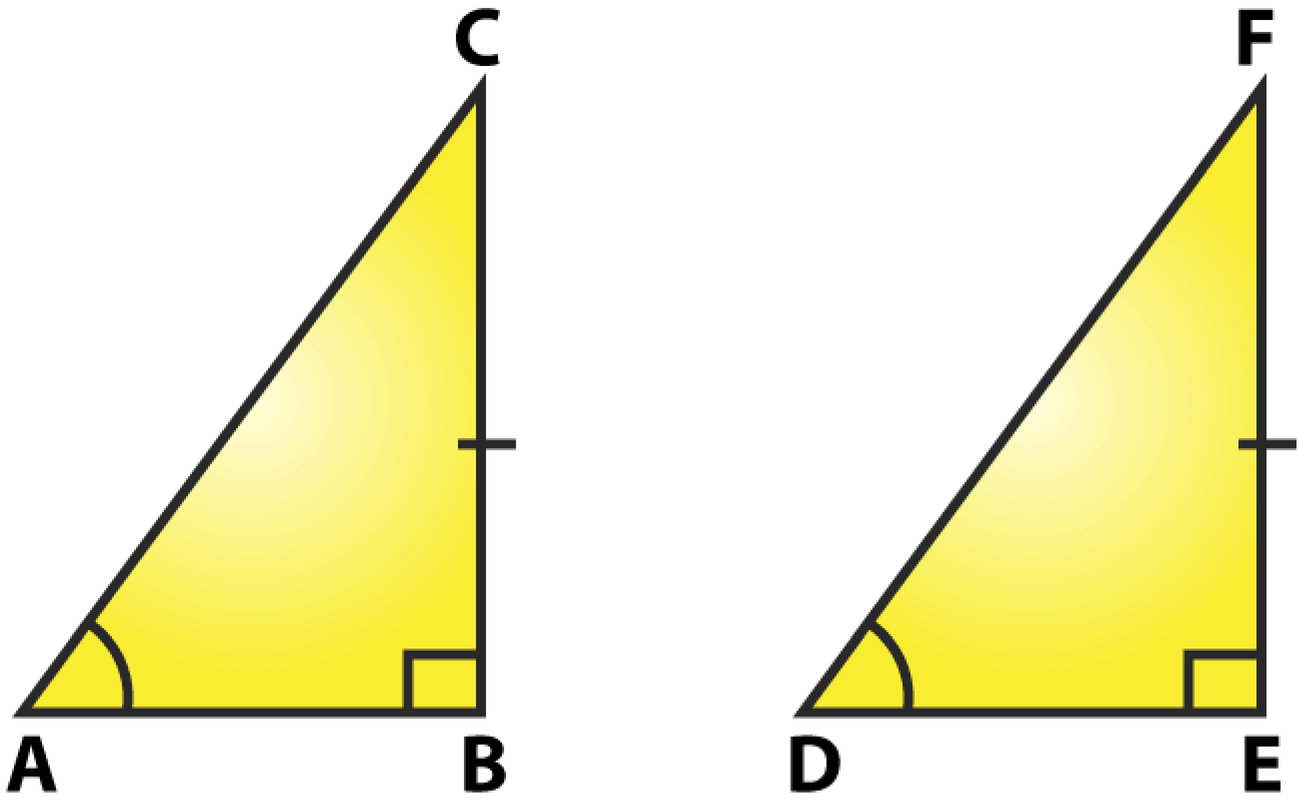
EXERCISE 10.3 PAGE NO.10.38

Question 1: In two right triangles, one side and an acute angle of one triangle are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

In two right triangles, one side and an acute angle of one triangle are equal to the corresponding side and angles of the other. (Given)



To prove: Both triangles are congruent.

Consider two right triangles such that

$\angle B = \angle E = 90^\circ$ …….(i)

AB = DE …….(ii)

$\angle C = \angle F$ ……(iii)

Here we have two right triangles, △ ABC and △ DEF

From (i), (ii) and (iii),

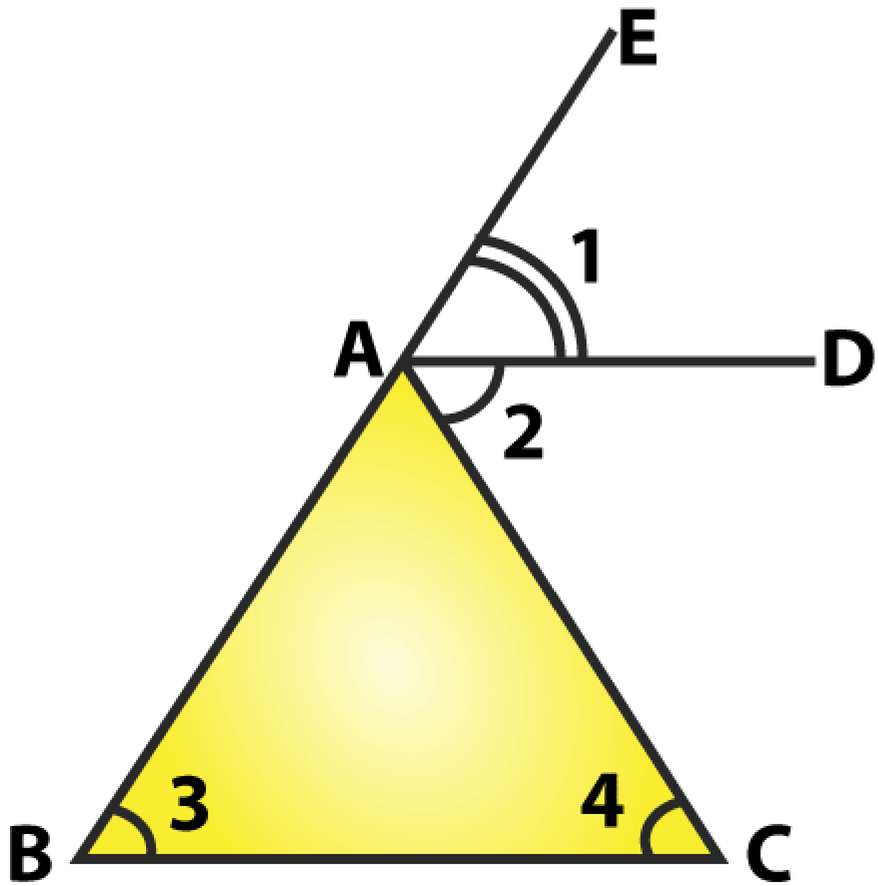
By the AAS congruence criterion, we have Δ ABC ≅ Δ DEF

Both triangles are congruent. Hence proved.

Question 2: If the bisector of the exterior vertical angle of a triangle is parallel to the base, show that the triangle is isosceles.

Solution:

Let ABC be a triangle such that AD is the angular bisector of the exterior vertical angle, ∠EAC and AD ‖ BC.



From figure,

∠1 = ∠2 [AD is a bisector of ∠ EAC]

∠1 = ∠3 [Corresponding angles]

and ∠2 = ∠4 [alternative angle]

From above, we have ∠3 = ∠4

This implies, AB = AC

Two sides, AB and AC, are equal.

=> Δ ABC is an isosceles triangle.

Question 3: In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution:

Let Δ ABC be isosceles where AB = AC and ∠ B = ∠ C

Given: Vertex angle A is twice the sum of the base angles B and C. i.e., ∠ A = 2(∠ B + ∠ C)

∠ A = 2(∠ B + ∠ B)

∠ A = 2(2 ∠ B)

∠ A = 4(∠ B)

Now, We know that the sum of angles in a triangle =180°

∠ A + ∠ B + ∠ C =180°

4 ∠ B + ∠ B + ∠ B = 180°

6 ∠ B =180°

∠ B = 30°

Since, ∠ B = ∠ C

∠ B = ∠ C = 30°

And ∠ A = 4 ∠ B

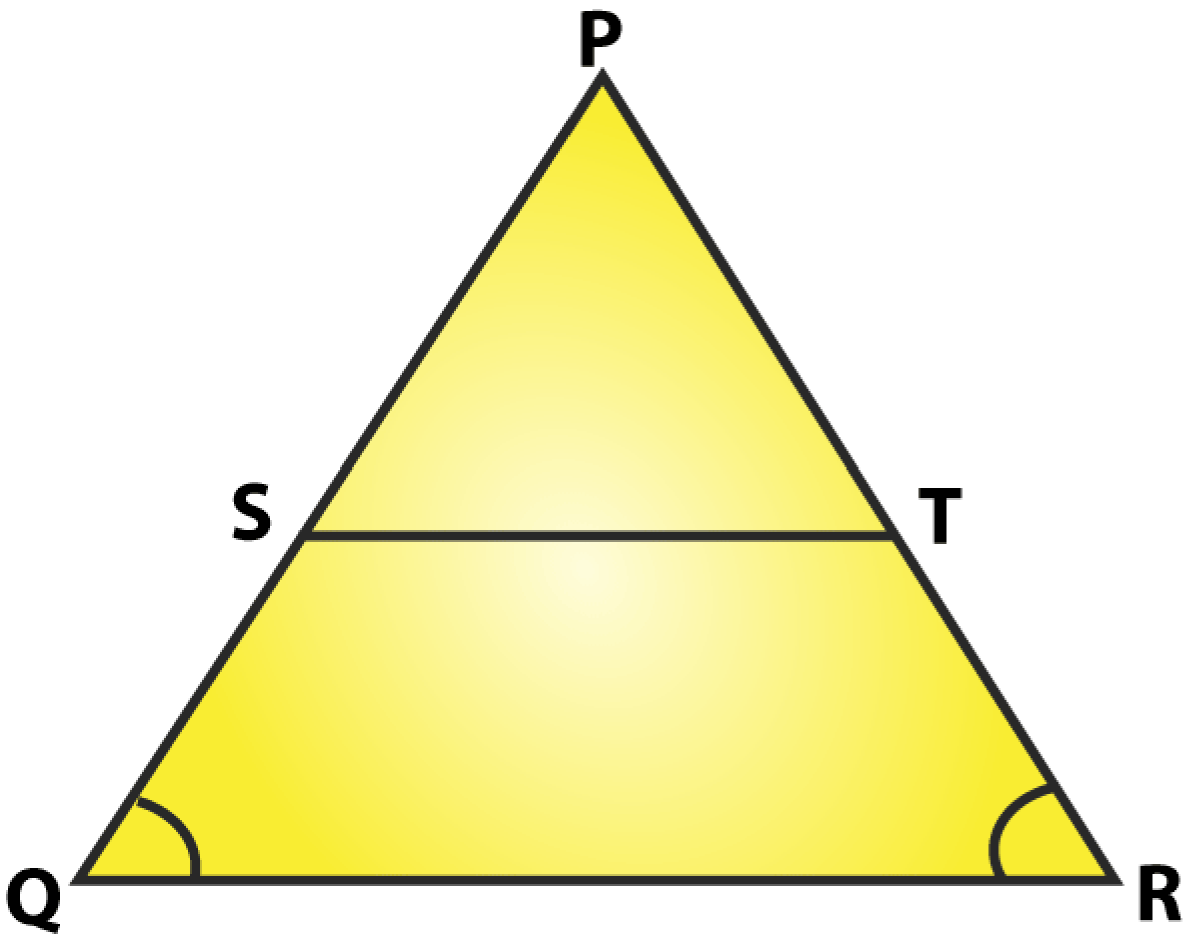
∠ A = 4 x 30° = 120°

Therefore, the angles of the given triangle are 30° and 30° and 120°.

Question 4: PQR is a triangle in which PQ = PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Solution: Given that PQR is a triangle such that PQ = PR and S is any point on the side PQ and ST ∥ QR.

To prove: PS = PT



Since, PQ= PR, so △PQR is an isosceles triangle.

∠ PQR = ∠ PRQ

Now, ∠ PST = ∠ PQR and ∠ PTS = ∠ PRQ

[Corresponding angles as ST parallel to QR]
Since, ∠ PQR = ∠ PRQ

∠ PST = ∠ PTS

In Δ PST,

∠ PST = ∠ PTS

Δ PST is an isosceles triangle.

Therefore, PS = PT.

Hence proved.