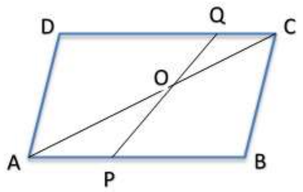
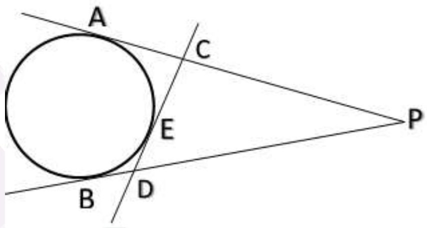
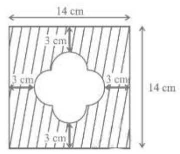
Marking Scheme
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	(b) xy^2	1
2.	(b) 1 zero and the zero is '3'	1
3.	(b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	1
4.	(c) 2 distinct real roots	1
5.	(c) 7	1
6.	(a) 1:2	1
7.	(d) infinitely many	1
8.	(b) $\frac{ac}{b+c}$	1
9.	(b) 100°	1
10.	(d) 11 cm	1
11.	(c) $\frac{\sqrt{b^2-a^2}}{b}$	1
12.	(d) cos A	1
13.	(d) 60°	1
14.	(a) 2 units	1
15.	(a) 10m	1
16.	(b) $\frac{4-\pi}{4}$	1
17.	(b) $\frac{22}{46}$	1
18.	(d) 150	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers *a* and *b* such that $\sqrt{2} = \frac{a}{b}$ where *a* and *b* are coprime. So, $b\sqrt{2} = a$. Squaring both sides, we get $2b^2 = a^2$. Therefore, 2 divides a^2 and so 2 divides a. So, we can write a = 2c for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2, and so 2 divides b Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	½ ½ ½ ½

22.	ABCD is a parallelogram. AB = DC = a Point P divides AB in the ratio 2:3 AP = $\frac{2}{5}$ a , BP = $\frac{3}{5}$ a point Q divides DC in the ratio 4:1. DQ = $\frac{4}{5}$ a , CQ = $\frac{1}{5}$ a Δ APO ~ Δ CQO [AA similarity] $\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$ $\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow OC = ½\, OA$ 	½ ½ ½ ½
23.	PA = PB; CA = CE; DE = DB [Tangents to a circle] Perimeter of ΔPCD = PC + CD + PD = PC + CE + ED + PD = PC + CA + BD + PD = PA + PB Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm 	½ 1 ½
24.	∵ $\tan(A+B) = \sqrt{3}$ ∴ $A + B = 60^0$ ...(1) ∵ $\tan(A-B) = \frac{1}{\sqrt{3}}$ ∴ $A - B = 30^0$ ...(2) Adding (1) & (2), we get 2A=$90^0 \Longrightarrow A = 45^0$ Also (1) –(2), we get $2B = 30^0 \Longrightarrow B = 45^0$	½ ½ ½ ½
	[or]	
	2 cosec²30 + x sin²60 – $\frac{3}{4}$ tan²30 = 10 $\Rightarrow \quad 2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$ $\Rightarrow \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$ $\Rightarrow \quad 8 + x\left(\frac{3}{4}\right) - \frac{1}{4} = 10$ $\Rightarrow \quad 32 + x(3) - 1 = 40$ $\Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$	1 ½ ½
25.	Total area removed = $\frac{\angle A}{360}\pi r^2 + \frac{\angle B}{360}\pi r^2 + \frac{\angle C}{360}\pi r^2$ $= \frac{\angle A + \angle B + \angle C}{360}\pi r^2$ $= \frac{180}{360}\pi r^2$ $= \frac{180}{360} X \frac{22}{7} X (14)^2$ ½ = 308 cm²	½ ½ ½
	[or]	
	The side of a square = Diameter of the semi-circle = a Area of the unshaded region = Area of a square of side 'a' + 4(Area of a semi-circle of diameter 'a') The horizontal/vertical extent of the white region = 14-3-3 = 8 cm Radius of the semi-circle + side of a square + Radius of the semi-circle = 8 cm 	½ ½

	2 (radius of the semi-circle) + side of a square = 8 cm $2a = 8$ cm $\Rightarrow a = 4$ cm Area of the unshaded region = Area of a square of side 4 cm + 4 (Area of a semi-circle of diameter 4 cm) $= (4)^2 + 4 \times \frac{1}{2}\pi(2)^2 = 16 + 8\pi$ cm^2	½ ½
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Number of students in each group subject to the given condition = HCF (60,84,108) HCF (60,84,108) = 12 Number of groups in Music $= \frac{60}{12} = 5$ Number of groups in Dance $= \frac{84}{12} = 7$ Number of groups in Handicrafts $= \frac{108}{12} = 9$ Total number of rooms required = 21	½ ½ ½ ½ ½ ½
27.	$P(x) = 5x^2 + 5x + 1$ $\alpha+\beta = \frac{-b}{a} = \frac{-5}{5} = -1$ $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= (-1)^2 - 2\left(\frac{1}{5}\right)$ $= 1 - \frac{2}{5} = \frac{3}{5}$ $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ $= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$	½ ½ ½ ½ ½ ½
28.	Let the ten's and the unit's digits in the first number be x and y, respectively. So, the original number = 10x + y When the digits are reversed, x becomes the unit's digit and y becomes the ten's Digit. So the obtain by reversing the digits= 10y + x According to the given condition. (10x + y) + (10y + x) = 66 i.e., 11(x + y) = 66 i.e., x + y = 6 ---- (1) We are also given that the digits differ by 2, therefore, either x – y = 2 ---- (2) or y – x = 2 ---- (3) If x – y = 2, then solving (1) and (2) by elimination, we get x = 4 and y = 2. In this case, we get the number 42. If y – x = 2, then solving (1) and (3) by elimination, we get x = 2 and y = 4. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	½ ½ ½ ½ ½ ½
	[or]	
	Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n', Then the given equations become 2m + 3n = 2 4m - 9n = -1	½ ½

$(2m + 3n = 2)$ X- 2 $\Rightarrow -4m - 6n = -4$ ...(1)

$4m - 9n = -1$ $4m - 9n = -1$ ...(2)

Adding (1) and (2)

We get $-15n = -5 \Rightarrow n = \frac{1}{3}$ ½

Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get ½

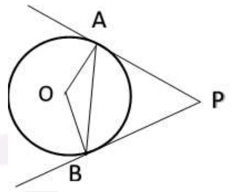
$2m + 1 = 2$

$2m = 1$

$m = \frac{1}{2}$ 1

$m = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ and $n = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$

29.



$\angle OAB = 30°$

$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle PAB = 90° - 30° = 60°$ ½

AP = BP [Tangents to a circle from an external point]

$\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle] ½

In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$ [Angle Sum Property]

$60° + 60° + \angle APB = 180°$

$\angle APB = 60°$ ½

$\therefore$ ΔABP is an equilateral triangle, where AP = BP = AB.

PA = 6 cm ½

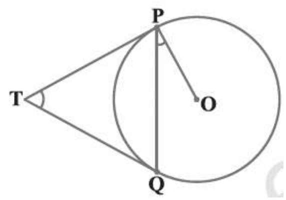
In Right ΔOAP, $\angle OPA = 30°$

$\tan 30° = \frac{OA}{PA}$

$\frac{1}{\sqrt{3}} = \frac{OA}{6}$ ½

$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3}cm$ ½

[or]



Let $\angle TPQ = \theta$

$\angle TPO = 90°$ [Angle between the tangent and the radius at the point of contact] ½

$\angle OPQ = 90° - \theta$

TP = TQ [Tangents to a circle from an external point] ½

$\angle TPQ = \angle TQP = \theta$ [Angles opposite to equal sides of a triangle] ½

In ΔPQT, $\angle PQT + \angle QPT + \angle PTQ = 180°$ [Angle Sum Property] ½

$\theta + \theta + \angle PTQ = 180°$

$\angle PTQ = 180° - 2\theta$ ½

$\angle PTQ = 2(90° - \theta)$ ½

$\angle PTQ = 2\angle OPQ$ [using (1)]

30. Given, $1 + \sin^2\theta = 3 \sin\theta \cos\theta$

Dividing both sides by $\cos^2\theta$,

$\frac{1}{\cos^2\theta} + \tan^2\theta = 3\tan\theta$

$\sec^2\theta + \tan^2\theta = 3\tan\theta$ ½

$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$ ½

$1 + 2\tan^2\theta = 3\tan\theta$ ½

$2\tan^2\theta - 3\tan\theta + 1 = 0$ ½

If $\tan\theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$

	$\Rightarrow (x-1)(2x-1) = 0$ x = 1 or $\frac{1}{2}$ tan θ = 1 or $\frac{1}{2}$	1
31.	(table below)	2 ½ ½

Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd
118 – 126	3	117.5- 126.5	122	-27	-81
127 – 135	5	126.5– 135.5	131	-18	-90
136 – 144	9	135.5– 144.5	140	-9	-81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

Mean = a + $\frac{\sum fd}{\sum f}$ = 149 + $\frac{-81}{40}$

= 149 – 2.025 = 146.975

Average length of the leaves = 146.975

SECTION D

Section D consists of 4 questions of 5 marks each

32. Let the speed of the stream be x km/h.

The speed of the boat upstream = (18 – x) km/h and

the speed of the boat downstream = (18 + x) km/h. 1

The time taken to go upstream = $\frac{distance}{speed} = \frac{24}{18-x}$ hours

the time taken to go downstream = $\frac{distance}{speed} = \frac{24}{18+x}$ hours 1

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ 1

24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

x^2 + 48x – 324 = 0

x = 6 or – 54 1

Since x is the speed of the stream, it cannot be negative.

Therefore, x = 6 gives the speed of the stream = 6 km/h. 1

[or]

Let the time taken by the smaller pipe to fill the tank = x hr.

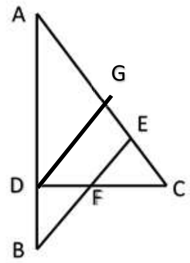
Time taken by the larger pipe = (x – 10) hr ½

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

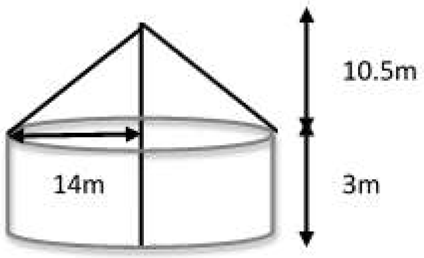
Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$ 1

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. ½

Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$ ½

	Therefore, $\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$ $8x^2 - 230x + 750 = 0$ $x = 25, \frac{30}{8}$ Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible. Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 =15 hours, respectively.	½ 1 ½ ½
33.	(a) Statement – ½ Given and To Prove – ½ Figure and Construction ½ Proof – 1 ½  [b] Draw DG ‖ BE In Δ ABE, $\frac{AB}{BD}=\frac{AE}{GE}$ [BPT] CF = FD [F is the midpoint of DC] ---(i) In Δ CDG, $\frac{DF}{CF}=\frac{GE}{CE}=1$ [Mid point theorem] GE = CE ---(ii) ∠CEF = ∠CFE [Given] CF = CE [Sides opposite to equal angles] ---(iii) From (ii) & (iii) CF = GE ---(iv) From (i) & (iv) GE = FD $\therefore \frac{AB}{BD}=\frac{AE}{GE} \Rightarrow \frac{AB}{BD}=\frac{AE}{FD}$	3 ½ ½ ½ ½
34.	Length of the pond, l= 50m, width of the pond, b = 44m Water level is to rise by, h = 21 cm = $\frac{21}{100}$ m Volume of water in the pond = lbh = 50 x 44 x $\frac{21}{100}$ m^3 =462 m^3 Diameter of the pipe = 14 cm Radius of the pipe, r = 7cm = $\frac{7}{100}$ m Area of cross-section of pipe = πr^2 $= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000}$ m^2 Rate at which the water is flowing through the pipe, h = 15km/h = 15000 m/h Volume of water flowing in 1 hour = Area of cross-section of pipe x height of water coming out of pipe $= \left(\frac{154}{10000} \times 15000\right) m^3$ Time required to fill the pond $= \frac{\textit{Volume of the pond}}{\textit{Volume of water flowing in 1 hour}}$ $= \frac{462 \times 10000}{154 \times 15000} = 2$ hours Speed of water if the rise in water level is to be attained in 1 hour = 30km/h	1 1 ½ ½ 1 1
	[or]	

Radius of the cylindrical tent (r) = 14 m
Total height of the tent = 13.5 m
Height of the cylinder = 3 m
Height of the Conical part = 10.5 m ½



Slant height of the cone (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$ 1

$= \sqrt{306.25} = 17.5$ m

Curved surface area of cylindrical portion

$= 2\pi rh$

$= 2x\frac{22}{7}\times 14\times 3$ 1

$= 264$ m^2

Curved surface area of conical portion

$=\pi rl$

$= \frac{22}{7}\times 14\times 17.5$ 1

$=770$ m^2 ½

Total curved surface area = 264 m^2 + 770 m^2 = 1034 m^2
Provision for stitching and wastage = 26 m^2
½
Area of canvas to be purchased = 1060 m^2
Cost of canvas = Rate × Surface area ½

= 500 x 1060 = ₹ 5,30,000/-

35.

Marks obtained	Number of students	Cumulative frequency
20 – 30	p	p
30 – 40	15	p + 15
40 – 50	25	p + 40
50 – 60	20	p + 60
60 – 70	q	p + q + 60
70 – 80	8	p + q + 68
80 - 90	10	p + q + 78
	90	

1
½
½

p + q + 78 = 90
p + q = 12

Median $=(l) + \frac{\frac{n}{2} - cf}{f} \cdot h$

$50 = 50 + \frac{45-(p+40)}{20} . 10$ ½

$\frac{45-(p+40)}{20} . 10 = 0$ ½

$45 - (p + 40) = 0$ ½

P = 5 ½

5 + q = 12

q = 7 1

Mode $= l + \frac{f1-f0}{2f1-f0-f2} . h$

	$= 40 + \frac{25-15}{2(25)-15-20} \cdot 10$ $= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$	
	SECTION E	
36.	(i) Number of throws during camp. a = 40; d = 12 $t_{11} = a + 10d$ $= 40 + 10 \times 12$ $= 160\ throws$	1
	(ii) a = 7.56 m; d = 9cm = 0.09 m n = 6 weeks $t_n = a + (n-1)\ d$ = 7.56 + 6(0.09) = 7.56 + 0.54 Sanjitha's throw distance at the end of 6 weeks = 8.1 m (or) a = 7.56 m; d = 9cm = 0.09 m t_n =11.16 m $t_n = a + (n-1)\ d$ 11.16 = 7.56 + (n-1) (0.09) 3.6 = (n-1) (0.09) $n-1 = \frac{3.6}{0.09} = 40$ n = 41 Sanjitha's will be able to throw 11.16 m in 41 weeks.	½ ½ ½ ½ ½ ½ ½ ½
	(iii) a = 40; d = 12; n = 15 $S_n = \frac{n}{2}\ [2a + (n-1)\ d]$ $S_n = \frac{15}{2}\ [2(40) + (15-1)\ (12)]$ $= \frac{15}{2}\ [80 + 168]$ $= \frac{15}{2}\ [248]$ =1860 throws	½ ½
37.	(i) Let D be (a,b), then Mid point of AC = Midpoint of BD $\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$ 4 + a = 7 3+ b = 8 a = 3 b = 5 Central midfielder is at (3,5)	½ ½

	(ii) GH = $\sqrt{(-3-3)^2+(5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ GK = $\sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$ HK = $\sqrt{(3-0)^2+(1-3)^2} = \sqrt{9+4} = \sqrt{13}$ GK +HK = GH ⇒G,H & K lie on a same straight line [or] CJ = $\sqrt{(0-5)^2+(1+3)^2} = \sqrt{25+16} = \sqrt{41}$ CI = $\sqrt{(0+4)^2+(1-6)^2} = \sqrt{16+25} = \sqrt{41}$ Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) Mid-point of IJ = $\left(\frac{5-4}{2}, \frac{-3+6}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$ C is NOT the mid-point of IJ	½ ½ ½ ½ ½ ½ ½ ½
	(iii) A,B and E lie on the same straight line and B is equidistant from A and E ⇒ B is the mid-point of AE $\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2,-3)$ $1+a = 4$; a = 3. 4+b = -6; b = -10 E is (3,-10)	 ½ ½
38.	(i) tan 45° = $\frac{80}{CB}$ ⇒ CB = 80m	1
	(ii) tan 30° = $\frac{80}{CE}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{80}{CE}$ ⇒ CE = $80\sqrt{3}$ Distance the bird flew = AD = BE = CE-CB = $80\sqrt{3} - 80 = 80(\sqrt{3}-1)$ m (or) tan 60° = $\frac{80}{CG}$ ⇒ $\sqrt{3} = \frac{80}{CG}$ ⇒ CG = $\frac{80}{\sqrt{3}}$ Distance the ball travelled after hitting the tree =FA=GB = CB -CG GB = 80 - $\frac{80}{\sqrt{3}} = 80\left(1 - \frac{1}{\sqrt{3}}\right)$ m	½ ½ ½ ½ ½ ½ ½ ½
	(iii) Speed of the bird = $\frac{Distance}{Time\ taken} = \frac{20(\sqrt{3}+1)}{2}$ m/sec $= \frac{20(\sqrt{3}+1)}{2}$ x 60 m/min = $600(\sqrt{3}+1)$ m/min	½ ½