MISCELLANEOUS EXERCISE

Solve the inequalities in Exercises 1 to 6

1. 2 ≤ 3x – 4 ≤ 5

Solution:

According to the question,

The inequality given is,

2 ≤ 3x – 4 ≤ 5

⇒ 2 ≤ 3x – 4 ≤ 5

⇒ 2 + 4 ≤ 3x – 4 + 4 ≤ 5 + 4

⇒ 6 ≤ 3x ≤ 9

⇒ 6/3 ≤ 3x/3 ≤ 9/3

⇒ 2 ≤ x ≤ 3

Hence, all real numbers x greater than or equal to 2, but less than or equal to 3 are solutions of given equality.

x ∈ [2, 3]

2. 6 ≤ –3 (2x – 4) < 12

Solution:

According to the question,

The inequality given is,

6 ≤ –3 (2x – 4) < 12

⇒ 6 ≤ -3 (2x – 4) < 12

Dividing the inequality by 3, we get.

⇒ 2 ≤ – (2x – 4) < 4

Multiplying the inequality by -1,

⇒ -2 ≥ 2x – 4 > -4 [multiplying the inequality with -1 changes the inequality sign.]

⇒ -2 + 4 ≥ 2x – 4 + 4 > -4 + 4

⇒ 2 ≥ 2x > 0

Dividing the inequality by 2,

⇒ 0 < x ≤ 1

Hence, all real numbers x greater than 0, but less than or equal to 1 are solutions of given equality.

x ∈ (0, 1]

3. – 3 ≤ 4 – 7x/2 ≤ 18

Solution:

According to the question,

The inequality given is,

– 3 ≤ 4 – 7x/2 ≤ 18

⇒ – 3 – 4 ≤ 4 – 7x/2 – 4 ≤ 18 – 4

⇒ – 7 ≤ – 7x/2 ≤ 18 – 14

Multiplying the inequality by -2,

$$\Rightarrow (-7) \times (-2) \geq -\frac{7x}{2} \times (-2) \geq 14 \times (-2)$$

⇒ 14 ≥ 7x ≥ -28

⇒ -28 ≤ 7x ≤ 14

Dividing the inequality by 7,

⇒ -4 ≤ x ≤ 2

Hence, all real numbers x greater than or equal to -4, but less than or equal to 2 are solutions of given equality.

x ∈ [-4, 2]

4. – 15 ≤ 3(x – 2)/5 ≤ 0

Solution:

According to the question,

The inequality given is,

– 15 ≤ 3(x – 2)/5 ≤ 0

⇒ – 15 < 3(x – 2)/5 ≤ 0

Multiplying the inequality by 5,

$$\Rightarrow -15 \times 5 < \frac{3(x-2)}{5} \times 5 \leq 0 \times 5$$

⇒ -75 < 3(x – 2) ≤ 0

Dividing the inequality by 3, we get,

$$\Rightarrow -\frac{75}{3} < \frac{3(x-2)}{3} \leq \frac{0}{3}$$

⇒ -25 < x – 2 ≤ 0

⇒ – 25 + 2 < x – 2 + 2 ≤ 0 + 2

⇒ – 23 < x ≤ 2

Hence, all real numbers x greater than -23, but less than or equal to 2 are solutions of given equality.

x ∈ (-23, 2]

5. – 12 < 4 – 3x/ (-5) ≤ 2

Solution:

According to the question,

The inequality given is,

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 - 4 < 4 - \frac{3x}{-5} - 4 \leq 2 - 4$$

$$\Rightarrow -16 < \frac{-3x}{-5} \leq -2$$

$$\Rightarrow -16 < \frac{3x}{5} \leq -2$$

Multiplying the inequality by 5.

$$\Rightarrow -16 \times 5 < \frac{3x}{5} \times 5 \leq -2 \times 5$$

$$\Rightarrow -80 < 3x \leq -10$$

$$\Rightarrow -\frac{80}{3} < x \leq -\frac{10}{3}$$

Hence, all real numbers x greater than -80/3, but less than or equal to -10/3 are solutions of given equality.

x ∈ (-80/3, -10/3]

6. 7 ≤ (3x + 11)/2 ≤ 11

Solution:

According to the question,

The inequality given is,

$$7 \leq \frac{(3x+11)}{2} \leq 11$$

$$\Rightarrow 7 \leq \frac{(3x+11)}{2} \leq 11$$

Multiplying the inequality by 2.

$$\Rightarrow 7 \times 2 \leq \frac{(3x+11)}{2} \times 2 \leq 11 \times 2$$

$$\Rightarrow 14 \leq 3x + 11 \leq 22$$

⇒ 14 – 11 ≤ 3x + 11 – 11 ≤ 22 – 11

⇒ 3 ≤ 3x ≤ 11

⇒ 1 ≤ x ≤ 11/3

Hence, all real numbers x greater than or equal to -4, but less than or equal to 2 are solutions of given equality.

x ∈ [1, 11/3]

Solve the inequalities in Exercises 7 to 11 and represent the solution graphically on number line.

7. 5x + 1 > – 24, 5x – 1 < 24

Solution:

According to the question,

The inequalities given are,

5x + 1 > -24 and 5x – 1 < 24

5x + 1 > -24

⇒ 5x > -24 – 1

⇒ 5x > -25

⇒ x > -5 …… (i)

5x – 1 < 24

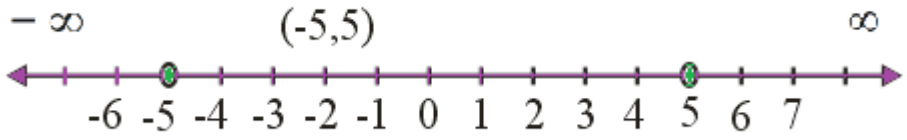
⇒ 5x < 24 + 1

⇒ 5x < 25

⇒ x < 5 …….(ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is (-5, 5).



8. 2 (x – 1) < x + 5, 3 (x + 2) > 2 – x

Solution:

According to the question,

The inequalities given are,

2 (x – 1) < x + 5 and 3 (x + 2) > 2 – x

2 (x – 1) < x + 5

⇒ 2x – 2 < x + 5

⇒ 2x – x < 5 + 2

⇒ x < 7 …… (i)

3 (x + 2) > 2 – x

⇒ 3x + 6 > 2 – x

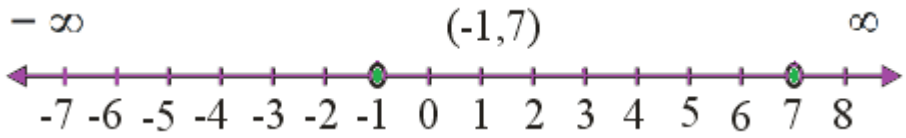
⇒ 3x + x > 2 – 6

⇒ 4x > -4

⇒ x > -1 ……. (ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is (-1, 7).



9. 3x – 7 > 2(x – 6), 6 – x > 11 – 2x

Solution:

According to the question,

The inequalities given are,

3x – 7 > 2(x – 6) and 6 – x > 11 – 2x

3x – 7 > 2(x – 6)

⇒ 3x – 7 > 2x – 12

⇒ 3x – 2x > 7 – 12

⇒ x > -5 …… (i)

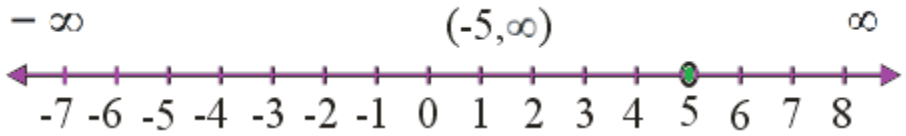
6 – x > 11 – 2x

⇒ 2x – x > 11 – 6

⇒ x > 5 …….(ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is (5, ∞).



10. 5(2x – 7) – 3(2x + 3) ≤ 0, 2x + 19 ≤ 6x + 47

Solution:

According to the question,

The inequalities given are,

5(2x – 7) – 3(2x + 3) ≤ 0 and 2x + 19 ≤ 6x + 47

5(2x – 7) – 3(2x + 3) ≤ 0

⇒ 10x – 35 – 6x – 9 ≤ 0

⇒ 4x – 44 ≤ 0

⇒ 4x ≤ 44

⇒ x ≤ 11 ……(i)

2x + 19 ≤ 6x +47

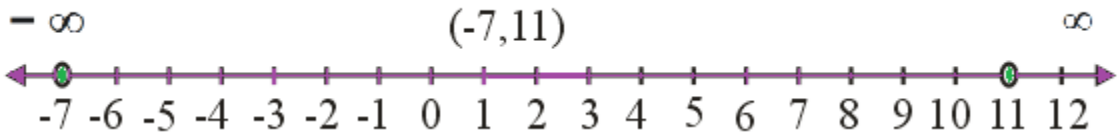
⇒ 6x – 2x ≥ 19 – 47

⇒ 4x ≥ -28

⇒ x ≥ -7 …….(ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is (-7, 11).



11. A solution is to be kept between 68° F and 77° F. What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by F = (9/5) C + 32?

Solution:

According to the question,

The solution has to be kept between 68° F and 77° F

So, we get, 68° < F < 77°

Substituting,

$$F = \frac{9}{5}C + 32$$

$$\Rightarrow 68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \frac{9}{5}C + 32 - 32 < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < \frac{9}{5}C \times \frac{5}{9} < 45 \times \frac{5}{9}$$

⇒ 20 < C < 25

Hence, we get,

The range of temperature in degree Celsius is between 20° C to 25° C.

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4%, but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution:

According to the question,

8% of solution of boric acid = 640 litres

Let the amount of 2% boric acid solution added = x litres

Then we have,

Total mixture = x + 640 litres

We know that,

The resulting mixture has to be more than 4% but less than 6% boric acid.

∴ 2% of x + 8% of 640 > 4% of (x + 640) and

2% of x + 8% of 640 < 6% of (x + 640)

2% of x + 8% of 640 > 4% of (x + 640)

⇒ (2/100) × x + (8/100) × 640 > (4/100) × (x + 640)

⇒ 2x + 5120 > 4x + 2560

⇒ 5120 – 2560 > 4x – 2x

⇒ 2560 > 2x

⇒ x < 1280 ….(i)

2% of x + 8% of 640 < 6% of (x + 640)

⇒ (2/100) × x + (8/100) × 640 < (6/100) × (x + 640)

⇒ 2x + 5120 < 6x + 3840

⇒ 6x – 2x > 5120 – 3840

⇒ 4x > 1280

⇒ x > 320 …….(i)

From (i) and (ii)

320 < x < 1280

Therefore, the number of litres of 2% of boric acid solution that has to be added will be more than 320 litres but less than 1280 litres.

13. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Solution:

According to the question,

45% of solution of acid = 1125 litres

Let the amount of water added = x litres

Resulting mixture = x + 1125 litres

We know that,

The resulting mixture has to be more than 25% but less than 30% acid content.

Amount of acid in resulting mixture = 45% of 1125 litres.

∴ 45% of 1125 < 30% of (x + 1125) and 45% of 1125 > 25% of (x + 1125)

45% of 1125 < 30% of (x + 1125)

$$\Rightarrow \frac{45}{100} \times 1125 < \frac{30}{100} \times (x + 1125)$$

⇒ 45 × 1125 < 30x + 30 × 1125

⇒ (45 – 30) × 1125 < 30x

⇒ 15 × 1125 < 30x

⇒ x > 562.5 ……..(i)

45% of 1125 > 25% of (x + 1125)

$$\Rightarrow \frac{45}{100} \times 1125 > \frac{25}{100} \times (x + 1125)$$

⇒ 45 × 1125 > 25x + 25 × 1125

⇒ (45 – 25) × 1125 > 25x

⇒ 25x < 20 × 1125

⇒ x < 900 …..(ii)

∴ 562.5 < x < 900

Therefore, the number of litres of water that has to be added will have to be more than 562.5 litres but less than 900 litres.

14. IQ of a person is given by the formula

$IQ = \frac{MA}{CA} \times 100,$ **, Where MA is mental age and CA is chronological age. If 80 ≤ IQ ≤ 140 for a group of 12-year-old children, find the range of their mental age.**

Solution:

According to the question,

Chronological age = CA = 12 years

IQ for age group of 12 is 80 ≤ IQ ≤ 140.

We get that,

80 ≤ IQ ≤ 140

Substituting,

$$IQ = \frac{MA}{CA} \times 100$$

We get,

$$\Rightarrow 80 \leq \frac{MA}{CA} \times 100 \leq 140$$

$$\Rightarrow 80 \leq \frac{MA}{12} \times 100 \leq 140$$

$$\Rightarrow 80 \times \frac{12}{100} \leq \frac{MA}{12} \times 100 \leq 140 \times \frac{12}{100}$$

⇒ 9.6 ≤ MA ≤ 16.8

∴ Range of mental age of the group of 12 year-old-children is 9.6 ≤ MA ≤ 16.8