MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Using all he letters of the word MATHS, then rank of the word THAMS is

(1) 101 (2) 102

(3) 103 (4) 104

Answer (3)

Sol. $\overset{5}{\text{T}}\,\overset{2}{\text{H}}\,\overset{1}{\text{A}}\,\overset{3}{\text{M}}\,\overset{4}{\text{S}}$

$\overset{4}{4!}\;\overset{1}{3!}\;\overset{0}{2!}\;\overset{0}{1!}\;\overset{0}{0!}$

$\therefore$ Rank = 4 × 4! + 1 × 3! + 1

= 96 + 6 + 1 = 103

2. $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then λ and $\frac{\lambda}{3}$ are roots of

(1) $4x^2 + 24x - 27 = 0$

(2) $4x^2 - 24x + 27 = 0$

(3) $4x^2 - 24x - 27 = 0$

(4) $4x^2 + 24x + 27 = 0$

Answer (2)

Sol. Put $x = 0$ in the given equation

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8}\times 81$$

$$\Rightarrow \lambda^3 = \frac{(3)^6}{2^3}$$

$$\lambda = \frac{9}{2}$$

$$\Rightarrow \frac{\lambda}{3} = \frac{3}{2}$$

$$x^2 - \left(\frac{9}{2}+\frac{3}{2}\right)x + \frac{9}{2}\times\frac{3}{2} = 0$$

$$\boxed{4x^2 - 24x + 27 = 0}$$

3. $\frac{dy}{dx} + \frac{5}{x(1+x^5)}y = \frac{(1+x^5)^2}{x^7}$. If $y(1) = 2$, then the value of $y(2)$ is

(1) $\frac{693}{128}$ (2) $\frac{697}{128}$

(3) $\frac{637}{128}$ (4) $\frac{627}{128}$

Answer (1)

Sol. I.F. $= e^{\int \frac{5}{x(1+x^5)}dx} = e^{\int \frac{5x^{-6}}{(x^{-5}+1)}dx}$

$$= e^{-\ln(x^{-5}+1)} = \frac{1}{x^{-5}+1} = \frac{x^5}{x^5+1}$$

$$y\cdot\frac{x^5}{x^5+1} = \int \frac{(1+x^5)^2}{x^7}\cdot\frac{x^5}{(1+x^5)}dx$$

$$= \int \frac{(1+x^5)}{x^2}dx$$

$$= \frac{-1}{x} + \frac{x^4}{4} + C$$

$$y(1) = 2 \Rightarrow 2\left(\frac{1}{2}\right) = -1 + \frac{1}{4} + C$$

$$\Rightarrow C = \frac{7}{4}$$

Put $x = 2$

$$\Rightarrow y\left(\frac{32}{33}\right) = \frac{-1}{2} + 4 + \frac{7}{4}$$

$$\Rightarrow y = \frac{693}{128}$$

4. The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is

(1) $(-\infty, 3] \cup [6, \infty)$ (2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-\infty, 3] \cup [5, \infty)$ (4) $(-\infty, -2) \cup [6, \infty)$

Answer (4)

Sol. $[x]^2 - 3[x] - 10 > 0$

$([x]+2)([x]-5) > 0$

$[x] < -2$ OR $[x] > 5$

$[x] \le -3$ OR $[x] \ge 6$

$x < -2$ OR $x \ge 6$

$x \in (-\infty, -2) \cup [6, \infty)$

5. Let mean and variance of the data 1, 2, 4, 5, x, y are 5 and 10 respectively. Then mean deviation about the mean of data is

(1) $\frac{8}{3}$ (2) $\frac{7}{2}$

(3) $\frac{5}{6}$ (4) $\frac{7}{6}$

Answer (1)

Sol. $12 + x + y = 30 \Rightarrow x + y = 18$

and $\frac{x^2+y^2+46}{6}-(5)^2=10$

$\therefore \frac{x^2+y^2+46}{6}=10+25$

$x^2 + y^2 = 164$

$\therefore x = 10, y = 8$

Now, mean deviation about mean

$$=\frac{4+3+1+0+5+3}{6}=\frac{16}{6}=\frac{8}{3}$$

6. If $a + b + c + d = 11$ ($a, b, c, d > 0$) then maximum value of $a^5b^3c^2d = 3750\beta$ the β is

(1) 90 (2) 115

(3) 120 (4) 85

Answer (1)

Sol. Assume numbers to be

$\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}, d.$

Now apply $AM \ge GM$

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11}\ge\left(\frac{a^5b^3c^2d}{5^53^32^21}\right)^{\frac{1}{11}}$$

$a^5b^3c^2d \le 5^53^32^2$

$\therefore$ Max of $a^5b^3c^2d = 5^53^32^2 = 3,37,500$

$= 90 \times 3750$

$\Rightarrow \beta = 90$

7. $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$ then $(1011)^{th}$ term from end is equal to (1024) times $(1011)^{th}$ term from starting then $|x|$ is

(1) $\frac{16}{7}$ (2) $\frac{16}{5}$

(3) $\frac{5}{16}$ (4) $\frac{8}{5}$

Answer (3)

Sol. 1011^{th} term from end = 1011 term from beginning

$\therefore r = 1010$ $\left(\frac{5}{2x}-\frac{4x}{5}\right)^{2022}$

$$T_{1011}={}^{2022}C_{1010}\left(\frac{5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$$

1011 term from starting $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$

$$T_{1011}={}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{5}{2x}\right)^{1010}$$

Now,

$$\frac{{}^{2022}C_{1010}\left(\frac{5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}}{{}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{5}{2x}\right)^{1010}}=1024$$

$\left(\frac{5\times5}{2x\times4x}\right)^2=2^{10}$

$\frac{25}{8x^2}=2^5$

$x^2=\frac{25}{2^8}$

$|x|=\frac{5}{2^4}$

8. A circle with center at (2, 0) and maximum radius "r" is inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{9}=1$. The value of $12r^2$ is

(1) 108 (2) 172

(3) 83 (4) 92

Answer (4)

Sol. Equation of normal at $P(6\cos\theta, 3\sin\theta)$ is

$(6\sec\theta)x-(3\text{cosec}\theta)y=27$

It passes through (2, 0)

$12\sec\theta = 27$

$\cos\theta=\frac{4}{9}, \sin\theta=\frac{\sqrt{65}}{9}$

$P\left(\frac{8}{3},\frac{\sqrt{65}}{3}\right)$

$$r=\sqrt{\left(\frac{8}{3}-2\right)^2+\left(\frac{\sqrt{65}}{3}\right)^2}=\frac{\sqrt{69}}{3}$$

$12r^2=12\times\frac{69}{9}=92$

9. $f : R \to R$ be a continuous non-constant function and $\int_0^{\pi/2} f(\sin 2x).\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$ then α is equal to

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $-\sqrt{2}$ (4) $-\sqrt{3}$

Answer (3)

Sol. $\int_0^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$

$$\int_0^{\pi/4} f(\sin 2x)\sin x\, dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x)\cos x\, dx = 0$$

Here $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let $x = t + \frac{\pi}{4}$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\sin\left(\frac{\pi}{4} - x\right)dx + \int_0^{\pi/4} f(\cos 2t)\sin\left(t + \frac{\pi}{4}\right)dx + \alpha \int_0^{\pi/4} f(\cos 2x)$$

$\cos x\, dx = 0$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\left\{\sin\left(\frac{\pi}{4} - x\right) + \sin\left(x + \frac{\pi}{4}\right) + \alpha\cos x\right\}dx = 0$$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\left\{\left(\left(\sqrt{2} + \alpha\right)\cos x\right)\right\}dx = 0$$

$$\because \left(\sqrt{2} + \alpha\right)\int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$$

$\because$ $f(\cos 2x)$ and $\cos x$ is not zero in $\left(0, \frac{\pi}{4}\right)$.

$\therefore \sqrt{2} + \alpha = 0$

$\Rightarrow \alpha = -\sqrt{2}.$

10. If the ratio of three consecutive terms is 1:3:5 in the expansion of $(1 + x)^{n+2}$. Then sum of consecutive terms is

(1) 41 (2) 64

(3) 63 (4) 43

Answer (3)

Sol. ${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} :: 1 : 3 : 5$

$$\therefore \frac{(n+2)!}{(r-1)!\,(n-r+3)!} \times \frac{r!(n+2-r)!}{(n+2)!} = \frac{1}{3}$$

$$\Rightarrow \frac{r}{(n-r+3)} = \frac{1}{3} \Rightarrow n - r + 3 = 3r$$

$n = 4r - 3$ ...(i)

and $\frac{(n+1)!}{r!(n+2-r)!} \times \frac{(r+1)!\,(n-r+1)!}{(n+2)!} = \frac{3}{5}$

$$\Rightarrow \frac{(r+1)}{n+2-r} = \frac{3}{5}$$

$\Rightarrow 5r + 5 = 3n + 6 - 3r$

$\Rightarrow 8r - 1 = 3n$ ...(ii)

By (i) and (ii)

$$4r - 3 = \frac{8r - 1}{3}$$

$\Rightarrow 4r = 8 \Rightarrow r = 2$

$n = 5$

$\therefore$ Sum $= {}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + 21 + 35 = 63$

11. The converse of the statement $(\sim p \wedge q) \Rightarrow r$ is

(1) $r \Rightarrow (\sim p \wedge q)$ (2) $r \Rightarrow (p \vee \sim q)$

(3) $\sim r \Rightarrow (p \vee \sim q)$ (4) $\sim r \Rightarrow (\sim p \wedge q)$

Answer (1)

Sol. Converse of $(\sim p \wedge q) \Rightarrow r$ is

$r \Rightarrow (\sim p \wedge q)$

12. If $\vec{a}, \vec{b}, \vec{c}, \vec{b}$ are coplanar reactor then value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

(1) $\left[\vec{b}\ \vec{d}\ \vec{c}\right] + \left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(2) $\left[\vec{b}\ \vec{d}\ \vec{c}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(3) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(4) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

Answer (3)

Sol. $\left[\vec{b} - \vec{a}\ \ \vec{c} - \vec{a}\ \ \vec{d} - \vec{a}\right] = 0$

$$\left(\vec{b} - \vec{a}\right)\cdot\left((\vec{c} - \vec{a}) \times (\vec{d} - \vec{a})\right) = 0$$

$$\left(\vec{b} - \vec{a}\right)\cdot\left(\vec{c} \times \vec{d} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d}\right) = 0$$

$$\left[\vec{b}\ \vec{c}\ \vec{d}\right] - \left[\vec{b}\ \vec{c}\ \vec{a}\right] - \left[\vec{b}\ \vec{a}\ \vec{d}\right] - \left[\vec{a}\ \vec{c}\ \vec{d}\right] = 0$$

$$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{b}\ \vec{c}\ \vec{d}\right] - \left[\vec{b}\ \vec{a}\ \vec{d}\right] - \left[\vec{a} \cdot \vec{c}\ \vec{d}\right]$$

13. $f(x)=\begin{cases} e^{\min(x^2,\alpha x^3)}, & x\in(0,1) \\ e^{[x-\ln x]}, & x\in[1,2) \end{cases}$ then find $\int_0^2 xf(x)dx$

(1) $2e-\frac{1}{2}$ (2) $2e+\frac{1}{2}$

(3) $4e-\frac{1}{2}$ (4) $4e+\frac{1}{2}$

Answer (1)

Sol. $f(x)=\begin{cases} e^{x^2}, & x\in(0,1) \\ e, & x\in[1,2) \end{cases}$

$$\int_0^2 xf(x)dx=\int_0^1 x\cdot e^{x^2}dx+\int_1^2 x\times e\,dx$$

$x^2 = t$

$2xdx = dt$

$$=\frac{1}{2}\int_0^1 e^t dt+e\int_1^2 xdx$$

$$=\frac{1}{2}\left[e^t\right]_0^1+e\left[\frac{x^2}{2}\right]_1^2$$

$$=\frac{1}{2}\times(e-1)+\frac{3}{2}e$$

$$=2e\frac{-1}{2}$$

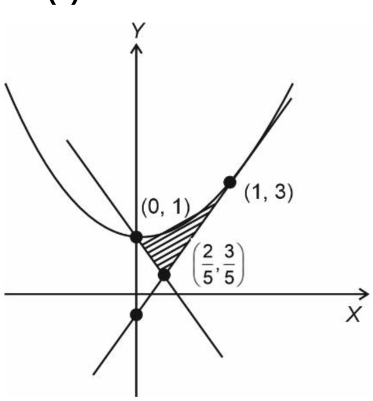
14. The area between the curve $y = 2x^2 + 1$ and tangent to it at (1, 3) and $x + y = 1$ is

(1) $\frac{1}{15}$ (2) $\frac{1}{60}$

(3) $\frac{4}{15}$ (4) $\frac{8}{3}$

Answer (3)

Sol.



Tangent at (1, 3) $\frac{y+3}{2}=2x+1$

$y = 4x - 1$

$\therefore$ Area

$$\int_0^{2/5}\left(2x^2+1-(1-x)\right)dx+\int_{2/5}^{1}\left(2x^2+1\right)-(4x-1)dx$$

$$=\int_0^{2/5}\left(2x^2+x\right)dx+\int_{2/5}^{1}\left(2x^2-4x+2\right)dx$$

$$=\left(\frac{2x^3}{3}+\frac{x^2}{2}\right)_0^{2/5}+\left[\frac{2x^3}{3}-\frac{4x^2}{2}+2x\right]_{2/5}^{1}$$

$$=\frac{92}{750}+\frac{144}{1000}=\frac{368+432}{3000}=\frac{800}{3000}=\frac{4}{15}$$

15. Angle between line $x=\frac{y-1}{2}=\frac{z-3}{r}$ and plane $x + 2y + 3z + 4 = 0$ is $\cos^{-1}\sqrt{\frac{5}{14}}$ then point of intersection of line and plane is

(1) (–15, –23, –11) (2) $\left(\frac{15}{7},\frac{-23}{7},\frac{11}{7}\right)$

(3) (15, 23, 11) (4) $\left(\frac{-15}{7},\frac{-23}{7},\frac{11}{7}\right)$

Answer (4)

Sol. $\sin\theta=\frac{1+4+3r}{\sqrt{14}\sqrt{5+r^2}}$

$$\cos^{-1}\frac{\sqrt{5}}{\sqrt{14}}=\sin^{-1}\frac{3}{\sqrt{14}}=\sin^{-1}\left(\frac{5+3r}{\sqrt{14}\sqrt{5+r^2}}\right)$$

$$\frac{3}{\sqrt{14}}=\frac{5+3r}{\left(\sqrt{5+r^2}\right)\sqrt{14}}$$

$3\sqrt{5+r^2}=5+3r$

$9(5 + r^2) = 25 + 9r^2 + 30r$

$\Rightarrow$ $45 = 25 + 30r$

$\Rightarrow$ $30r = 30$

$r=\frac{2}{3}$

Let the point on line is $P(3k, 6k + 1, 2k + 3)$

$3k + 12k + 2 + 6k + 9 + 4 = 0$

$\Rightarrow$ $21k = -15$

$\Rightarrow$ $k=-\frac{5}{7}$

$\therefore$ $P\left(\frac{-15}{7},\frac{-23}{7},\frac{11}{7}\right)$

16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$, then number of solutions of above equation is

Answer (2)

Sol. $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$,

$$\Rightarrow \left(e^{4x} + \frac{1}{e^{4x}}\right) - \left(e^{2x} + \frac{1}{e^{2x}}\right) = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right)^2 - \left(e^{2x} + \frac{1}{e^{2x}}\right) = 5$$

$$\Rightarrow t^2 - t - 5 = 0$$

$$t = \frac{1 \pm \sqrt{1+20}}{2}$$

$$= \frac{1 \pm \sqrt{21}}{2}$$

$\frac{1-\sqrt{21}}{2}$ is rejected

$$\therefore \quad t = \frac{1+\sqrt{21}}{2}$$

$\Rightarrow \quad e^{2x} + \frac{1}{e^{2x}} = \frac{1+\sqrt{21}}{2} \Rightarrow$ 2 values of e^{2x} possible

$\therefore$ 2 real solution

22. If $f(1) + f(2) = f(4) - 1$ and a function from A to B is defined where $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 5, 6\}$. Find the numbers of function with such relation.

Answer (360)

Sol. $f(4) = f(1) + f(2) + 1$

$$\Rightarrow f(1) + f(2) + 1 \le 6$$

$$f(1) + f(2) \le 5$$

Possible cases

1	$\{1, 2, 3, 4\}$	$\rightarrow$	4
2	$\{1, 2, 3\}$	$\rightarrow$	3
3	$\{1, 2\}$	$\rightarrow$	2
4	$\{1\}$	$\rightarrow$	$\frac{1}{10}$

$f(5)$, $f(3)$ can be filled in 6 ways

Total functions = 10 × 6 × 6 = 360

23. For a biased coin, the probability of getting head is $\frac{1}{4}$. It is tossed n times till we get head. Given a quadratic equation $64x^2 + 2nx + 1 = 0$. If the probability that the quadratic equation has no real roots is $\frac{P}{Q}$ (where P and Q are coprime), then the value of $Q - P$ is

Answer (2187)

Sol. $(2n)^2 - 4 \times 64 < 0 \Rightarrow n < 8 \Rightarrow n \le 7$

Required probability

$$= \frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \ldots + \left(\frac{3}{4}\right)^6 \cdot \frac{1}{4}$$

$$= \frac{1}{4}\frac{\left(1-\left(\frac{3}{4}\right)^7\right)}{1-\frac{3}{4}} = \frac{4^7 - 3^7}{4^7} = \frac{P}{Q}$$

$Q - P = 3^7 = 2187$

24.
25.
26.
27.
28.
29.
30.

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