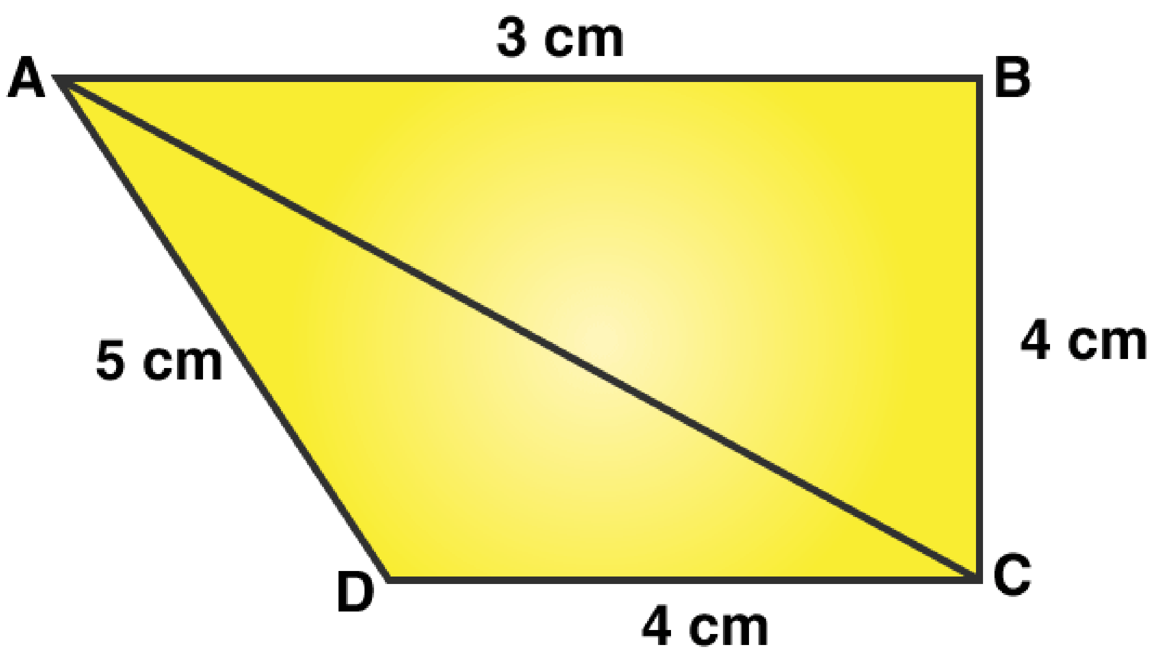
EXERCISE 12.2 PAGE NO: 12.19

Question 1: Find the area of the quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Solution:



Area of the quadrilateral ABCD = Area of △ABC + Area of △ADC ….(1)

△ABC is a right-angled triangle, which is right-angled at B.

Area of △ABC = 1/2 x Base x Height

= 1/2×AB×BC

= 1/2×3×4

= 6

Area of △ABC = 6 cm^2 ……(2)

Now, In △CAD,

Sides are given, apply Heron's Formula.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter = 2s = AC + CD + DA

2s = 5 cm + 4 cm + 5 cm

2s = 14 cm

s = 7 cm

Area of the $\triangle$CAD = $\sqrt{7 \times (7-5) \times (7-4) \times (7-5)}$

$= \sqrt{7 \times 2 \times 3 \times 2}$

$= 2\sqrt{21}$

$= 9.16$

Area of the $\triangle$CAD = 9.16 cm^2 …(3)

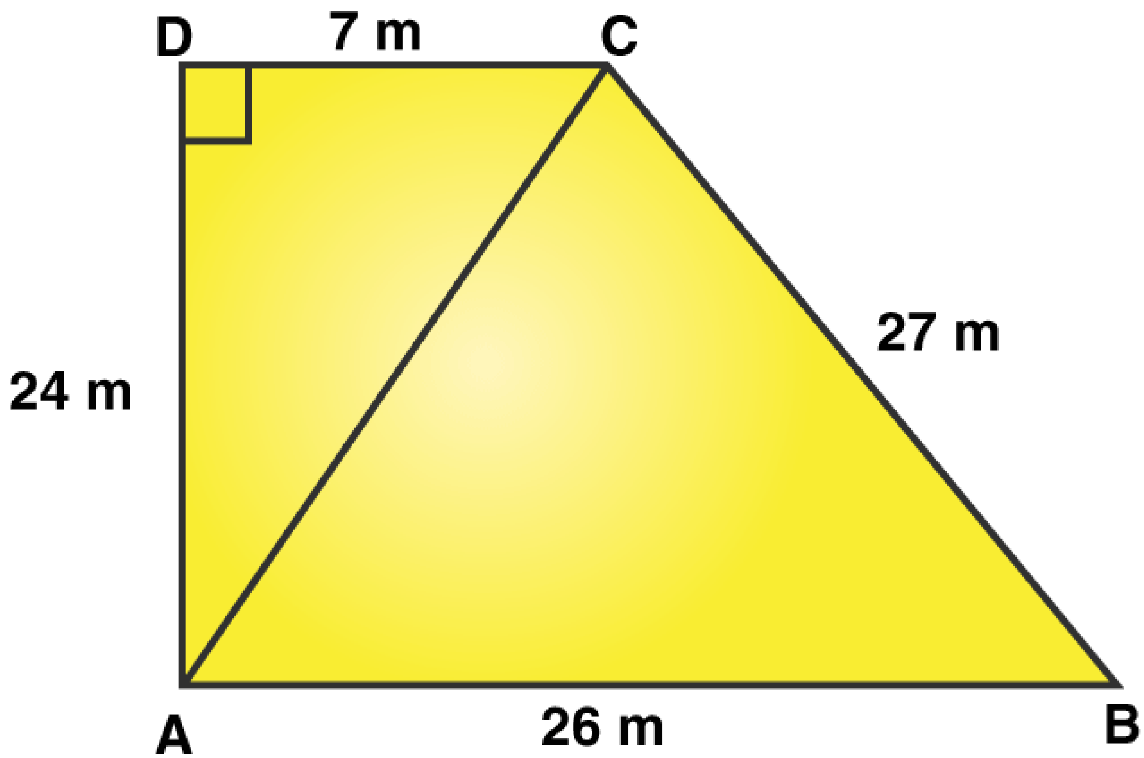
Using equations (2) and (3) in (1), we get

Area of quadrilateral ABCD = (6 + 9.16) cm^2

= 15.16 cm^2.

Question 2: The sides of a quadrilateral field, taken in order, are 26 m, 27 m, 7 m, and 24 m, respectively. The angle contained by the last two sides is a right angle. Find its area.

Solution:



Here,

AB = 26 m, BC = 27 m, CD = 7 m, DA = 24 m

AC is the diagonal joined at A to C point.

Now, in △ADC,

From Pythagoras theorem,

$AC^2 = AD^2 + CD^2$

$AC^2 = 14^2 + 7^2$

AC = 25

Now, area of △ABC

All the sides are known, Apply Heron's Formula.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of △ABC= 2s = AB + BC + CA

2s = 26 m + 27 m + 25 m

s = 39 m

Area of a triangle = $\sqrt{39 \times (39-25) \times (39-26) \times (39-27)}$

$= \sqrt{39 \times 14 \times 13 \times 12}$

$= \sqrt{85176}$

= 291.84

Area of a triangle ABC = 291.84 m^2

Now, for the area of △ADC, (Right angle triangle)

Area = 1/2 x Base X Height

= 1/2 x 7 x 24

= 84

Thus, the area of a △ADC is 84 m^2

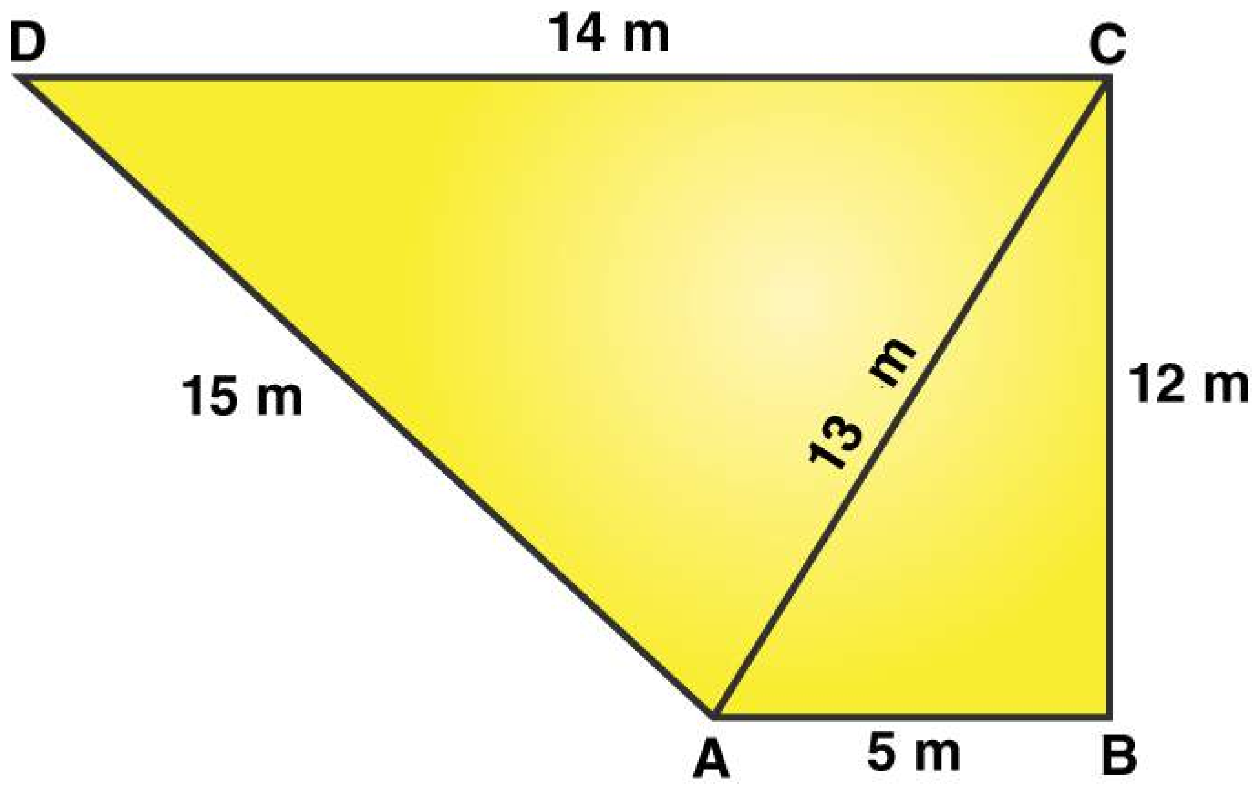
Therefore, the area of rectangular field ABCD = Area of △ABC + Area of △ADC

= 291.84 m^2 + 84 m^2

= 375.8 m^2

Question 3: The sides of a quadrilateral, taken in order as 5, 12, 14, and 15 meters, respectively, and the angle contained by the first two sides is a right angle. Find its area.

Solution:



Here, AB = 5 m, BC = 12 m, CD =14 m and DA = 15 m

Join the diagonal AC.

Now, the area of △ABC = 1/2 ×AB×BC

= 1/2×5×12 = 30

The area of △ABC is 30 m^2

In △ABC, (right triangle).

From Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 5^2 + 12^2$

$AC^2 = 25 + 144 = 169$

or AC = 13

Now in △ADC,

All sides are known, apply Heron's Formula:

Area of triangle = $\sqrt{s(s-a)(s-b)\,(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of $\triangle$ADC = 2s = AD + DC + AC

2s = 15 m +14 m +13 m

s = 21 m

Area of $\triangle$ADC = $\sqrt{21 \times (21-13) \times (21-14) \times (21-15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$

= 84

Area of $\triangle$ADC = 84 m^2

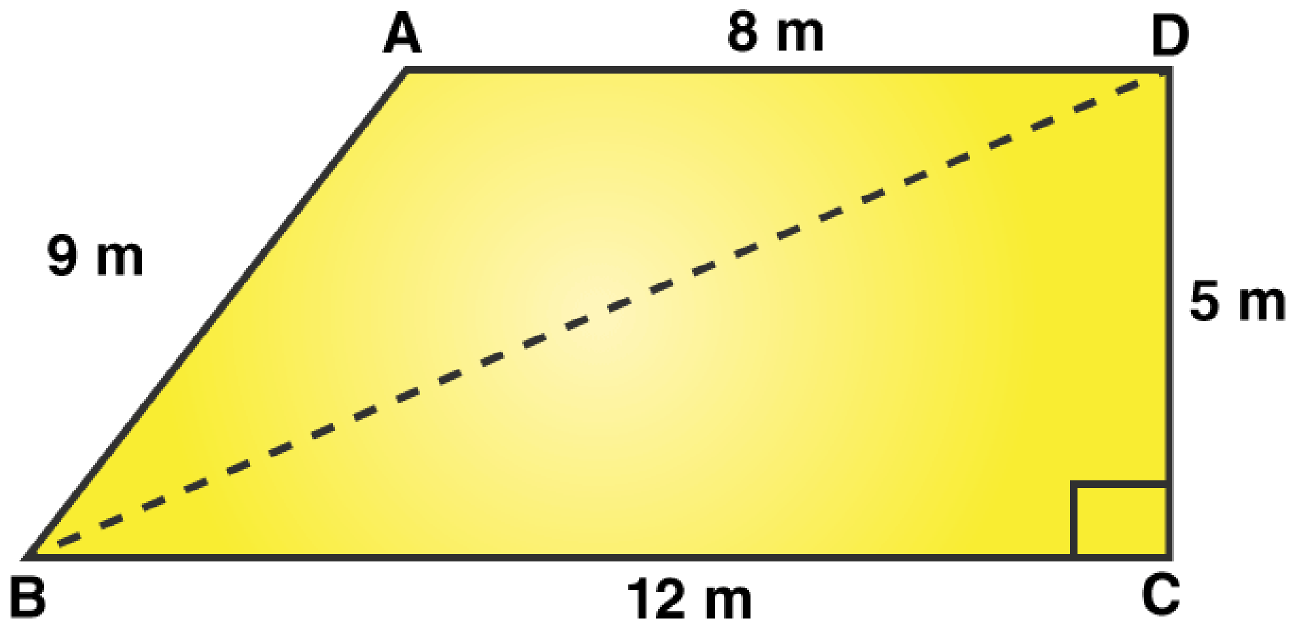
Area of quadrilateral ABCD = Area of $\triangle$ABC + Area of $\triangle$ADC

= (30 + 84) m^2

= 114 m^2

Question 4: A park in the shape of a quadrilateral ABCD has ∠ C = 90°, AB = 9 m, BC = 12 m, CD = 5 m, AD = 8 m. How much area does it occupy?

Solution:



Here, AB = 9 m, BC = 12 m, CD = 5 m, DA = 8 m.

And BD is a diagonal of ABCD.

In the right △BCD,

From Pythagoras theorem;

$BD^2 = BC^2 + CD^2$

$BD^2 = 12^2 + 5^2 = 144 + 25 = 169$

BD = 13 m

Area of △BCD = 1/2×BC×CD

= 1/2×12×5

= 30

Area of △BCD = 30 m^2

Now, In △ABD,

All sides are known, Apply Heron's Formula:

Area of triangle = $\sqrt{s(s-a)(s-b)\,(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of △ABD = 2s = 9 m + 8m + 13m

s = 15 m

Area of the △ABD = $\sqrt{15 \times (15-9) \times (15-8) \times (15-13)}$

= $\sqrt{15 \times 6 \times 7 \times 2}$

= $6\sqrt{35}$

= 35.49

Area of the △ABD = 35.49 m^2

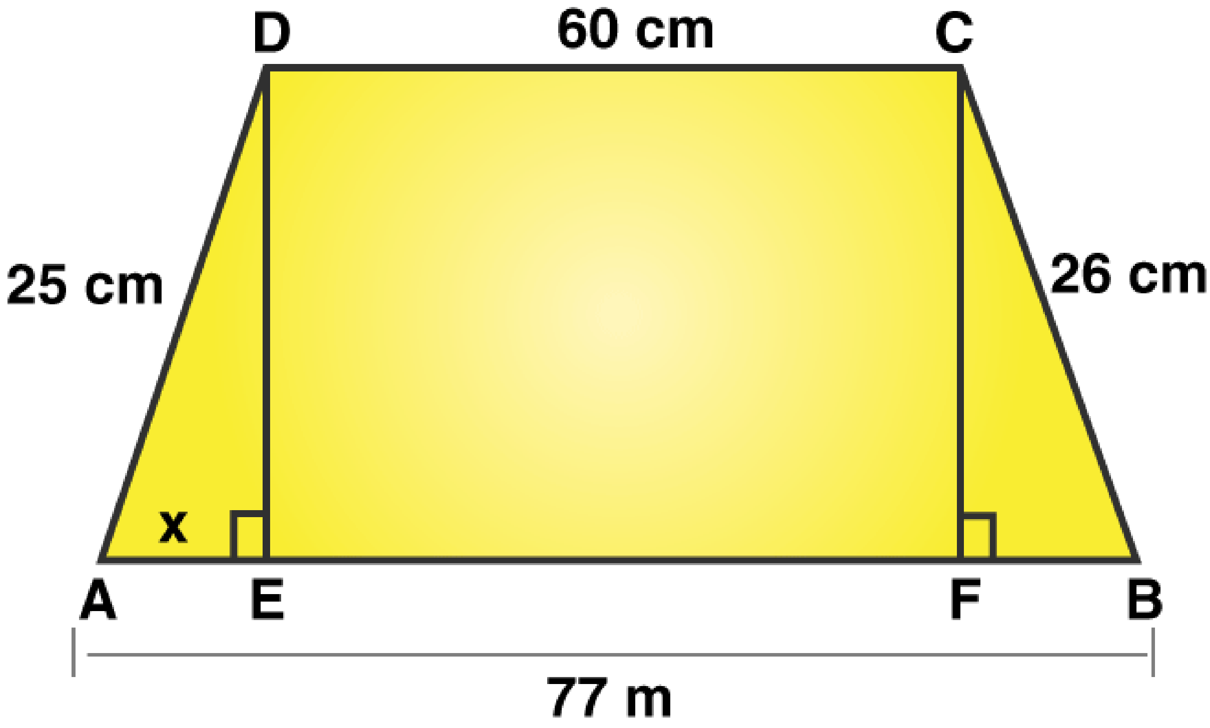
Area of quadrilateral ABCD = Area of △ABD + Area of △BCD

= (35.496 + 30) m^2

= *65.5*m^2.

Question 5: Two parallel sides of a trapezium are 60 m and 77 m and the other sides are 25 m and 26 m. Find the area of the trapezium.

Solution:



Given: AB = 77 m , CD = 60 m, BC = 26 m and AD = 25m

AE and CF are diagonals.

DE and CF are two perpendiculars on AB.

Therefore, we get, DC = EF = 60 m

Let's say, AE = x

Then BF = 77 – (60 + x)

BF = 17 – x …(1)

In the right △ADE,

From Pythagoras theorem,

$DE^2 = AD^2 - AE^2$

$DE^2 = 25^2 - x^2$ ….(2)

In right △BCF

From Pythagoras theorem,

$CF^2 = BC^2 - BF^2$

$CF^2 = 26^2 - (17-x)^2$

[Uisng (1)]

Here, DE = CF

So, $DE^2 = CF^2$

(2) ⇒ $25^2 - x^2 = 26^2 - (17-x)^2$

$625 - x^2 = 676 - (289 - 34x + x^2)$

$625 - x^2 = 676 - 289 + 34x - x^2$

238 = 34x

x = 7

(2) ⇒ $DE^2 = 25^2 - (7)^2$

$DE^2 = 625 - 49$

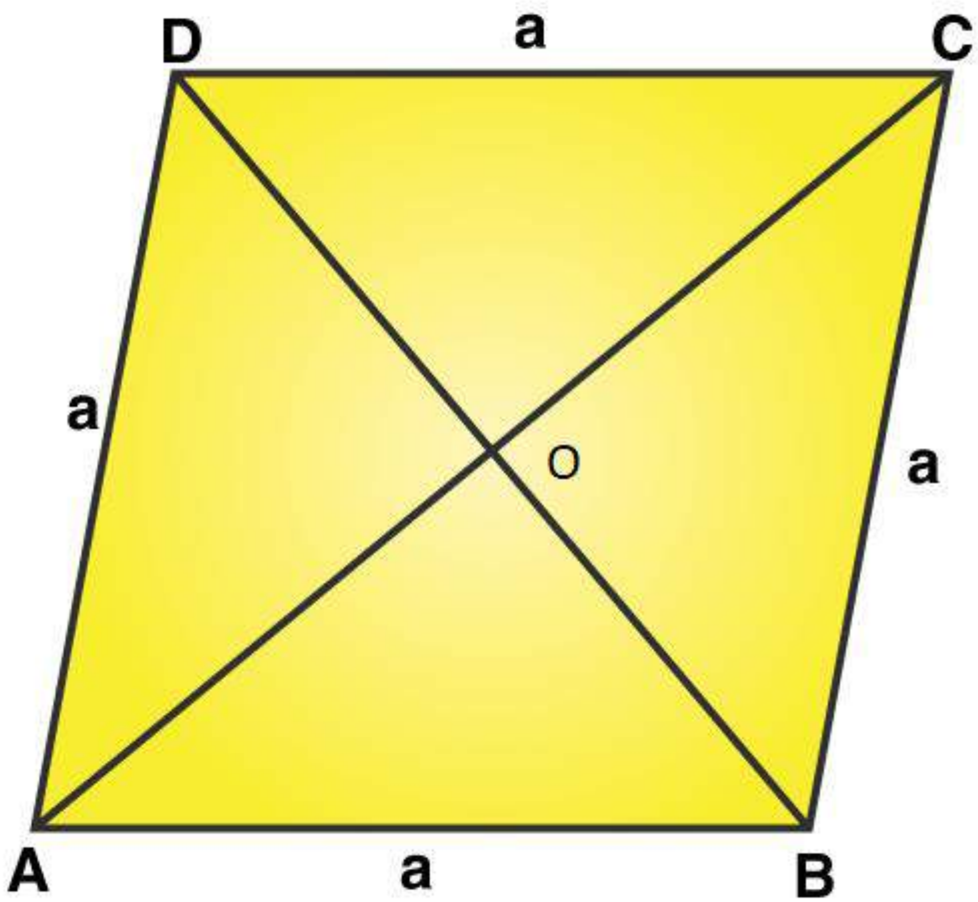
DE = 24

Area of trapezium = 1/2×(60+77)×24 = 1644

Area of trapezium is 1644 m^2 (Answer)

Question 6: Find the area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m.

Solution:



The perimeter of a rhombus = 80 m (given)

We know, Perimeter of a rhombus = 4×side

Let a be the side of a rhombus.

4×a = 80

or a = 20

One of the diagonal, AC = 24 m (given)

Therefore OA = 1/2×AC

OA = 12

In △AOB,

Using Pythagoras theorem:

$OB^2 = AB^2 - OA^2 = 20^2 - 12^2 = 400 - 144 = 256$

or OB = 16

Since the diagonal of the rhombus bisect each other at 90 degrees.

And OB = OD

Therefore, BD = 2 OB = 2 x 16 = 32 m

Area of rhombus = 1/2×BD×AC = 1/2×32×24 = 384

Area of rhombus = 384 m^2.

Question 7: A rhombus sheet, whose perimeter is 32 m and whose diagonal is 10 m long, is painted on both the sides at the rate of Rs 5 per m^2. Find the cost of painting.

Solution:

The perimeter of a rhombus = 32 m

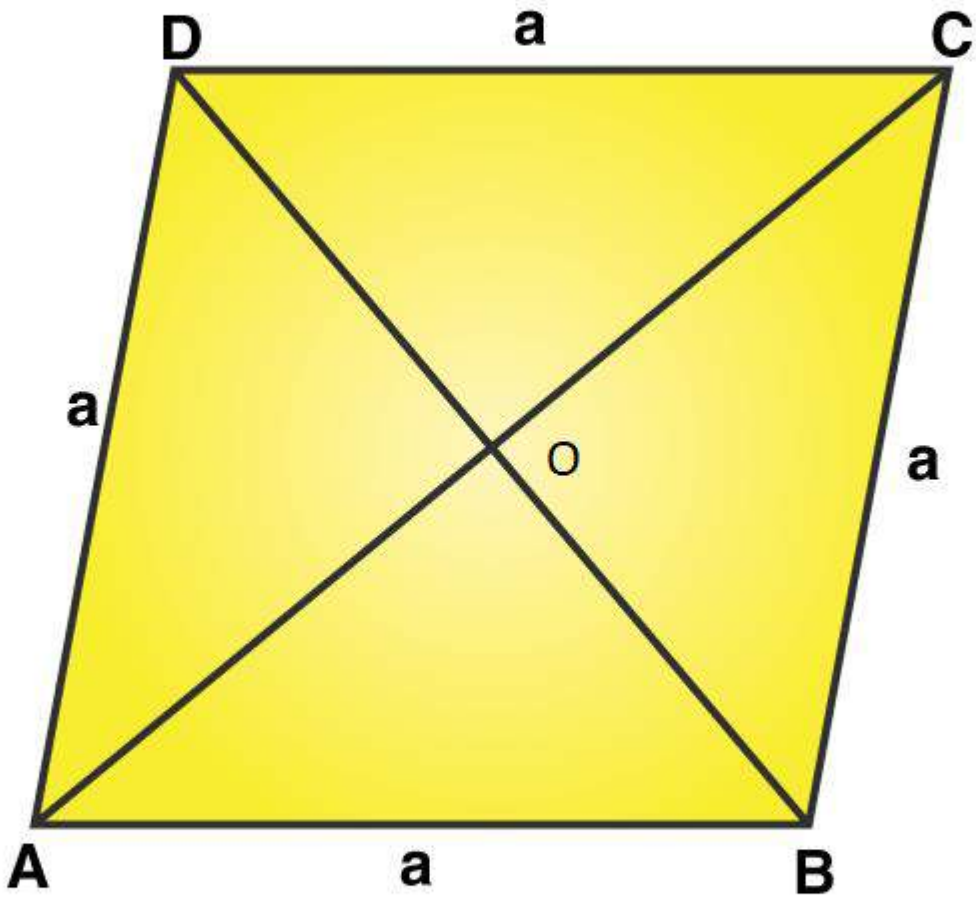
We know, Perimeter of a rhombus = 4×side

⇒ 4×side = 32

side = a = 8 m

Each side of the rhombus is 8 m

AC = 10 m (Given)



Then, OA = 1/2×AC

OA = 1/2×10

OA = 5 m

In right triangle AOB,

From Pythagoras theorem;

$OB^2 = AB^2 – OA^2 = 8^2 – 5^2 = 64 – 25 = 39$

OB = √39 m

And, BD = 2 x OB

BD = 2√39 m

Area of the sheet = 1/2×BD×AC = 1/2 x (2√39 × 10) = 10√39

The area of the sheet is 10√39 m^2

Therefore, the cost of printing on both sides of the sheet, at the rate of Rs. 5 per m^2

= Rs. 2 x (10√39 x 5) = *Rs. 625.*