1$ OR $\dfrac{2x}{5x+3} \leq -1$

$\Rightarrow \dfrac{2x}{5x+3} - 1 \geq 0$

$\Rightarrow \dfrac{-3x-3}{5x+3} \geq 0$

$\Rightarrow \dfrac{x+1}{5x+3} \leq 0 \qquad \Rightarrow x \in \left[-1, \dfrac{-3}{5}\right)$

$\dfrac{2x}{5x+3} + 1 \leq 0$

$\Rightarrow \dfrac{7x+3}{5x+3} \leq 0 \qquad \Rightarrow x \in \left(\dfrac{-3}{5}, \dfrac{-3}{7}\right]$

$\therefore x \in \left[-1, \dfrac{-3}{5}\right) \cup \left(\dfrac{-3}{5}, \dfrac{-3}{7}\right]$

$\alpha = -1, \beta = \dfrac{-3}{5}, \gamma = \dfrac{-3}{5}, \delta = \dfrac{-3}{7}$

$|3\alpha + 10\beta + 5\gamma + 21\delta|$

= $|-3 - 6 - 3 - 9| = 21$

3. 8 persons has to travel from A to B in 3 allotted cars. If a car can carry maximum 3 persons. Then find the number of ways they can travel.

   (1) 1880   (2) 1800

   (3) 1680   (4) 1600

Answer (3)

Sol. $C_1 \quad C_2 \quad C_3 \rightarrow$
   $\quad 3 \quad\; 3 \quad\; 2$

Total $\dfrac{8!}{3!\,3!\,2!\,2!}$ groups

So, they can travel in

$\dfrac{8!}{3!\,3!\,2!\,2!} \times 31$ ways

= 1680

4. If $\dfrac{z+i}{4z+2i}$ is purely real and $z = x + iy$ ($x, y \in R$) then one of the possibility is

   (1) $x \neq 0, y \neq -1$ (2) $x \neq 0, y = -1$

   (3) $x = 0, 2y \neq -1$ (4) $x = 1, 2y \neq -1$

Answer (3)

Sol. $Im\left(\dfrac{x+i(y+1)}{4x+2i(2y+1)} \cdot \dfrac{4x-2i(2y+1)}{4x-2i(2y+1)}\right) = 0$

$\Rightarrow \dfrac{4x(y+1)-2x(2y+1)}{16x^2+4(2y+1)^2} = 0$

$\Rightarrow \dfrac{2x}{16x^2+4(2y+1)^2} = 0$

$\Rightarrow x = 0$

But at $x = 0, 2y \neq -1$

5. If $\int\left(\left(\dfrac{x}{e}\right)^{2x}+\left(\dfrac{e}{x}\right)^{2x}\right)\ln x\, dx = \alpha\left(\dfrac{x}{e}\right)^{2x}+\beta\left(\dfrac{e}{x}\right)^{2x}+c$

   where c is constant of integration, then

   (1) $\alpha+\beta = 0$ (2) $\alpha+\beta = 1$

   (3) $\alpha\beta = \dfrac{1}{2}$ (4) $\alpha\beta = \dfrac{1}{4}$

Answer (1)

Sol. Let $\left(\dfrac{x}{e}\right)^{2x} = t$

$2x(\ln x - 1) = \ln t$

$\left[2(\ln x - 1) + 2x\left(\dfrac{1}{x}\right)\right]dx = \dfrac{1}{t}\, dt$

$\ln x = \dfrac{1}{2t}\, dt$

$I = \int\left(t + \dfrac{1}{t}\right)\times\dfrac{1}{2t}\, dt = \dfrac{1}{2}\int\left(1 + \dfrac{1}{t^2}\right)dt$

$= \dfrac{1}{2}\left(t - \dfrac{1}{t}\right) + c$

$= \dfrac{1}{2}\left(\left(\dfrac{x}{e}\right)^{2x} - \left(\dfrac{e}{x}\right)^{2x}\right) + c$

$\Rightarrow \alpha = \dfrac{1}{2}, \beta = \dfrac{-1}{2}$

6. If a dice is thrown n-times and probability of getting 7 times odd is equal to 9 times even. Then P(2 even) is $\dfrac{K}{2^{15}}$ then K is

   (1) 58   (2) 60

   (3) 48   (4) 65

Answer (2)

Sol. P(getting odd 7 times) = P(getting even 9 times)

$^nC_7\left(\dfrac{1}{2}\right)^7\left(\dfrac{1}{2}\right)^{n-7} = {}^nC_9\left(\dfrac{1}{2}\right)^9\left(\dfrac{1}{2}\right)^{n-9}$

$^nC_7 = {}^nC_9$

$n = 9 + 7 = 16$

P(2 times even) $= {}^{16}C_2\left(\dfrac{1}{2}\right)^{14}\left(\dfrac{1}{2}\right)^2$

$= \dfrac{^{16}C_2}{2^{16}} = \dfrac{16!}{2!\times 14!}\times\dfrac{1}{2^{16}}$

$= \dfrac{15\times 16}{2\times 2^{16}}$

$= \dfrac{15\times 4}{2^{15}} = \dfrac{K}{2^{15}}$

$\Rightarrow K = 60$

7. If $\displaystyle\int_0^{t^2}\left(f(x)+x^2\right)dx = \dfrac{4}{3}t^3$, then $f(x)$ is

   (1) $x^2 - 2\sqrt{x}$ (2) $x^2 + 2\sqrt{x}$

   (3) x^2 (4) $-x^2 + 2\sqrt{x}$

Answer (4)

Sol. $\left(f(t^2) + t^4\right)2t = 4t^2$

$f(t^2) + t^4 = 2t$

$f(x^2) = -x^4 + 2x$

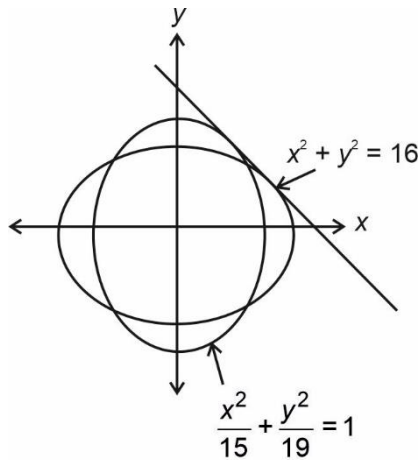Let $x^2 = u$

$f(u) = -u^2 + 2\sqrt{u}$

$f(x) = -x^2 + 2\sqrt{x}$

8. A tangent drawn to an ellipse $19x^2 + 15y^2 = 285$ is also a tangent to a circle. This circle is concentric with the conic and its radius is 4 units. Then the angle made by the tangent with minor axis of ellipse is

   (1) $\dfrac{\pi}{3}$ (2) $\dfrac{\pi}{2}$

   (3) $\dfrac{\pi}{6}$ (4) $\dfrac{\pi}{4}$

Answer (1)

Sol.

$$19x^2 + 15y^2 = 285$$

$$\Rightarrow \quad \frac{x^2}{15} + \frac{y^2}{19} = 1$$

Let equation of tangent of ellipse be

$$y = mx \pm \sqrt{15m^2 + 19} \qquad \dots(i)$$

If it is tangent to $x^2 + y^2 = 16$ then

$$\left| \frac{\sqrt{15m^2 + 19}}{\sqrt{m^2 + 1}} \right| = 4$$

$$15m^2 + 19 = 16m^2 + 16$$

$$m^2 = 3$$

$$\boxed{m = \pm\sqrt{3}}$$

∴ angle made by the tangent by minor axis (x- axis) will be $= \dfrac{\pi}{3}$

9. $y = f(x)$ is a quadratic function passing through $(-1, 0)$ and tangent to it at $(1, 1)$ is $y = x$. Find x intercept by normal at point $(\alpha, \alpha + 1)$, $(\alpha > 0)$

(1) 11   (2) −11

(3) 5   (4) −5

Answer (1)

Sol. Let $f(x) = ax^2 + bx + (b - a)$

$f'(1) = 2a + b = 1$ and $f(1) = 1 = 2b$

$$b = \frac{1}{2}, \quad a = \frac{1}{4}$$

$$\frac{-dx}{dy} = \frac{-1}{2ax + b} = \frac{-2}{x + 1}$$

and $\alpha + 1 = \dfrac{1}{4}\alpha^2 + \dfrac{1}{2}\alpha + \dfrac{1}{4}$

$4(\alpha + 1) = (\alpha + 1)^2$

$\Rightarrow \quad \alpha + 1 = 4$ OR $\alpha = 3$

Slope of normal $= \dfrac{-2}{4} = \dfrac{-1}{2}$

Equation of normal

$$y - 4 = \frac{-1}{2}(x - 3)$$

$2y - 8 = -x + 3$

$2y + x = 11$

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, −00.33, −00.30, 30.27, −27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The sum of all 4 digit number using the digits 1, 2, 2, 3 is

Answer (26,664)

Sol. Sum of all unit place numbers

$$= 3! \times 2 + \frac{3!}{2!} \times 3 + \frac{3!}{2!} \times 1$$

$$= 12 + 9 + 3 = 24$$

∴ Sum of all 4 digit no.

$= 1 \times 24 + 10 \times 24 + 100 \times 24 + 1000 \times 24$

$= 24000 + 2400 + 240 + 24$

$= 26,6764$

22. 4, 11, 21, 34 ……. then find the value of $\dfrac{S_{29} - S_9}{60}$

Answer (223)

Sol. $S = 4 + 11 + 21 + 34 + ….. + T_n$

$\underline{S = \qquad 4 + 11 + 21 + ….. + T_{n-1} + T_n}$

$T_n = 4 + \underbrace{7 + 10 + 13 + …. + (T_n + T_{n-1})}_{n-1 \text{ term}}$

$T_n = 4 + \dfrac{(n-1)}{2}\left[14 + (n-2)3\right]$

$\quad = 4 + \dfrac{(n-1)}{2}\left[8 + 3n\right]$

$T_n = 4 + \dfrac{1}{2}\left(3n^2 + 5n - 8\right)$

$\sum T_n = S_n = \dfrac{3}{2}\sum n^2 + \dfrac{5}{2}\sum n$

$\quad = \dfrac{3}{2}\dfrac{n(n+1)(2n+1)}{6} + \dfrac{5}{2}\dfrac{n(n+1)}{2}$

$\dfrac{S_{29} - S_9}{60} = 223$

23. In ΔABC, P is circumcentre, ΔQ is orthocentre, then $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is

(1) $2\overrightarrow{PQ}$ (2) $\overrightarrow{PQ}$

(3) $3\overrightarrow{PQ}$ (4) $\dfrac{1}{2}\overrightarrow{PQ}$

Answer (2)

Sol.
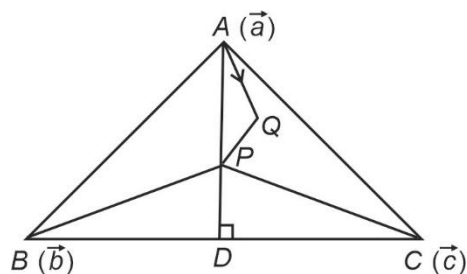
Let P be origin then

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{a} + \vec{b} + \vec{c}$

$\overrightarrow{PD} = \dfrac{\vec{b} + \vec{c}}{2}$

$\overrightarrow{PQ} = 2\overrightarrow{PD}$

$\quad = \vec{b} + \vec{c}$

$\overrightarrow{PQ} = \vec{a} + \left(\vec{b} + \vec{c}\right)$

$\quad = \vec{a} + \vec{b} + \vec{c}$

24. Let $S = \left\{ x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] : 9^{1 - \tan^2 x} + 9^{\tan^2 x} = 10 \right\}$ and $\beta = \sum\limits_{x \in S}\left(\dfrac{x}{3}\right)$. Then $\dfrac{1}{7}(\beta - 14)^2$ is

Answer (28)

Sol. Let $9^{\tan^2 x} = t$

$\dfrac{9}{t} + t = 10$

$t^2 - 10t + 9 = 0$

$\Rightarrow \quad t = 9 \quad \text{or} \quad 1$

$9^{\tan^2 x} = 9$

$\Rightarrow \quad \tan^2 x = 1$

$\quad = \tan x = \pm 1$

$x = \pm \dfrac{\pi}{4}$

or

$\tan x = 0$

$\Rightarrow \quad x = 0$

$\beta = \dfrac{0}{3} + \dfrac{\pi}{12} - \dfrac{\pi}{12} = 0$

$\therefore \quad \dfrac{1}{7}(\beta - 14)^2 = \dfrac{14^2}{7} = 28$

25. The coefficient of x and x^2 in $(1 + x)^p (1 - x)^q$ are 4 and -5, then $2p + 3q$ is

Answer (63)

Sol. $(1 + x)^p (1 - x)^q = (1 + px + \dfrac{p(p-1)}{2}x^2 + …)$

$\qquad\qquad (1 - qx + q\dfrac{(q-1)}{2}x^2 + …)$

∴ Coefficient of $x = p - q = 4$ …(i)

Coefficient of $x^2 = \dfrac{q(q-1)}{2} - pq + \dfrac{p(p-1)}{2}$

$= -5$

⇒ $(p - q)^2 - (p + q) = -10$

∴ $p + q = 26$ …(ii)

By (i) and (ii) $p = 15$, $q = 11$

So, $2p + 3q = 30 + 33 = 63$

26. Let α be the remainder $(22)^{2022} + (2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7 then $\alpha^2 + \beta^2$ is

Answer (05)

Sol. $(22)^{2022} + (2022)^{22}$

For α

$(21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$

$= (3k_1 + 1)$

For β

$(21+1)^{2022} + (2023-1)^{22}$

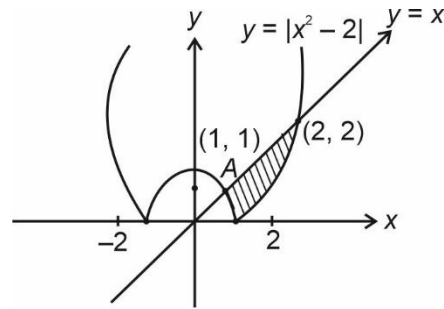$= (7\lambda + 1) + (7\mu + 1)$

$= 7k_2 + 2$

So, $\alpha = 1$, $\beta = 2$

$\alpha^2 + \beta^2 = 5$

27. If area bounded by region $\{x, y\}|\ |x^2 - 2| \le y \le x\}$ is A, then $6A + 16\sqrt{2}$ is

Answer (27)

Sol.

∴ Required area

$= \displaystyle\int_1^{\sqrt{2}} x - \{-(x^2 - 2)\}dx + \int_{\sqrt{2}}^{2} \{x - (x^2 - 2)\}dx$

$= \displaystyle\int_1^{\sqrt{2}} (x^2 + x - 2)dx + \int_{\sqrt{2}}^{2} (-x^2 + x + 2)dx$

$= \left(\dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x\right)_1^{\sqrt{2}} + \left(\dfrac{-x^3}{3} + \dfrac{x^2}{2} + 2x\right)_{\sqrt{2}}^{2}$

$= \left(\dfrac{2\sqrt{2}}{3} + 1 - 2\sqrt{2}\right) - \left(\dfrac{1}{3} + \dfrac{1}{2} - 2\right)$

$\qquad + \left(\dfrac{-8}{3} + 2 + 4\right) - \left(\dfrac{-2\sqrt{2}}{3} + 1 + 2\sqrt{2}\right)$

∴ $6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2}$

$= 27$

28.

29.

30.

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