MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. There are 5 black and 3 white balls in a bag. A die is rolled, we need to pick the number of balls appearing on a die. The probability that all balls are white is

(1) $\frac{1}{12}$ (2) $\frac{1}{18}$

(3) $\frac{2}{9}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $\frac{1}{6}\times\frac{{}^3C_1}{{}^8C_1}+\frac{1}{6}\times\frac{{}^3C_2}{{}^8C_2}+\frac{1}{6}\times\frac{{}^3C_3}{{}^8C_3}$

$$=\frac{1}{6}\left(\frac{3}{8}+\frac{3}{28}+\frac{1}{56}\right)$$

$$=\frac{1}{6}\left(\frac{21+6+1}{56}\right)=\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$$

2. The mean and variance of 15 observations is 20 and 64, respectively. If 55 is wrongly read as 40 as one of the observation, then the correct variance is ______ .

(1) $\frac{243}{3}$ (2) $\frac{167}{2}$

(3) $\frac{247}{3}$ (4) 96

Answer (3)

Sol. $64=\frac{\sum x_i^2}{15}-(20)^2$

$$\Rightarrow \sum x_i^2=6950$$

$$\sigma^2=\frac{6950-40^2+50^2}{15}-(21)^2$$

$$=\frac{7850}{15}-441$$

$$=\frac{1235}{15}$$

$$=\frac{247}{3}$$

3. Matrix A having order m has the value of its determinant as $(m)^{-n}$. The value of $\det(n\,\text{adj}(\text{adj}(mA)))$ is

(1) $n^m(m^{m-n})^{(m-1)^2}$ (2) $n^m(m^{m-n})^{(m-1)}$

(3) $m^n(m^{m-n})$ (4) $n^m(m^{n-m})^2$

Answer (1)

Sol. $\det(n\,adj(adj\,(mA)))$

$$=n^m\det(adj(adj\,mA))$$

$$=n^m\cdot(\det(mA))^{(m-1)^2}$$

$$=n^m\cdot\left(m^m\det(A)\right)^{(m-1)^2}$$

$$=n^m\cdot m^{n(m-1)^2}\cdot(m^{-n})^{(m-1)^2}$$

$$=n^m\cdot(m^n)^{(n-1)^2}(m^{-n})^{(m-1)^2}$$

$$=n^m(m^{m-n})^{(m-1)^2}$$

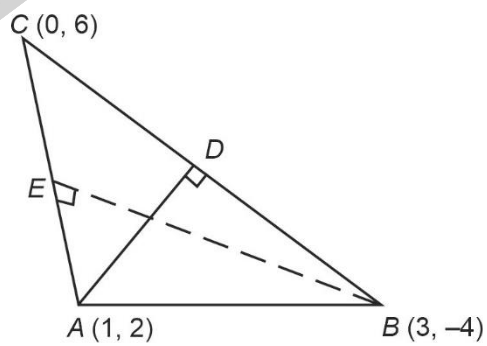
4. The orthocentre of a triangle having vertices as $A(1, 2)$, $B(3, -4)$, $C(0, 6)$ is

(1) (–129, –37) (2) (9, –1)

(3) (7, –3) (4) (28, –16)

Answer (1)

Sol.



$AD:(y-2)=\frac{3}{10}(x-1)$

$3x - 10y + 17 = 0$ ...(i)

$BE:(y+4)=\frac{1}{4}(x-3)$

$x - 4y = 19$ ...(ii)

Solving (i) and (ii)

(–129, –37) is orthocentre

5. The statement $p \wedge (q \wedge \sim (p \wedge q))$ is

(1) Tautology

(2) Fallacy

(3) Is equivalent to $p \wedge q$

(4) Is equivalent to $p \vee q$

Answer (2)

Sol. $p \wedge (q \wedge \sim (p \wedge q))$

$= p \wedge (q \wedge (\sim p \vee \sim q))$

$= p \wedge ((q \wedge \sim p) \vee (q \wedge \sim q))$

$= p \wedge \; (q \wedge \sim p)$

$=$ F

6. If we have a ATM pin of 4 digit. The Sum of first two digits is equal to sum of last two digits and the greatest integer used is 7. Then the number of trials used to get the pin if all digits are different

(1) 194

(2) 192

(3) 200

(4) 220

Answer (2)

Sol. $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}$

According to condition $a + b = c + d$.

If sum is 3 $\rightarrow$ (0, 3) (1, 2)

If sum is 4 $\rightarrow$ (0, 4), (1, 3)

If sum is 5 $\rightarrow$ (0, 5), (1, 4), (2, 3)

If sum is 6 $\rightarrow$ (0, 6), (1, 5), (2, 4)

If sum is 7 $\rightarrow$ (0, 7), (1, 6), (2, 5), (3, 4)

If sum is 8 $\rightarrow$ (1, 7), (2, 6), (3, 5)

If sum is 9 $\rightarrow$ (2, 7), (3, 6), (4, 5)

If sum is 10 $\rightarrow$ (3, 7), (4, 6)

If sum is 11 $\rightarrow$ (4, 7), (5, 6)

Now total trials $= 4 \times 2! \times 2!\ 2! + 4 \times {}^3C_2 \times 2! \times 2! \times 2! + {}^4C_2 \times 2! \times 2! \times 2!$

$= 32 + 32 \times 3 + 64$

$= 32 + 96 + 64$

$= 192$

7. 3 points $A(1, 1, 1)$, $B(-2, 3, 2)$ and $C(0, 3, 0)$ lie on a plane. Line $\frac{x-1}{-2} = \frac{y+2}{-1} = \frac{z}{4}$ intersects the plane at P. The distance OP is (O is origin) ______.

(1) $\sqrt{349}$

(2) $\sqrt{231}$

(3) $\sqrt{341}$

(4) $\sqrt{168}$

Answer (3)

Sol. Equation of plane : $\begin{vmatrix} x & y-3 & z \\ 3 & -2 & -1 \\ 1 & -2 & 1 \end{vmatrix} = 0$

$\Rightarrow\ x(-2-2) - (y-3)(3+1) + z(-6+2) = 0$

$\Rightarrow\ -4x - (y-3)4 - 4z = 0$

$\Rightarrow\ x + y - 3 + z = 0$

$\Rightarrow\ x + y + z = 3$

Point on a line : $(-2k + 1, -k - 2, 4k)$

$(-2k + 1) + (-k - 2) + 4k = 3$

$\Rightarrow\ k = 4$

$\therefore\ P(-7, -6, 16)$

$OP = \sqrt{49 + 36 + 256}$

$= \sqrt{341}$

8. $A(5, -3)$, $C(7, 8)$ and $B(t, 0)$, $0 \le t \le 4$. The perimeter is maximum at $t = \alpha$ and minimum at $t = \beta$, then $\alpha^2 + \beta^2$ is ____

(1) 12

(2) 9

(3) 16

(4) 25

Answer (3)

Sol. perimeter $= AC + BC + AB$

$(\text{perimeter})^2 = 5\sqrt{5} + (t-7)^2 + 64 + (t-5)^2 + 9$

$= 73 + 5\sqrt{5} + 2t^2 - 24t + 74$

$\Rightarrow\ 2t^2 - 24t + 147 + 5\sqrt{5}$

$\Rightarrow\ 2(t-6)^2 + 75 + 5\sqrt{5}$

$(\text{perimeter})^2_{\text{max}}$ at $t = 0 = \alpha$

$(\text{perimeter})^2_{\text{min}}$ at $t = 4 = \beta$

$\therefore\ \alpha^2 + \beta^2 = 16$

9. Consider the circles $x^2 + y^2 - 13x - 15y + 13 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, then number of common tangents is

(1) 2 (2) 0

(3) 1 (4) 4

Answer (1)

Sol : $c_1 \equiv \left(\frac{13}{2}, \frac{15}{2}\right)$ $c_2 \equiv (3, 3)$

$$r_1 = \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{15}{2}\right)^2 - 13}\ ;\ \ 9$$

$$r_2 = \sqrt{9+9+7} = 5$$

and $c_1c_2 = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{9}{2}\right)^2}$

So, $|c_1c_2| < r_1 + r_2$

$\therefore$ Total common tangents = 2

10. $f(x) = \int \frac{dx}{\sqrt{4-3x^2}\left(4x^2+3\right)}$, then $f(x)$ =

(1) $-\frac{1}{25}\left(\frac{\log\left(\frac{4}{x^2}-3\right)}{2} - \frac{\log\left(\frac{12}{x^2}+16\right)}{6}\right)+c$

(2) $\frac{1}{25}\left(\frac{\log\left(4-x^2\right)}{4} - \frac{\log\left(x^2-16\right)}{6}\right)$

(3) $-\frac{1}{25}\left[\log\left(4-3x^2\right)+\log\left(3x^2-16\right)\right]$

(4) $-\frac{1}{25}\left(\frac{\log\left(4-3x^2\right)}{2} + \frac{\log\left(12-16x^2\right)}{6}\right)$

Answer (1)

Sol. Let $x = \frac{1}{t}$

$$dx = -\frac{1}{t^2}dt$$

$$\int \frac{\frac{-1}{t^2}dt}{\left(\sqrt{4-\frac{3}{t^2}}\right)\left(\frac{4}{t^2}+3\right)}$$

$$\Rightarrow \quad -\int \frac{tdt}{\sqrt{4t^2-3}\left(4+3t^2\right)}$$

$4t^2 - 3 = m^2$

$\Rightarrow$ $8t\,dt = 2\,mdm$

$$= -\frac{1}{4}\int \frac{dm}{m\left(4+3\left(\frac{m^2+3}{4}\right)\right)}$$

$$= -\frac{1}{4}\int \frac{4dm}{m\left(3m^2+25\right)}$$

$$-\frac{1}{25}\int \frac{\left(3m^2+25\right)-m^2}{m\left(3m^2+25\right)}dm$$

$$= -\frac{1}{25}\int\left(\frac{1}{m} - \frac{m}{3m^2+25}\right)dm =$$

$$-\frac{1}{25}\left(\log m - \frac{\log\left(3m^2+25\right)}{6}\right)+c$$

$$\Rightarrow \quad -\frac{1}{25}\left(\log\sqrt{4t^2-3} - \frac{\log\left(3\left(4t^2-3\right)+25\right)}{6}\right)+c$$

$$\Rightarrow \quad -\frac{1}{25}\left(\log\sqrt{4t^2-3} - \frac{\log\left(12t^2+16\right)}{6}\right)+c$$

$$= -\frac{1}{25}\left[\frac{\log\left(\frac{4}{x^2}-3\right)}{2} - \frac{\log\left(\frac{12}{x^2}+16\right)}{6}\right]+c$$

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The number of solution of equation $x|x| + 5|x + 2| + 6 = 0$ is

Answer (01)

Sol. **Case I**

$x < -2$

$-x^2 - 5(x + 2) + 6 = 0$

$x^2 + 5x + 4 = 0$

$(x + 1)(x + 4) = 0$

$x = \underset{\text{(rejected)}}{-1}$ or -4

$\therefore x = -4$ is solution

Case II

$-2 < x < 0$

$-x^2 + 5(x + 2) + 6 = 0$

$x^2 - 5x - 16 = 0$

$$x = \frac{5 \pm \sqrt{25 + 64}}{2} = \frac{5 \pm \sqrt{89}}{2}$$

$\therefore$ No solution between $-2 < x < 0$

Case III

For $x > 0$

$x^2 + 5(x + 2) + 6 = 0$

$x^2 + 5x + 16 = 0$

$D < 0$

$\therefore$ No solution

$\Rightarrow$ Only one solution *i.e.*, $x = -4$

22. Let $f(x) = \log(4x^2 + 11x + 9) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ and if domain of $f(x)$ is $[\alpha, \beta]$, then $|10[\alpha - \beta]|$ is

Answer (04)

Sol. $4x^2 + 11x + 9 > 0$ $\quad (\because\ 0 = 121 - 144 < 0)$

So, $-1 \le 4x + 3 \le 1$ and $-1 \le \dfrac{10x+6}{3} \le 1$

$-4 \le 4x \le -2$ $\qquad$ $-9 \le 10x \le -3$

$-1 \le x \le -\dfrac{1}{2}$ $\qquad$ $-\dfrac{9}{10} \le x \le \dfrac{-3}{10}$

So, $D_f = \left[\dfrac{-9}{10}, \dfrac{-1}{2}\right]$

$\therefore \quad \alpha = \dfrac{-9}{10} \quad \beta = \dfrac{-1}{2}$

So, $\left|10\left(\dfrac{-9}{10} + \dfrac{1}{2}\right)\right| = 4$

23. 23. How many three-digit number can be formed which are divisible by 3 using the digits 1, 3, 5, 8 and repeatation is allowed

Answer (22)

Sol. I : All three digits are alike

1 1 1, 3 3 3, 5 5 5, 8 8 8 $\rightarrow$ 4

II : 2 digits are alike

5 5 8 $\rightarrow \dfrac{3!}{2!} = 3$

8 8 5 $\rightarrow \dfrac{3!}{2!} = 3$

III : All three digits are different

1, 3, 5 $\rightarrow$ 6

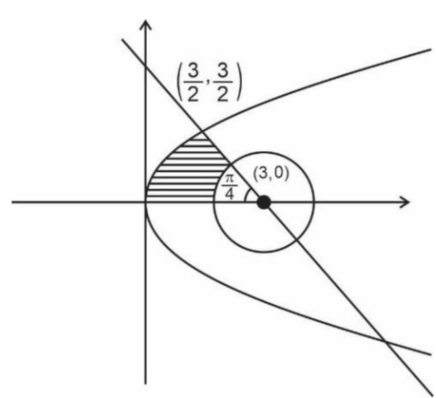
1, 3, 8 $\rightarrow$ 6

$\therefore$ Total numbers = 22

24. Area bounded by the curve $2y^2 = 3x$ and the line $x + y = 3$ outside the circle $(x - 3)^2 + y^2 = 2$ and above x-axis is A. The value of $4(\pi + 4A)$ is

Answer (42)

Sol.



A = required area

$$= \int_0^{\frac{3}{2}} \left[(3-y) - \left(\frac{2y^2}{3}\right)\right] dy - \pi\left(\sqrt{2}\right)^2 \cdot \frac{1}{8}$$

$$\Rightarrow \left(3y - \frac{y^2}{2} - \frac{2}{9}y^3\right)\Bigg|_0^{\frac{3}{2}} - \frac{\pi}{4}$$

$$\Rightarrow 3\cdot\frac{3}{2} - \frac{9}{8} - \frac{2}{9}\cdot\frac{27}{8} - \frac{\pi}{4}$$

$$\Rightarrow \frac{36-9-6}{8} - \frac{\pi}{4} = \frac{21}{8} - \frac{\pi}{4}$$

$\Rightarrow$ $4(\pi + 4A)$

$$= 4\left(\frac{21}{2}\right) = 42$$

25. If $n \in [10, 100]$ and $n \in N$, then how many such n are possible where $3^n - 3$ is divisible by 7?

Answer (15)

Sol. $3^n - 3 = 7K, \quad K \in I$

$3^n = 7K + 3$

Now,

$3 \equiv 3 \pmod 7$

$3^2 \equiv 2 \pmod 7$

$3^3 \equiv -1 \pmod 7$

$3^6 \equiv 1 \pmod 7$

$3^7 \equiv 3 \pmod 7$

Since,

$3^{13} \equiv 3 \pmod 7$

$\vdots$

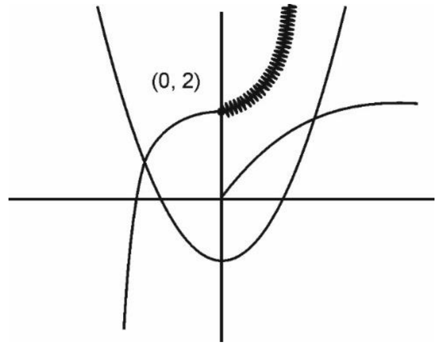
$\therefore$ n can be 13, 19, ……97

$\therefore$ Total $\boxed{n = 15}$

26. If $y = \max\left\{\sqrt{x}, x^2 - 4, x^3 + 2\right\}$, then number of solution(s) of $y = 1$ is/are ____.

Answer (0)

Sol.



As domain of y is $[0, \infty)$

$\therefore$ $y = \max\left(\sqrt{x}, x^2 - 4, x^3 + 2\right) = x^3 + 2$

$\forall x \in [0, \infty)$

$\therefore$ $x^3 + 2 = 1$

$\Rightarrow$ $x^3 = -1$

No solution in $[0, \infty)$

27. Let $A = \{1, 2, 3, 4\}$ if R on a set $A \times A$ such that $(a, b)\ R\ (c, d)$ iff $2a + 3b = 6c + 5d$, then number of elements in R is

Answer (04)

Sol. Maximum value of $2a + 3b = 20$ at (4, 4)

Minimum value of $6c + 5d = 11$ at (1, 1)

So, $6c + 5d$ can be 11, 16, 17

So, $2a + 3b = 11$

$(a, b) \equiv (4, 1), (1, 3)$

and

$2a + 3b = 16$ $\qquad$ $(6c + 5d = 16)$ (1, 2)

$(a, b) \equiv (2, 4)$

$2a + 3b = 17$

$(a, b) \equiv (4, 3)$

So, total elements = 4

28. If $f(x) = \max.\{1 + x + [x], x + 1, 1 - x + [x]\}$, $0 \le x \le 2$, then number of points where $f(x)$ is non-differentiable is

Answer (01)

Sol. $f(x) = \max.\{1 + x + [x], x + 1, 1 - x + [x]\}$

$$= \begin{cases} 1 + x, & 0 \ge x < 1 \\ 1 + x + [x] & 1 \le x \le 2 \end{cases}$$

$\therefore$ Number of points of non-differentiability = 1 (at $x = 1$)

29.

30.

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