MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

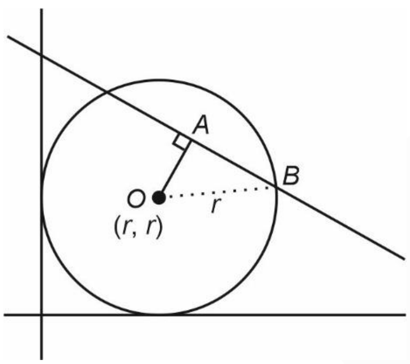
1. Two circles having radius r_1 and r_2 touch both the coordinate axes. Line $x + y = 2$ makes intercept as 2 on both the circles. The value of $r_1^2 + r_2^2 - r_1 \cdot r_2$ is

(1) $\frac{9}{2}$ (2) 6

(3) 7 (4) 8

Answer (3)

Sol.



$AB = 1$

$OA = \sqrt{r^2 - 1}$

$\Rightarrow \left|\frac{2r-2}{\sqrt{2}}\right| = \sqrt{r^2-1}$

$\Rightarrow \sqrt{2}(r-1) = \sqrt{r^2-1}$

$\Rightarrow 2(r-1)^2 = r^2 - 1$

$\Rightarrow 2r^2 - 4r + 2 = r^2 - 1$

$\Rightarrow r^2 - 4r + 3 = 0$

$\Rightarrow (r-1)(r-3) = 0$

$\Rightarrow r = 1, 3$

$\therefore r_1 = 1$ and $r_2 = 3$

$\therefore r_1^2 + r_2^2 - r_1 \cdot r_2 = 1 + 9 - 3 = 7$

2. Area of region enclosed by curve $y = x^3$ and its tangent at (–1, –1)

(1) 4 (2) 27

(3) $\frac{4}{27}$ (4) $\frac{27}{4}$

Answer (4)

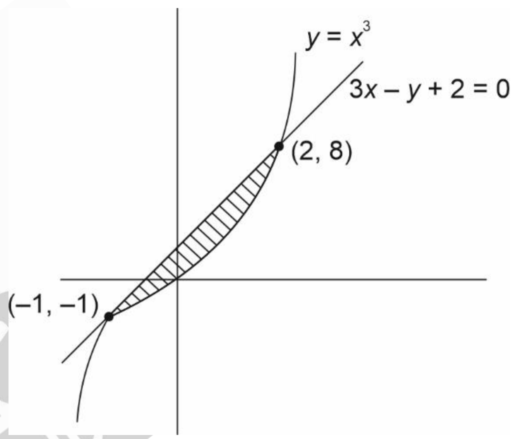
Sol. $y = x^3$

$y' = 3x^2$

$y'_{(-1,-1)} = 3$

$T : y + 1 = 3(x + 1)$

$T : 3x - y + 2 = 0$



$$\text{Area} = \int_{-1}^{2} (3x+2) - x^3 dx$$

$$= \left.\frac{3x^2}{2} + 2x - \frac{x^4}{4}\right]_{-1}^{2}$$

$$= \left|\frac{3}{2} \times 3 + 2 \times 3 - \frac{1}{4} \times 15\right|$$

$$= \frac{9}{2} + 6 - \frac{15}{4}$$

$$= \frac{27}{4} \text{ sq. units}$$

3. If $(1 + x^2)dy = y(y - x)dx$ and $y(1) = 1$. Then $y(2\sqrt{2})$ is

(1) $\frac{4}{\sqrt{2}}$ (2) $\frac{3}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Answer (3)

Sol. $\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{y^2}{1+x^2}$

$\frac{1}{y^2}\frac{dy}{dx}+\frac{x}{1+x^2}\times\frac{1}{y}=\frac{1}{1+x^2}$

Let $\frac{1}{y}=t$

$-\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{-dt}{dx}+\left(\frac{x}{1+x^2}\right)dt=\frac{1}{1+x^2}$

$\frac{dt}{dx}-\left(\frac{x}{1+x^2}\right)dt=-\frac{1}{1+x^2}$

$\text{IF}=e^{-\int\frac{x}{1+x^2}dx}=e^{-\frac{1}{2}\log|1+x^2|}=\frac{1}{\sqrt{1+x^2}}$

$\frac{t}{\sqrt{1+x^2}}=-\int\underbrace{\frac{1}{(1+x^2)\sqrt{1+x^2}}dx}_{I}$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I=\int\frac{\sec^2\theta}{\sec^2\theta\cdot\sec\theta}d\theta=\int\cos\theta=\sin\theta+C$

$\therefore\quad \frac{1}{y\sqrt{1+x^2}}=-\frac{x}{\sqrt{1+x^2}}+C$

$\because\quad y(1) = 1$

$\Rightarrow\quad C=\sqrt{2}$

$\therefore\quad \frac{1}{y\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}=\sqrt{2}$

$1+xy=\sqrt{2}y\sqrt{1+x^2}$

Now

$y(2\sqrt{2})$

$1+2\sqrt{2}y=3\sqrt{2}y$

$\sqrt{2}y=1$

$y=\frac{1}{\sqrt{2}}$

4. For the expression $(1 - x)^{100}$. Then sum of coefficient of first 50 terms is

(1) $^{99}C_{49}$ (2) $-\frac{^{100}C_{50}}{2}$

(3) $-^{99}C_{49}$ (4) $-^{101}C_{50}$

Answer (2)

Sol. Sum of coefficient of first 50 terms

$(t)={}^{100}C_0-{}^{100}C_1+...+{}^{100}C_{49}$

Now

${}^{100}C_0-{}^{100}C_1+...+{}^{100}C_{100}=0$

$2\left[{}^{100}C_0-{}^{100}C_1+...\right]+{}^{100}C_{50}=0$

$\therefore\quad t=-\frac{1}{2}\,{}^{100}C_{50}$

5. Positive numbers a_1, a_2, …. a_5 are in geometric progression. Their mean and variance are $\frac{31}{10}$ and $\frac{m}{n}$ respectively. The mean of the reciprocals is $\frac{31}{40}$, then $m + n$ is

(1) 209 (2) 211

(3) 113 (4) 429

Answer (2)

Sol. $a\left(\frac{1}{r^2}+\frac{1}{r}+1+r+r^2\right)=\frac{31}{2}$

$\frac{1}{a}\left(\frac{1}{r^2}+\frac{1}{r}+1+r+r^2\right)=\frac{31}{8}$

$\Rightarrow\quad a^2 = 4$

$\Rightarrow\quad a = 2$

$\Rightarrow\quad \frac{1}{r^2}+\frac{1}{r}+1+r+r^2=\frac{31}{4}$

$\Rightarrow\quad \left(r+\frac{1}{r}\right)^2+\left(r+\frac{1}{r}\right)=\frac{31}{4}+1=\frac{35}{4}$

$4t^2+4t-35=0$

$\Rightarrow\quad t=\frac{5}{2}$

$\Rightarrow\quad r = 2$

$\therefore\quad$ numbers are $=\frac{1}{2}, 1, 2, 4, 8$

$$\therefore \quad \sigma^2 = \frac{\frac{1}{4}+1+4+16+64}{5} - \left(\frac{31}{10}\right)^2$$

$$= \frac{341}{20} - \frac{961}{100}$$

$$= \frac{1705-961}{100}$$

$$= \frac{744}{100} = \frac{186}{25}$$

$\therefore \quad m + n = 186 + 25$

$= 211$

6. If $\Delta(k) = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$, then

$\sum_{k=1}^{n} \Delta(k) =$

(1) n

(2) 1

(3) $\frac{n^2}{2}$

(4) 0

Answer (4)

Sol. $\sum_{k=1}^{n} \Delta(k) = \begin{vmatrix} n & n^2 & n(n+1) \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix} = 0$

7. Given A, B, C represents angles of a ΔAB and $\cos A + 2\cos B + \cos C = 2$ and $AB = 3$ and $BC = 7$ then $\cos A - \cos C$ is

(1) $-\frac{10}{7}$

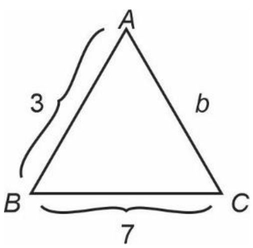
(2) $\frac{10}{7}$

(3) $\frac{5}{7}$

(4) $-\frac{5}{7}$

Answer (1)

Sol.



$\cos A + 2\cos B + \cos C = 2.$

$$\frac{9+b^2-49}{6b} + 2\left(\frac{49+9-b^2}{42}\right) + \left(\frac{49+b^2-9}{14b}\right) = 2$$

$$\frac{b^2-40}{6b} + \frac{58-b^2}{21} + \frac{40+b^2}{14b} = 2$$

$\Rightarrow \quad b = -4$ or 4 or 5

b cannot be –4 and 4

$\Rightarrow \quad b = 5.$

Now,

$\cos A - \cos C$

$$\frac{9+25-49}{2\times3\times5} - \frac{49+25-9}{2\times7\times5}$$

$$-\frac{1}{2} - \frac{13}{14} = \frac{-20}{14} = \frac{-10}{7}$$

8. Let $x^2 + \sqrt{6x} + 4 = 0$ be any quadratic equation and α, β are the roots of that equation then

$$\frac{\alpha^{34}\beta^{24} + \alpha^{32}\beta^{26} + 2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20} + \alpha^{28}\beta^{23} + 3\alpha^{30}\beta^{21} + 3\alpha^{29}\beta^{22}}$$ is

(1) $\frac{-2^7}{3}\sqrt{6}$

(2) $\frac{2^7}{3}\sqrt{6}$

(3) $\frac{-2^8}{3}\sqrt{6}$

(4) $\frac{2^8}{3}\sqrt{6}$

Answer (1)

Sol. $x^2 + \sqrt{6}x + 4 = 0$ (roots α, β)

$\therefore \quad \alpha + \beta = -\sqrt{6}, \quad \alpha\beta = 4$

Now $$\frac{\alpha^{34}\beta^{24} + \alpha^{32}\beta^{26} + 2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20} + \alpha^{28}\beta^{23} + 3\alpha^{30}\beta^{21} + 3\alpha^{29}\beta^{22}}$$

$$= \frac{\alpha^{32}\beta^{24}\left[\alpha^2 + \beta^2 + 2\alpha\beta\right]}{\alpha^{28}\beta^{20}\left[\alpha^3 + \beta^3 + 3\alpha^2\beta + 3\alpha\beta^2\right]}$$

$$= (\alpha\beta)^4 \frac{\left[(\alpha+\beta)^2\right]}{(\alpha+\beta)^3} = \frac{4^4}{-\sqrt{6}} = \frac{-2^7}{3}\sqrt{6}$$

9. If a plane $4x - 3y + z = 2$ is rotated by an angle of $\frac{\pi}{2}$ at intersection point of another plane $3x + 11z - 4y = 12$, then $P(2, 3, 4)$ is at what distance from resultant plane?

(1) $\frac{250}{\sqrt{63245}}$

(2) $\frac{641}{\sqrt{66846}}$

(3) $\frac{925}{\sqrt{66215}}$

(4) $\frac{24}{\sqrt{11235}}$

Answer (2)

Sol : Equation of required plane:

$4x - 3y + z - 2 + \lambda(3x - 4y + 11z - 12) = 0$

If is perpendicular to $4x - 3y + z = 2$

$\therefore$ $(4 + 3\lambda)\cdot 4 + (-3 - 4\lambda)(-3) + (1 + 11\cdot\lambda)1 = 0$

$\Rightarrow$ $16 + 12\lambda + 9 + 12\lambda + 1 + 11\lambda = 0$

$\Rightarrow$ $35\lambda + 26 = 0$

$\Rightarrow$ $\lambda = -\frac{26}{35}$

$\therefore$ $x(4 + 3\lambda) - y(3 + 4\lambda) + z(1 + 11\lambda) - 2 - 12\lambda = 0$

$\Rightarrow$ $\frac{62x}{35} - y\left(\frac{1}{35}\right) + z\left(\frac{-251}{35}\right) + \left(\frac{242}{35}\right) = 0$

$\Rightarrow$ $62x - y - 251z + 242 = 0$

Distance from (2, 3, 4)

$= \left|\frac{124 - 3 - 1004 + 242}{\sqrt{66846}}\right|$

$= \frac{641}{\sqrt{66846}}$

10. A circle with centre $z_0 = \frac{1}{2} + \frac{3i}{2}$ exists in an argand plane. A point $z_1 = 1 + i$ and z_2 lies outside the circle, such that $|z_0 - z_1|\,|z_0 - z_2| = 1$. Then the largest value of $|z_2|$ is

(1) $\sqrt{5} - \sqrt{2}$

(2) $\sqrt{\frac{5}{2}} - \sqrt{2}$

(3) $\sqrt{\frac{5}{2}}$

(4) $\sqrt{\frac{5}{2}} + \sqrt{2}$

Answer (4)

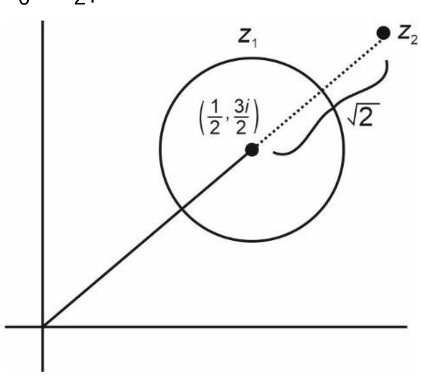
Sol. $\left|z - \frac{1}{2} - \frac{3i}{2}\right| = r \rightarrow$ Circle

Now,

$|z_0 - z_1|\,|z_0 - z_2| = 1$

$\frac{1}{\sqrt{2}}|z_0 - z_2| = 1$

$|z_0 - z_2| = \sqrt{2}$



Max $|z_2| = \sqrt{\frac{1}{4} + \frac{9}{4}} + \sqrt{2}$

$= \sqrt{\frac{10}{4}} + \sqrt{2}$

$= \left(\sqrt{\frac{5}{2}} + \sqrt{2}\right)$ unit

11. Let $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}$ is a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0$ and $\vec{a}\cdot\vec{c} = -17$, $\vec{b}\cdot\vec{c} = -20$. Find $\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2$ given $(\lambda > 0)$

(1) 46

(2) 61

(3) 48

(4) 51

Answer (1)

Sol. $k(\vec{a} + \vec{b}) = \vec{c}$

$\vec{a}\cdot\vec{c} = -17$, $\vec{b}\cdot\vec{c} = -20$

$k(\lambda^2 + 3\lambda - 1) = -17$, $k(3\lambda + 11) = -20$

$\Rightarrow$ $\lambda = -\frac{69}{20}, 3$

$\lambda = 3$, $k = -1$

$\vec{c} = -1(\vec{a} + \vec{b})$

$= -((\lambda + 3)\hat{i} + \hat{k}) = -6\hat{i} - \hat{k}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 0 \end{vmatrix} = \hat{i}(1) - \hat{j}(3) + \hat{k}(-6)$

$= \hat{i} - 3\hat{j} - 6\hat{k}$

$\left|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})\right|^2 = 46$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If $\frac{{}^nC_n}{n+1}+\frac{{}^nC_{n-1}}{n}+\text{.........}+\frac{1}{2}\,{}^nC_1+{}^nC_0=\frac{255}{8}$. Then value of n is

Answer (07)

Sol. $\int_0^1 (1+x)^n=\int_0^1\left({}^nC_0+{}^nC_1x+{}^nC_2x^2+\text{.........}+{}^nC_nx^n\right)dx$

$$\left.\frac{(1+x)^{n+1}}{n+1}\right]_0^1={}^nC_0x+{}^nC_1\frac{x^2}{2}+{}^nC_2\frac{x^3}{3}+\text{........}+\left.\frac{{}^nC_nx^{n+1}}{n+1}\right]_0^1$$

$$\frac{2^{n+1}}{n+1}-\frac{1}{n+1}={}^nC_0+\frac{{}^nC_1}{2}+\frac{{}^nC_2}{3}+\text{.........}+\frac{{}^nC_n}{n+1}$$

Now

$$\frac{2^{n+1}-1}{n+1}=\frac{255}{8}$$

$\Rightarrow \quad n = 7$

22. If the value of $\int_{-0.15}^{0.15}\left|100x^2-1\right|dx=\frac{k}{3000}$, then the value of k is _______.

Answer (575)

Sol. $I=2\int_0^{0.15}\left|100x^2-1\right|dx$

$$=2\left[\int_0^{0.1}-(100x^2-1)dx+\int_{0.1}^{0.15}(100x^2-1)dx\right]$$

$$=2\left[\left[x-\frac{100x^3}{3}\right]_0^{0.1}+\left[\frac{100x^3}{3}-x\right]_{0.1}^{0.15}\right]$$

$$=\frac{575}{3000}$$

$\Rightarrow \quad k = 575$

23. N > 40000, where N is divisible by 5. How many such 5 digits numbers using 0, 1, 3, 5, 7, 9?

Answer (120)

Sol. **Case I :** Number starts with 5

5 _ _ _ 0

4 ways, 3 ways, 2 ways $= 4 \times 3 \times 2 = 24$

Case II : Number starts with 7

7 _ _ _ _

4, 3, 2, 2 $= 4 \times 3 \times 2 \times 2 = 48$

Case III : Number starts with 9

9 _ _ _ _

4, 3, 2, 2 $= 4 \times 3 \times 2 \times 2 = 48$

Total ways = 120

24. Three numbers a, b, c are in A.P. and they are used to make a 9-digits number using each digit thrice, such that at least 3 consecutive digits are in A.P. then number of such numbers is

Answer (1260)

Sol.

a	b	c

or

c	b	a

So, total number $\frac{{}^7C_1\times2\times6!}{2!\,2!\,2!}=\frac{7!}{4}$

$= 7 \times 6 \times 5 \times 3 \times 2 = 1260$

25. If $\begin{matrix}a\hat{i}+\hat{j}+k\\ \hat{i}+b\hat{j}+k\\ \hat{i}+\hat{j}+ck\end{matrix}$ are co-planar,

then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

Answer (1)

Sol. $\begin{vmatrix}a&1&1\\1&b&1\\1&1&c\end{vmatrix}=0$

$R_1\to R_1-R_2,\ R_2\to R_2-R_3$

$$\begin{vmatrix}a-1&1-b&0\\0&b-1&1-c\\1&1&c\end{vmatrix}=0$$

$(a – 1)[c(b – 1) – (1 – c)] + 1[(1 – b) (1 – c)] = 0$

$c(a – 1) (b – 1) – (a – 1) (1 – c) + (1 – b) (1 – c) = 0$

Multiply and divide by $(1 – a) (1 – b) (1 – c)$

$$-\frac{1-c-1}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

$$-1+\frac{1}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

$\therefore \quad \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

26. $f(x)=\|[x]\|+\sqrt{x-[x]}$. The number of points of discontinuity of $f(x)$ in [–2, 1] is.

Answer (2)

Sol. $f(x)=\|[x]\|+\sqrt{\{x\}}$

$x = -2$

$f(-2) = 2$

$f(-2^+) = 2 + 0 = 2$

$x = -1$

$f(-1) = 1 + 0 = 1$

$f(-1^-) = 2 + 1 = 3$

$\therefore$ discontinuous at $x = -1$

$x = 0$

$f(0) = 0$

$f(0^-) = 1 + 1 = 2$

$\therefore$ discontinuous at $x = 0$

$x = 1$

$f(1) = 1$

$f(1^-) = 0 + 1 = 1$

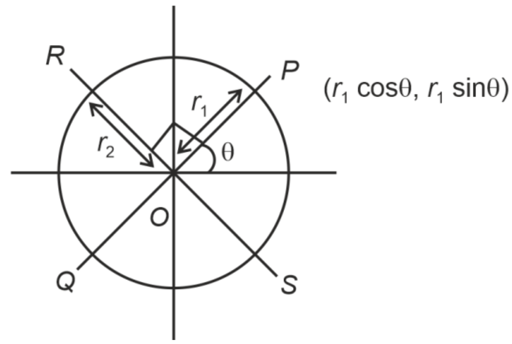
$\therefore$ discontinuous at $x = -1$ and at $x = 0$

$\therefore$ 2 points of discontinuity

27. Given $9x^2 + 4y^2 = 36$ and a point $P\left(\frac{2\sqrt{3}}{\sqrt{7}},\frac{6}{\sqrt{7}}\right)$ lie on ellipse. PQ is a diameter of ellipse and RS is a diameter which is perpendicular to PQ. If $\frac{1}{PQ^2}+\frac{1}{RS^2}=\frac{p}{m}$ in simplest form, then $p + m$ is

Answer (157)

Sol. $r_1=\sqrt{\frac{48}{7}}$



$$\frac{r_1^2\cos^2\theta}{4}+\frac{r_1^2\sin^2\theta}{9}=1$$

$$\frac{\cos^2\theta}{4}+\frac{\sin^2\theta}{9}=\frac{7}{48} \quad \text{...(i)}$$

$$\frac{r_2^2\sin^2\theta}{4}+\frac{r_2^2\cos^2\theta}{9}=1$$

$$\frac{\sin^2\theta}{4}+\frac{\cos^2\theta}{9}=\frac{1}{r_2^2}$$

From (i), $\frac{1}{r_2^2}=\frac{1}{4}+\frac{1}{9}-\frac{7}{48}=\frac{31}{144}$

$$\frac{1}{PQ^2}+\frac{1}{RS^2}=\frac{1}{4}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)$$

$$=\frac{1}{4}\left(\frac{7}{48}+\frac{31}{144}\right)=\frac{13}{144}$$

28.

29.

30.

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