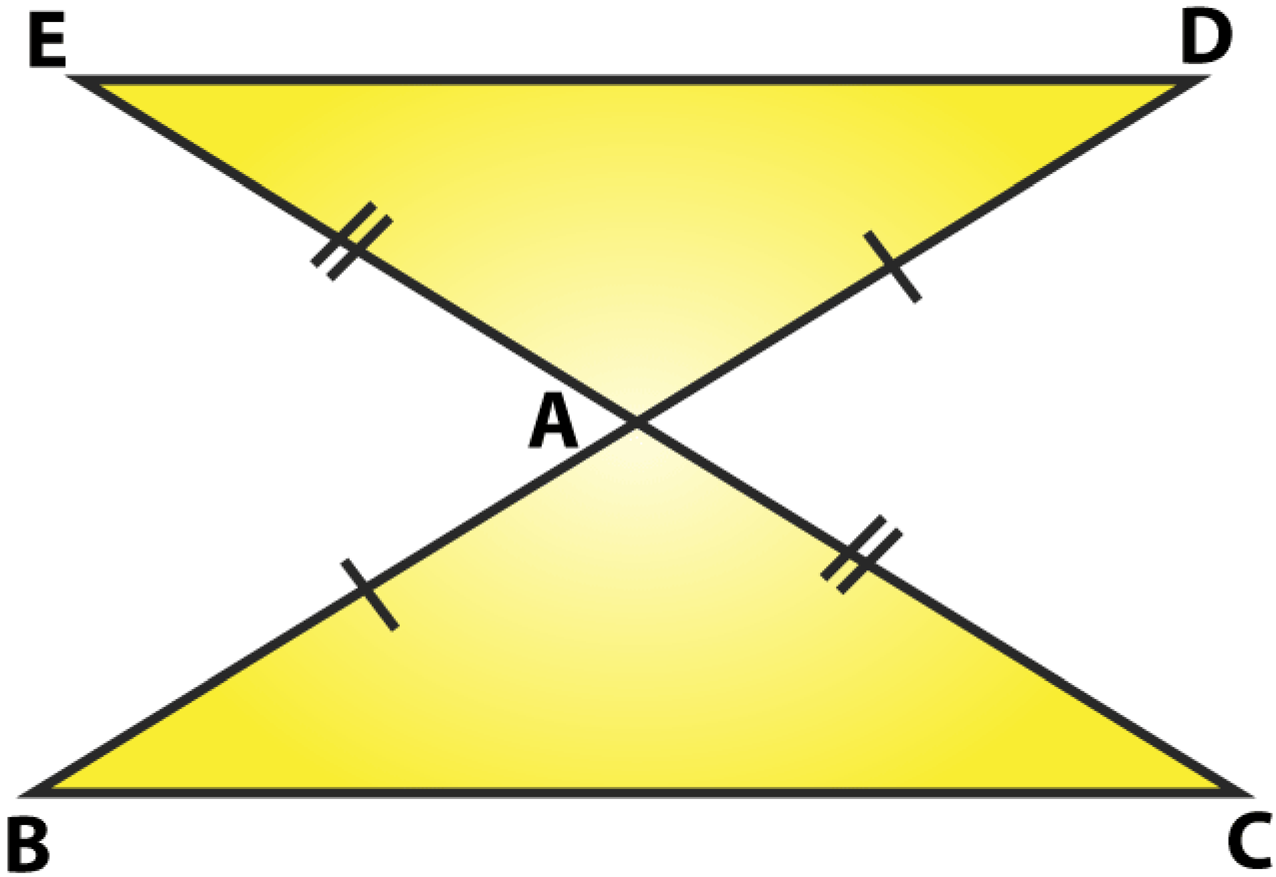
EXERCISE 10.1 PAGE NO. 10.12

Question 1: In the figure, the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that segment DE ∥ BC.



Solution:

Sides BA and CA have been produced such that BA = AD and CA = AE.

To prove: DE ∥ BC

Consider △ BAC and △DAE,

BA = AD and CA= AE (Given)

∠BAC = ∠DAE (vertically opposite angles)

By the SAS congruence criterion, we have

△ BAC ≃ △ DAE

We know corresponding parts of congruent triangles are equal

So, BC = DE and ∠DEA = ∠BCA, ∠EDA = ∠CBA

Now, DE and BC are two lines intersected by a transversal DB s.t.

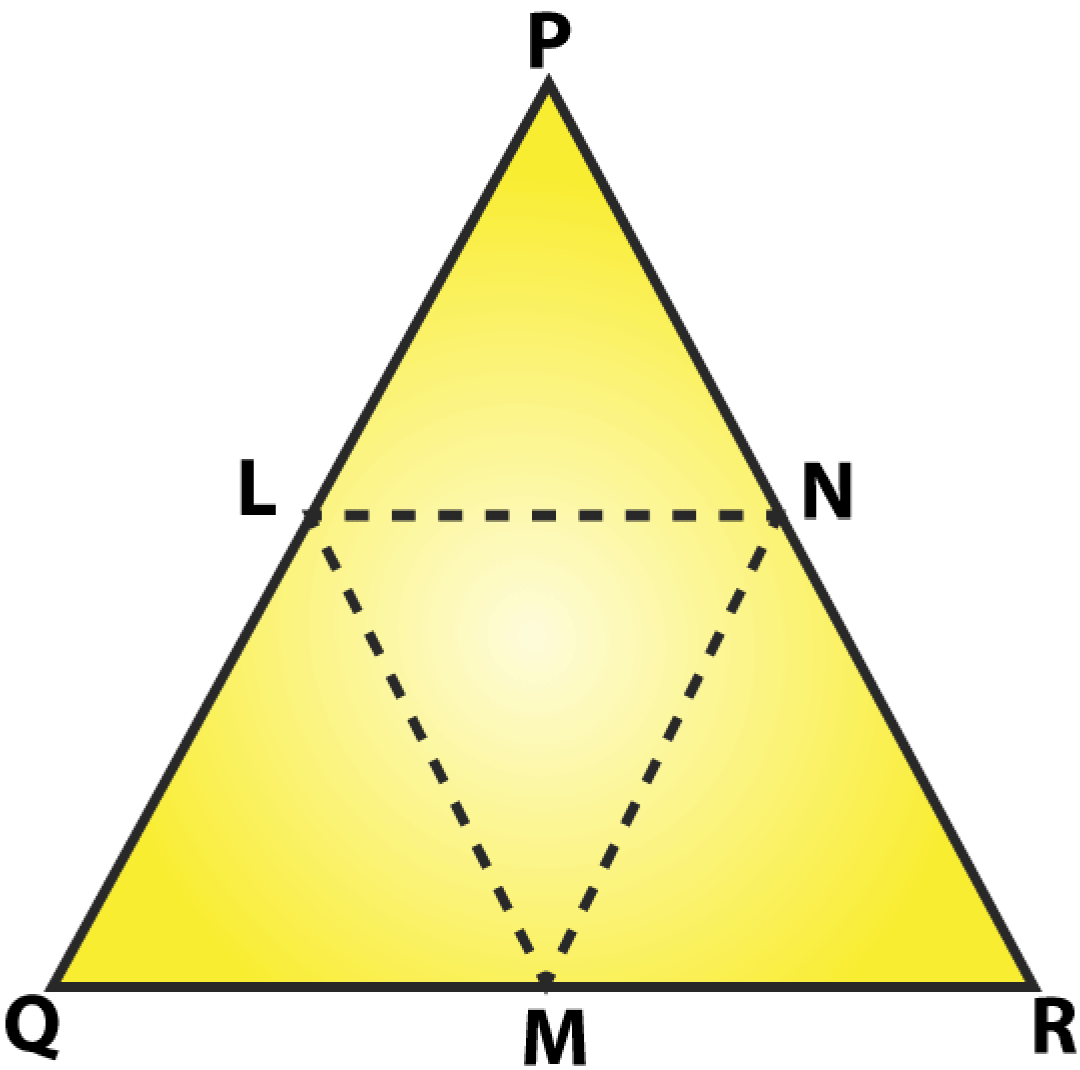
∠DEA=∠BCA (alternate angles are equal)

Therefore, DE ∥ BC. Proved.

Question 2: In a PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP, respectively. Prove that LN = MN.

Solution:

Draw a figure based on the given instruction,



In △PQR, PQ = QR and L, M, N are midpoints of the sides PQ, QP and RP, respectively (Given)

To prove: LN = MN

As two sides of the triangle are equal, so △ PQR is an isosceles triangle

PQ = QR and ∠QPR = ∠QRP ……. (i)

Also, L and M are midpoints of PQ and QR, respectively

PL = LQ = QM = MR = QR/2

Now, consider ∆ LPN and ∆ MRN,

LP = MR

∠LPN = ∠MRN [From (i)]

∠QPR = ∠LPN and ∠QRP = ∠MRN

PN = NR [N is the midpoint of PR]

By SAS congruence criterion,

∆ LPN ≃ ∆ MRN

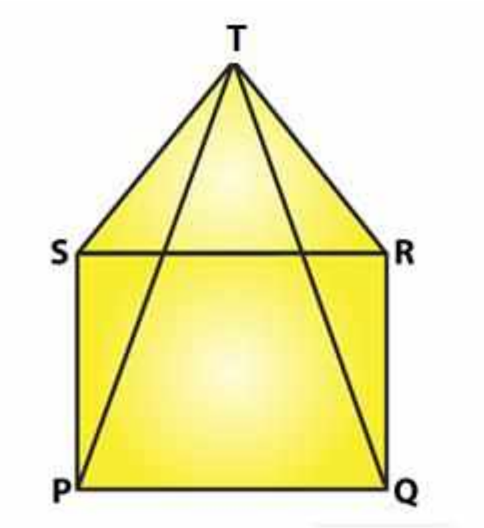
We know that the corresponding parts of congruent triangles are equal.

So LN = MN

Proved.

Question 3: In the figure, PQRS is a square, and SRT is an equilateral triangle. Prove that

(i) PT = QT (ii) ∠ TQR = 15^0



Solution:

Given: PQRS is a square, and SRT is an equilateral triangle.

To prove:

(i) PT =QT and (ii) ∠ TQR =15°

Now,

PQRS is a square:

PQ = QR = RS = SP …… (i)

And ∠ SPQ = ∠ PQR = ∠ QRS = ∠ RSP = 90°

Also, △ SRT is an equilateral triangle:

SR = RT = TS …….(ii)

And ∠ TSR = ∠ SRT = ∠ RTS = 60°

From (i) and (ii)

PQ = QR = SP = SR = RT = TS ……(iii)

From figure,

∠TSP = ∠TSR + ∠ RSP = 60° + 90° = 150° and

∠TRQ = ∠TRS + ∠ SRQ = 60° + 90° = 150°

=> ∠ TSP = ∠ TRQ = 150^{0} ………………… (iv)

By SAS congruence criterion, Δ TSP ≃ Δ TRQ

We know that the corresponding parts of congruent triangles are equal

So, $PT = QT$

Proved part (i).

Now, consider Δ TQR.

QR = TR [From (iii)]

Δ TQR is an isosceles triangle.

∠ QTR = ∠ TQR [angles opposite to equal sides]

The sum of angles in a triangle = 180°

=> ∠QTR + ∠ TQR + ∠TRQ = 180°

=> 2 ∠ TQR + 150° = 180° [From (iv)]

=> 2 ∠ TQR = 30°

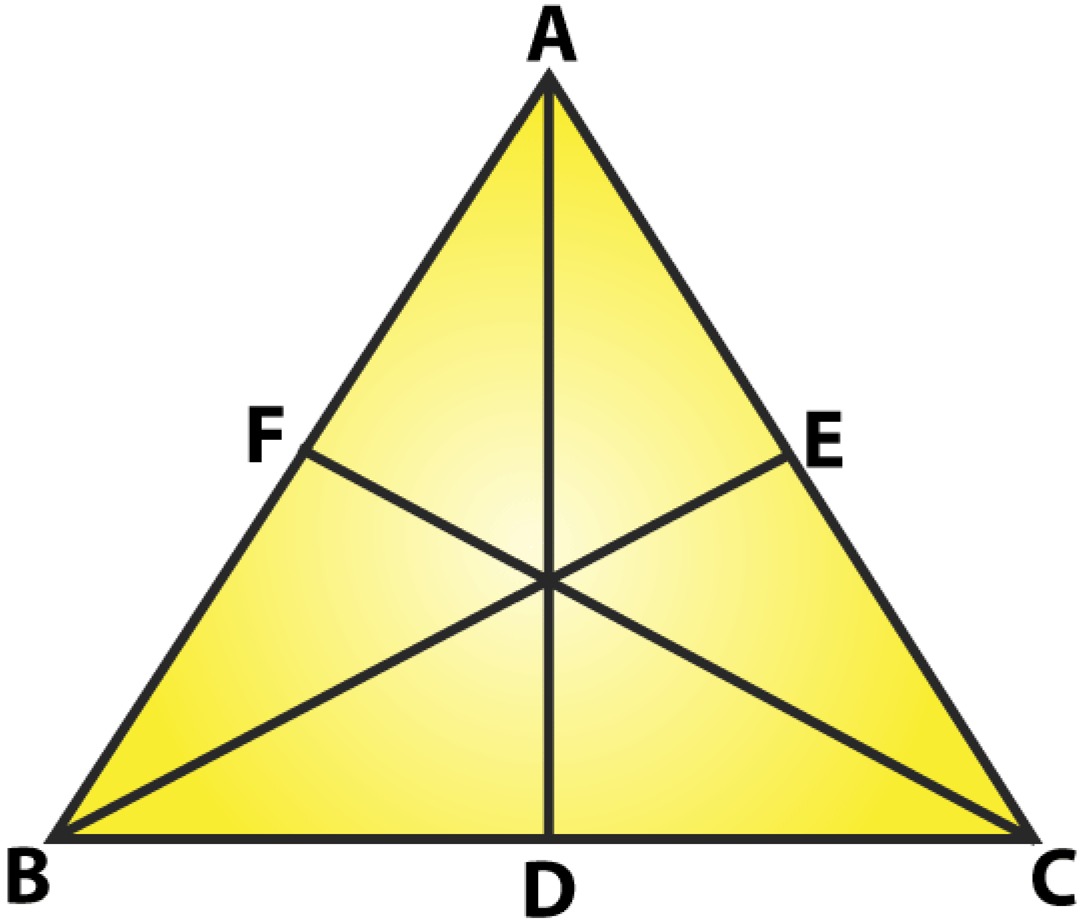
=> $\angle TQR = 15^{0}$

Hence proved part (ii).

Question 4: Prove that the medians of an equilateral triangle are equal.

Solution:

Consider an equilateral △ABC, and Let D, E, and F are midpoints of BC, CA and AB.



Here, AD, BE and CF are medians of △ABC.

Now,

D is the midpoint of BC => BD = DC

Similarly, CE = EA and AF = FB

Since ΔABC is an equilateral triangle

AB = BC = CA …..(i)

BD = DC = CE = EA = AF = FB …………(ii)

And also, ∠ ABC = ∠ BCA = ∠ CAB = 60° ……….(iii)

Consider Δ ABD and Δ BCE

AB = BC [From (i)]

BD = CE [From (ii)]

∠ ABD = ∠ BCE [From (iii)]

By SAS congruence criterion,

Δ ABD ≃ Δ BCE

=> AD = BE ……..(iv)

[Corresponding parts of congruent triangles are equal in measure]
Now, consider Δ BCE and Δ CAF,

BC = CA [From (i)]

∠ BCE = ∠ CAF [From (iii)]

CE = AF [From (ii)]

By SAS congruence criterion,

Δ BCE ≃ Δ CAF

=> BE = CF …………..(v)

[Corresponding parts of congruent triangles are equal]
From (iv) and (v), we have

AD = BE = CF

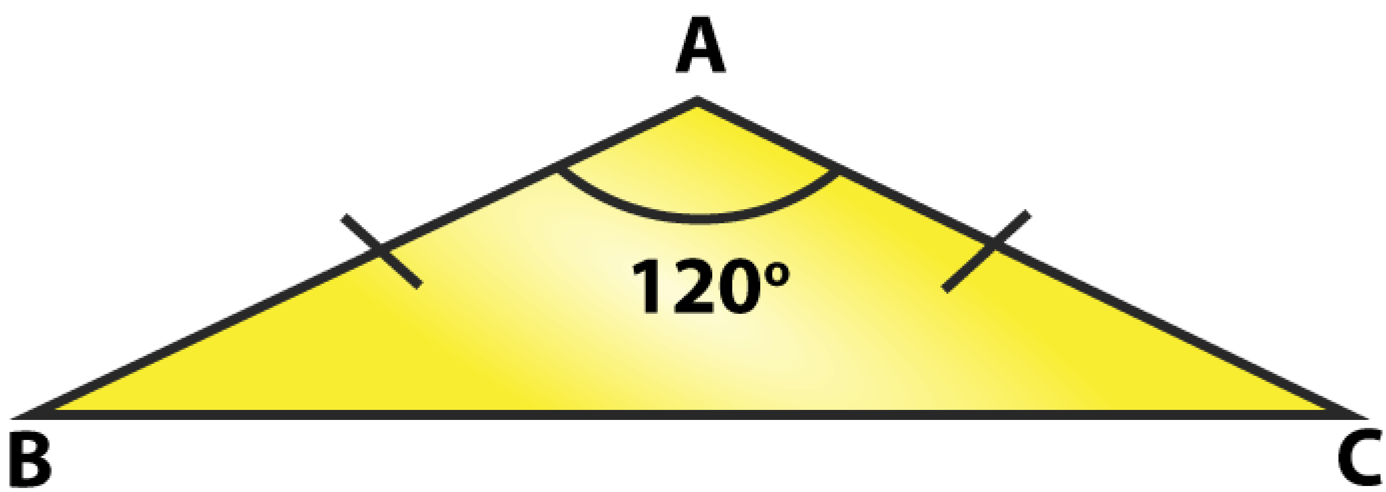
Median AD = Median BE = Median CF

The medians of an equilateral triangle are equal.

Hence proved

Question 5: In a Δ ABC, if ∠A = 120° and AB = AC. Find ∠B and ∠C.

Solution:



To find: ∠ B and ∠ C.

Here, Δ ABC is an isosceles triangle since AB = AC

∠ B = ∠ C ……… (i)

[Angles opposite to equal sides are equal]

We know that the sum of angles in a triangle = 180°

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + \angle B = 180°$ (using (i)

$120^0 + 2\angle B = 180^0$

$2\angle B = 180^0 - 120^0 = 60^0$

$\angle B = 30^o$

Therefore, $\angle B = \angle C = 30°$

Question 6: In a Δ ABC, if AB = AC and ∠ B = 70°, find ∠ A.

Solution:

Given: In a Δ ABC, AB = AC and ∠B = 70°

$\angle B = \angle C$ [Angles opposite to equal sides are equal]

Therefore, $\angle B = \angle C = 70^\circ$

The sum of angles in a triangle = 180∘

$\angle A + \angle B + \angle C = 180^\circ$

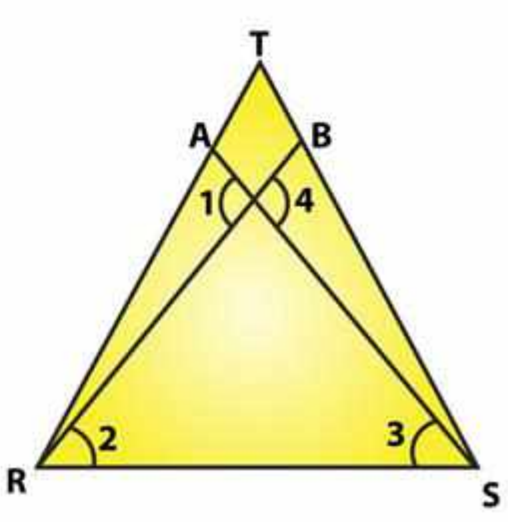
$\angle A + 70^\circ + 70^\circ = 180^\circ$

$\angle A = 180^\circ - 140^\circ$

$\angle A = 40^o$

EXERCISE 10.2 PAGE NO.10.21

Question 1: In the figure, it is given that RT = TS, ∠ 1 = 2 ∠ 2 and ∠4 = 2(∠3). Prove that ΔRBT ≅ ΔSAT.



Solution:

In the figure,

RT = TS ……(i)

∠ 1 = 2 ∠ 2 ……(ii)

And ∠ 4 = 2 ∠ 3 ……(iii)

To prove: ΔRBT ≅ ΔSAT

Let the point of intersection RB and SA be denoted by O

∠ AOR = ∠ BOS [Vertically opposite angles]

or ∠ 1 = ∠ 4

2 ∠ 2 = 2 ∠ 3 [From (ii) and (iii)]

or ∠ 2 = ∠ 3 ……(iv)

Now in Δ TRS, we have RT = TS

=> Δ TRS is an isosceles triangle

∠ TRS = ∠ TSR ……(v)

But, ∠ TRS = ∠ TRB + ∠ 2 ……(vi)

∠ TSR = ∠ TSA + ∠ 3 ……(vii)

Putting (vi) and (vii) in (v) we get

∠ TRB + ∠ 2 = ∠ TSA + ∠ 3

=> ∠ TRB = ∠ TSA [From (iv)]

Consider Δ RBT and Δ SAT

RT = ST [From (i)]

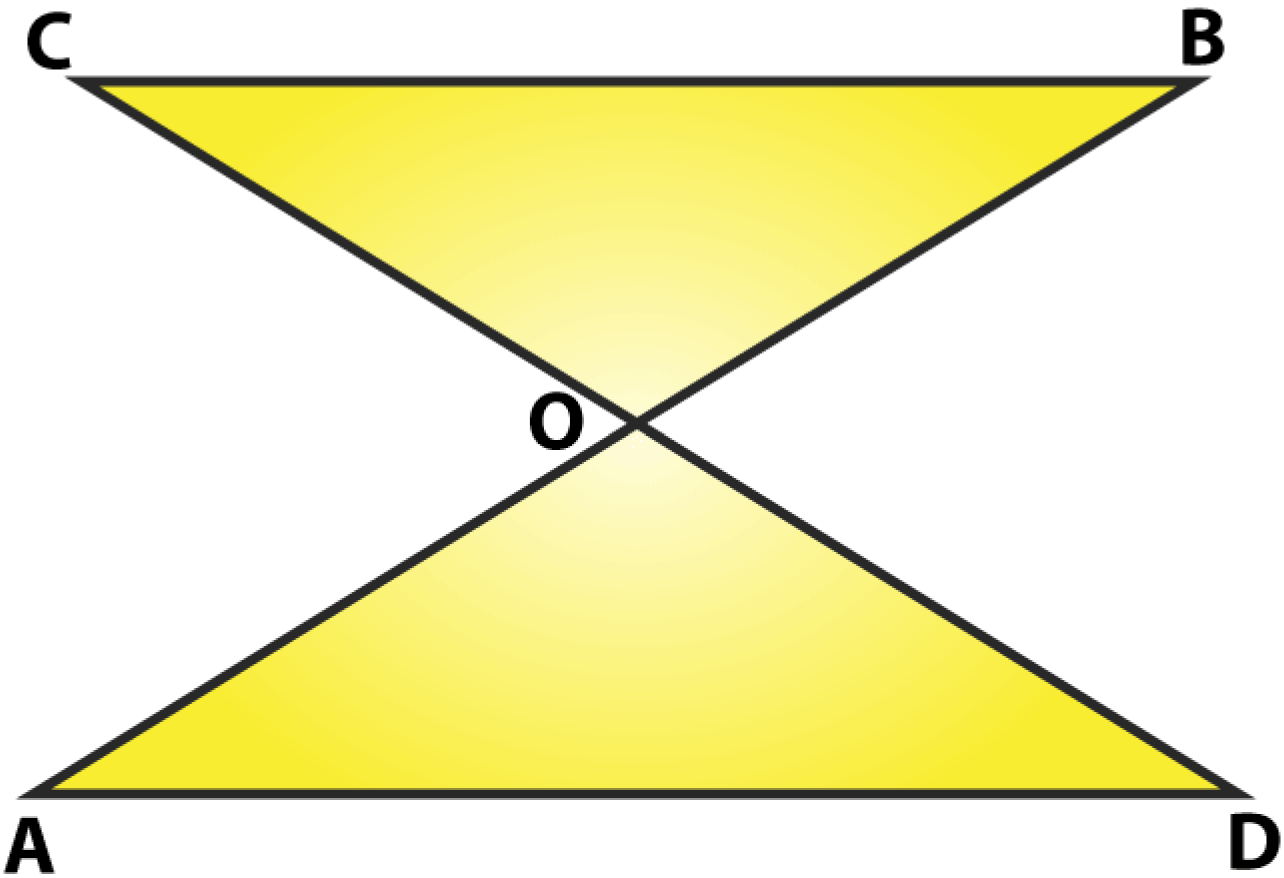
∠ TRB = ∠ TSA [From (iv)]

By the ASA criterion of congruence, we have

Δ RBT ≅ Δ SAT

Question 2: Two lines, AB and CD, intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Lines AB and CD Intersect at O



Such that BC ∥ AD and

BC = AD …….(i)

To prove: AB and CD bisect at O.

First, we have to prove that Δ AOD ≅ Δ BOC

∠OCB =∠ODA [AD||BC and CD is transversal]

AD = BC [from (i)]

∠OBC = ∠OAD [AD||BC and AB is transversal]

By ASA Criterion:

Δ AOD ≅ Δ BOC

OA = OB and OD = OC (By c.p.c.t.)

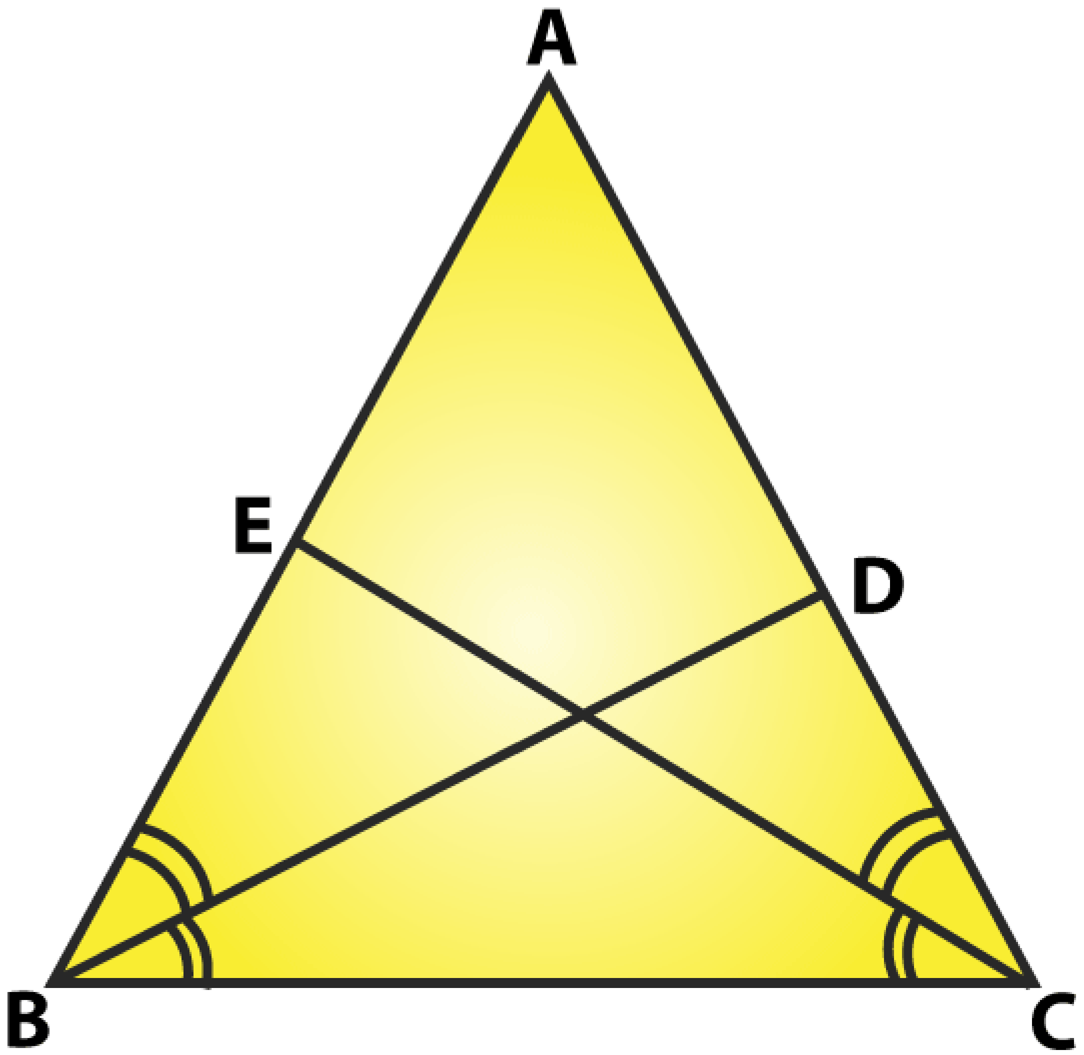
Therefore, AB and CD bisect each other at O.

Hence Proved.

Question 3: BD and CE are bisectors of ∠ B and ∠ C of an isosceles Δ ABC with AB = AC. Prove that BD = CE.

Solution:

Δ ABC is isosceles with AB = AC, and BD and CE are bisectors of ∠ B and ∠ C. We have to prove BD = CE. (Given)



Since AB = AC

=> ∠ABC = ∠ACB ……(i)

[Angles opposite to equal sides are equal]
Since BD and CE are bisectors of ∠ B and ∠ C

∠ ABD = ∠ DBC = ∠ BCE = ECA = ∠B/2 = ∠C/2 …(ii)

Now, Consider Δ EBC = Δ DCB

∠ EBC = ∠ DCB [From (i)]

BC = BC [Common side]

∠ BCE = ∠ CBD [From (ii)]

By ASA congruence criterion, Δ EBC ≅ Δ DCB

Since corresponding parts of congruent triangles are equal.

=> CE = BD

or, BD = CE

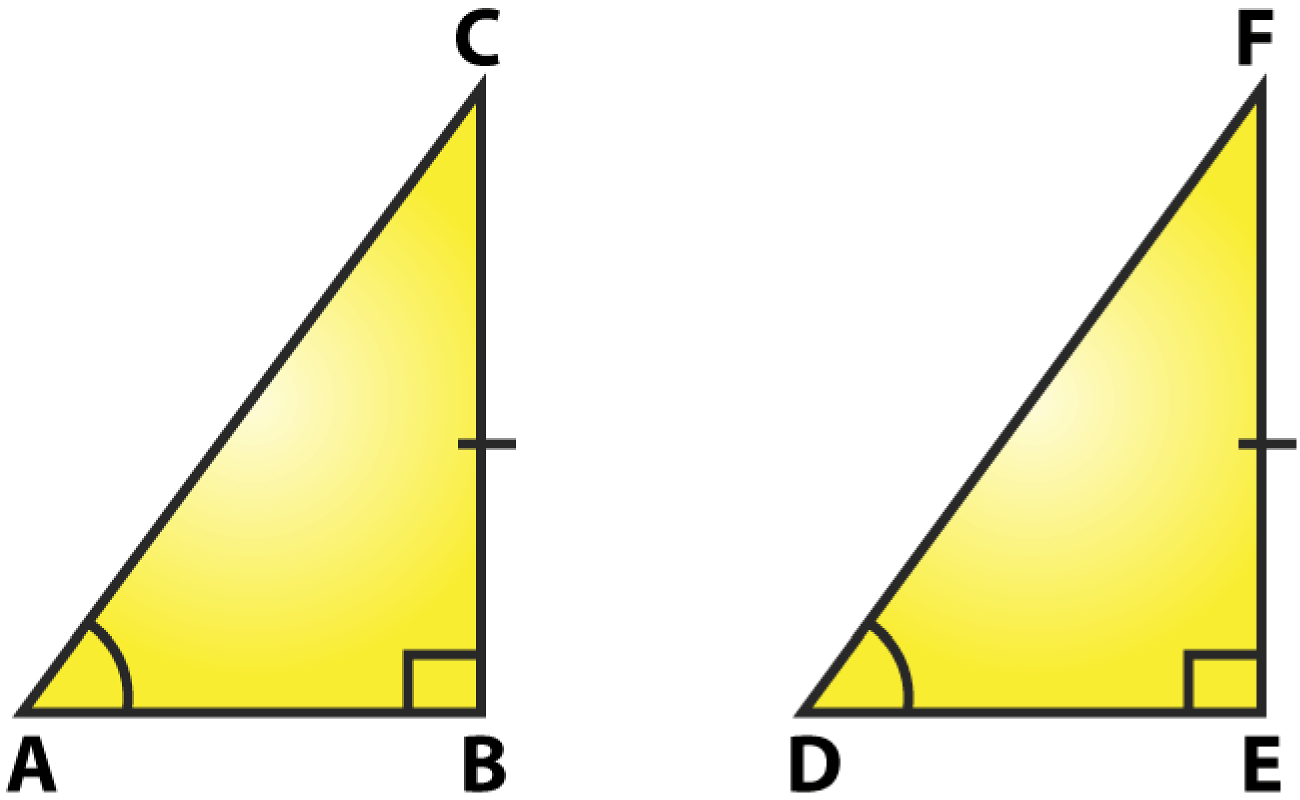
Hence proved.

EXERCISE 10.3 PAGE NO.10.38

Question 1: In two right triangles, one side and an acute angle of one triangle are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

In two right triangles, one side and an acute angle of one triangle are equal to the corresponding side and angles of the other. (Given)



To prove: Both triangles are congruent.

Consider two right triangles such that

$\angle B = \angle E = 90^\circ$ …….(i)

AB = DE …….(ii)

$\angle C = \angle F$ ……(iii)

Here we have two right triangles, △ ABC and △ DEF

From (i), (ii) and (iii),

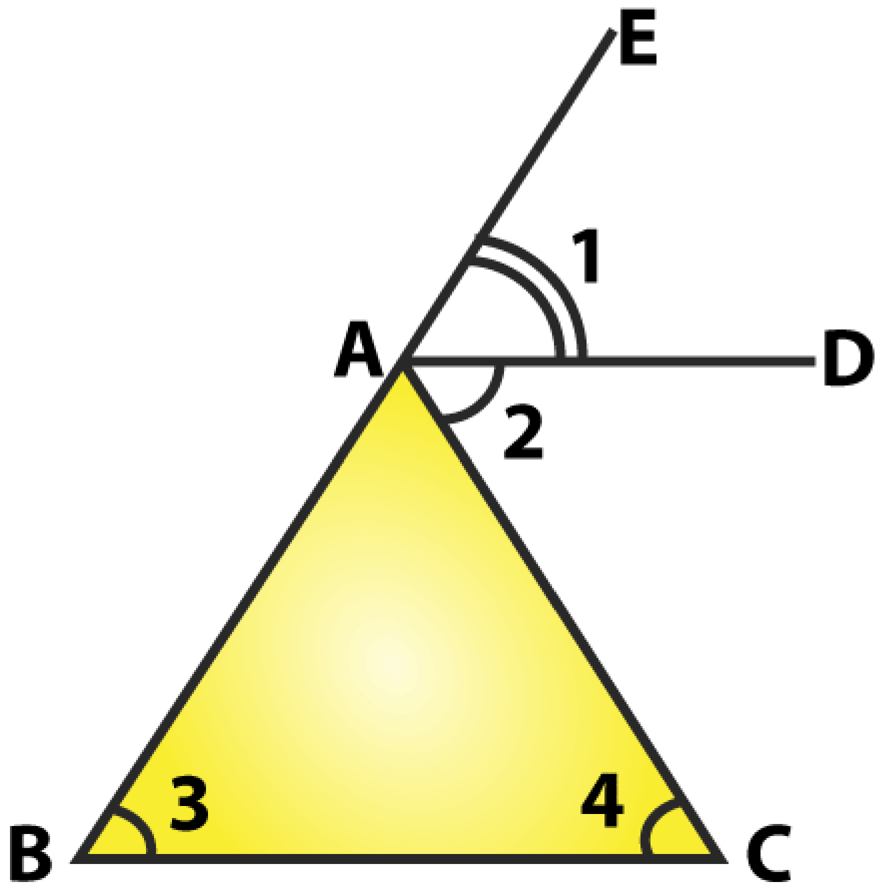
By the AAS congruence criterion, we have Δ ABC ≅ Δ DEF

Both triangles are congruent. Hence proved.

Question 2: If the bisector of the exterior vertical angle of a triangle is parallel to the base, show that the triangle is isosceles.

Solution:

Let ABC be a triangle such that AD is the angular bisector of the exterior vertical angle, ∠EAC and AD ∥ BC.



From figure,

∠1 = ∠2 [AD is a bisector of ∠ EAC]

∠1 = ∠3 [Corresponding angles]

and ∠2 = ∠4 [alternative angle]

From above, we have ∠3 = ∠4

This implies, AB = AC

Two sides, AB and AC, are equal.

=> Δ ABC is an isosceles triangle.

Question 3: In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution:

Let Δ ABC be isosceles where AB = AC and ∠ B = ∠ C

Given: Vertex angle A is twice the sum of the base angles B and C. i.e., ∠ A = 2(∠ B + ∠ C)

∠ A = 2(∠ B + ∠ B)

∠ A = 2(2 ∠ B)

∠ A = 4(∠ B)

Now, We know that the sum of angles in a triangle =180°

∠ A + ∠ B + ∠ C =180°

4 ∠ B + ∠ B + ∠ B = 180°

6 ∠ B =180°

∠ B = 30°

Since, ∠ B = ∠ C

∠ B = ∠ C = 30°

And ∠ A = 4 ∠ B

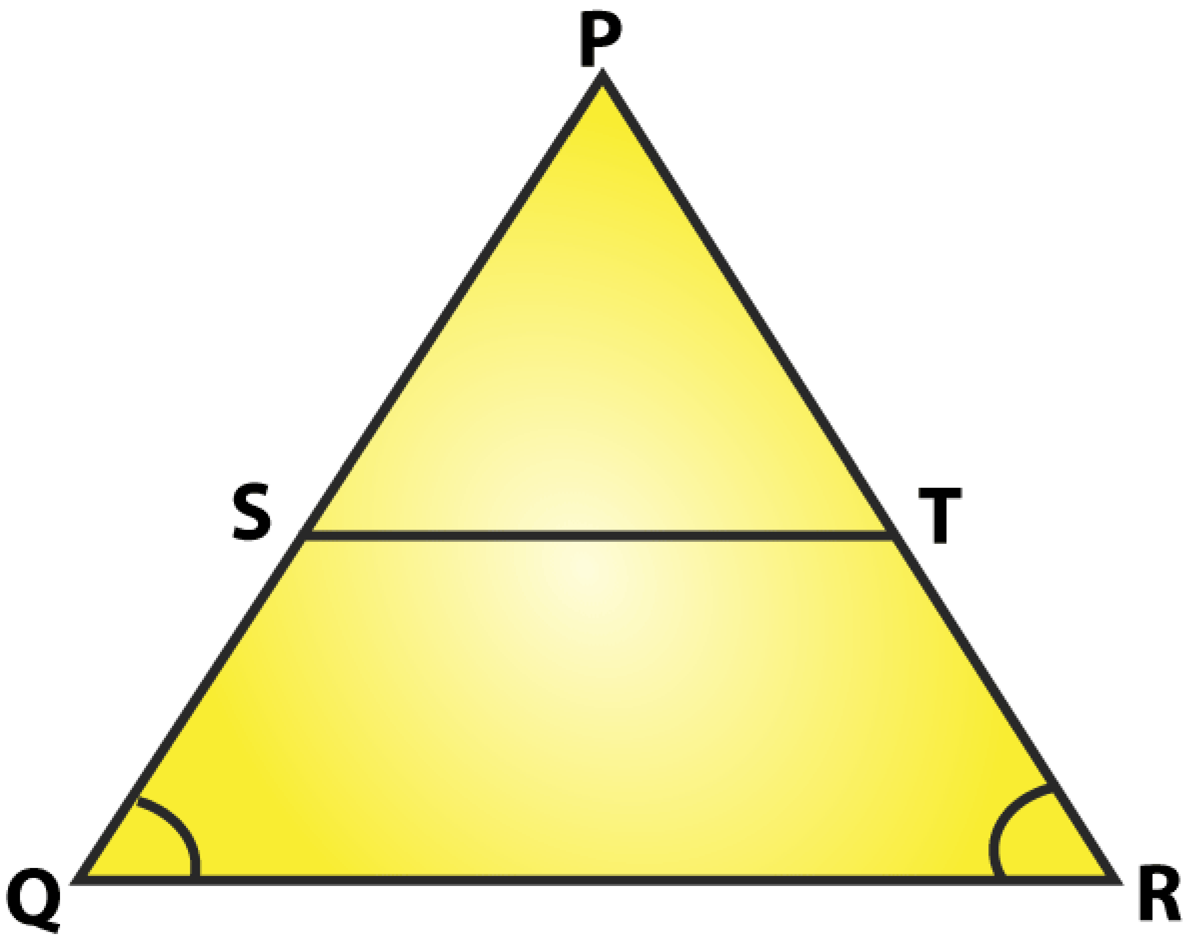
∠ A = 4 x 30° = 120°

Therefore, the angles of the given triangle are 30° and 30° and 120°.

Question 4: PQR is a triangle in which PQ = PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Solution: Given that PQR is a triangle such that PQ = PR and S is any point on the side PQ and ST ∥ QR.

To prove: PS = PT



Since, PQ= PR, so △PQR is an isosceles triangle.

∠ PQR = ∠ PRQ

Now, ∠ PST = ∠ PQR and ∠ PTS = ∠ PRQ

[Corresponding angles as ST parallel to QR]
Since, ∠ PQR = ∠ PRQ

∠ PST = ∠ PTS

In Δ PST,

∠ PST = ∠ PTS

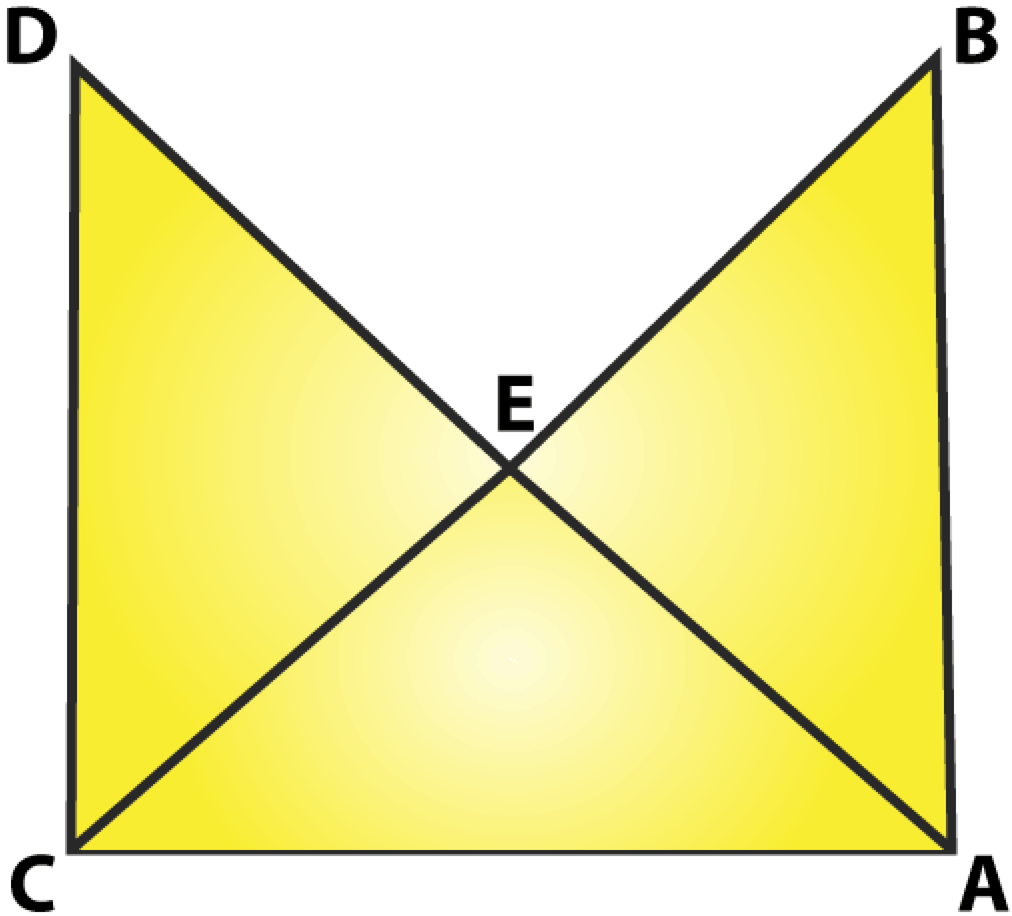
Δ PST is an isosceles triangle.

Therefore, PS = PT.

Hence proved.

EXERCISE 10.4 PAGE NO. 10.47

Question 1: In the figure, It is given that AB = CD and AD = BC. Prove that ΔADC ≅ ΔCBA.



Solution:

From the figure, AB = CD and AD = BC.

To prove: ΔADC ≅ ΔCBA

Consider Δ ADC and Δ CBA.

AB = CD [Given]

BC = AD [Given]

And AC = AC [Common side]

So, by the SSS congruence criterion, we have

ΔADC≅ΔCBA

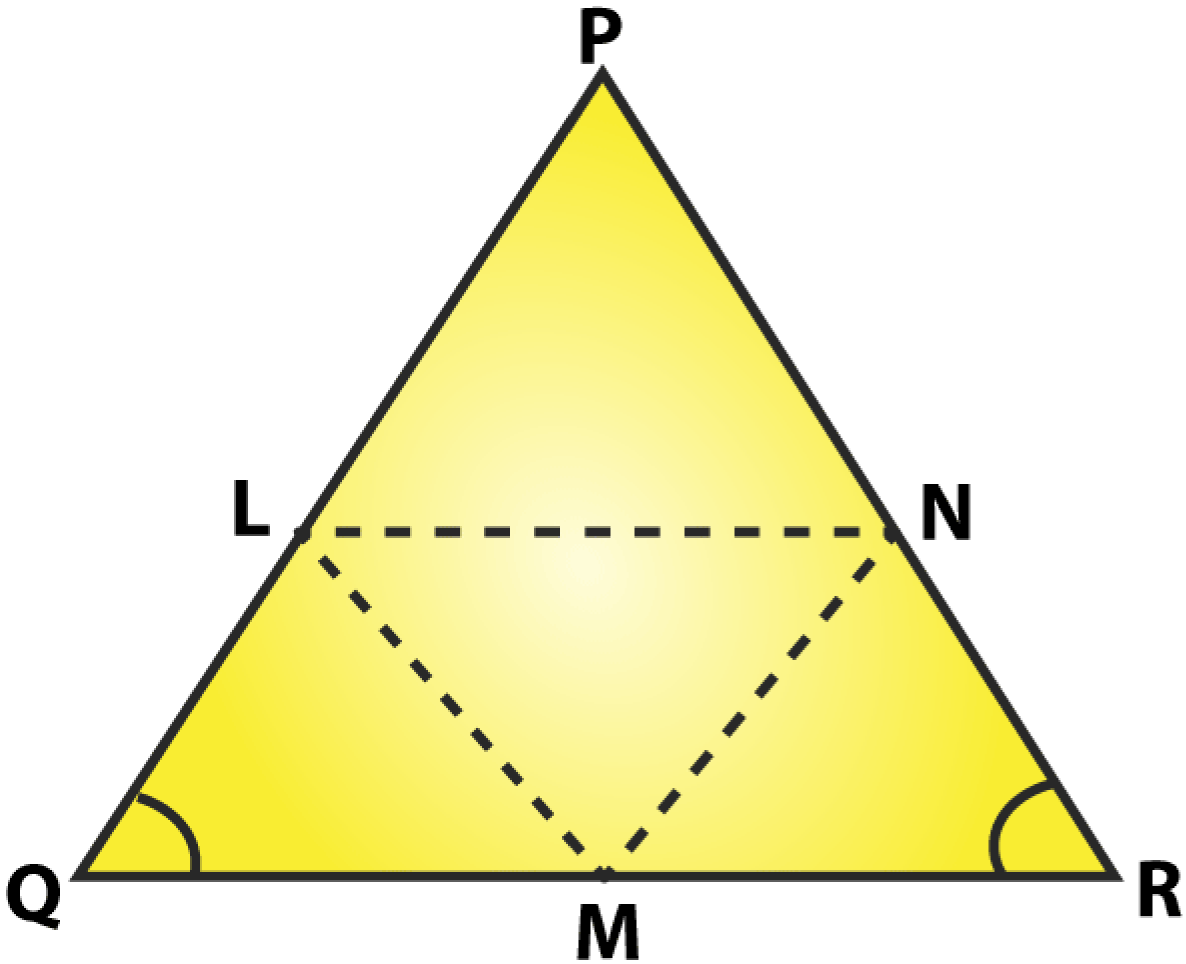
Hence proved.

Question 2: In a Δ PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP, respectively. Prove that LN = MN.

Solution:

Given: In Δ PQR, PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively

To prove: LN = MN



Join L and M, M and N, N and L

We have PL = LQ, QM = MR and RN = NP

[Since L, M and N are mid-points of PQ, QR and RP, respectively]
And also, PQ = QR

PL = LQ = QM = MR = PN = LR …….(i)

[Using mid-point theorem]
MN ∥ PQ and MN = PQ/2

MN = PL = LQ ……(ii)

Similarly, we have

$LN \parallel QR$ and $LN = (1/2)QR$

$LN = QM = MR$ ……(iii)

From equations (i), (ii) and (iii), we have

$PL = LQ = QM = MR = MN = LN$

This implies, $LN = MN$

Hence Proved.

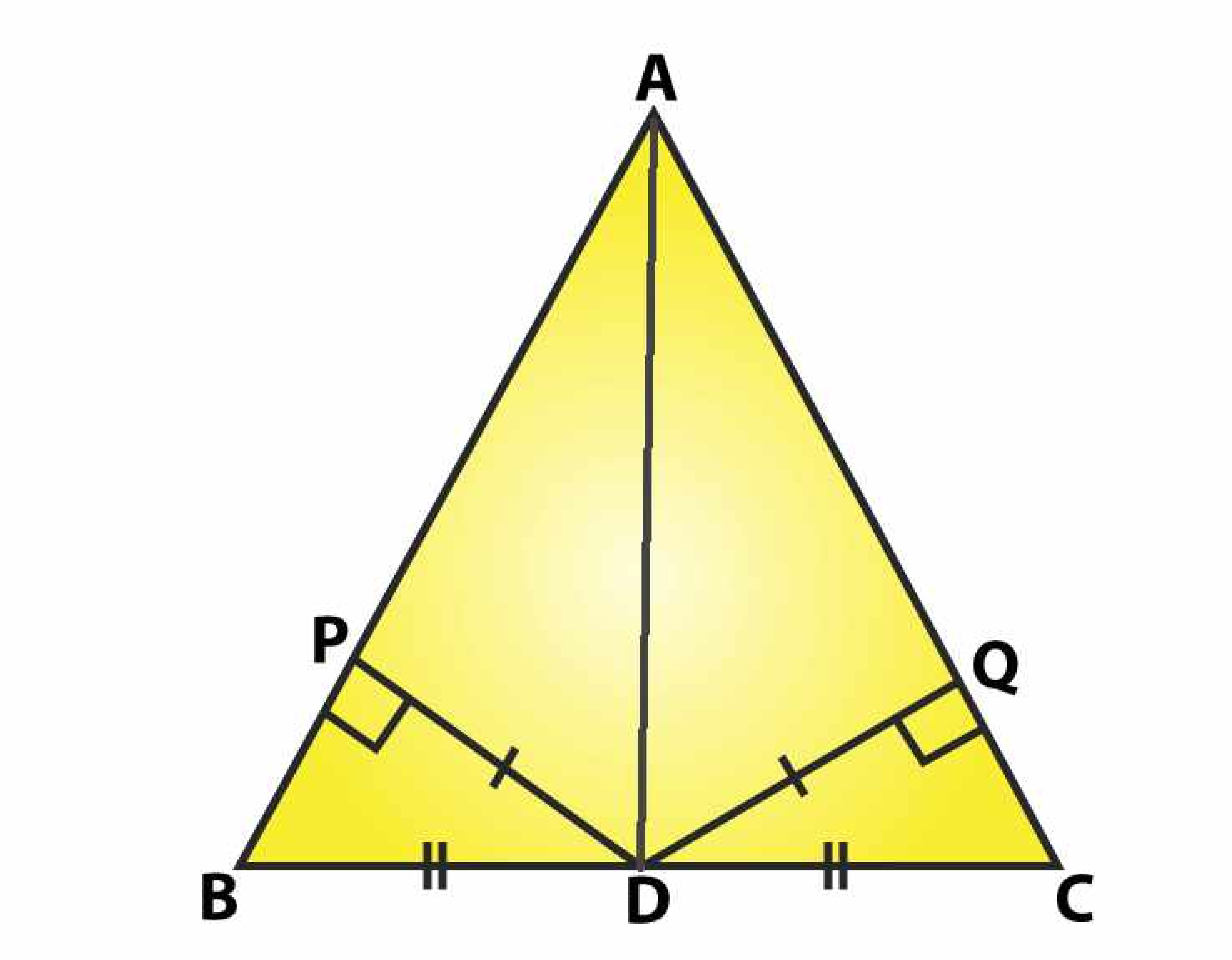
EXERCISE 10.5 PAGE NO. 10.51

Question 1: ABC is a triangle, and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Solution:

Given: D is the midpoint of BC and PD = DQ in a triangle ABC.

To prove: ABC is isosceles triangle.



In △BDP and △CDQ

PD = QD (Given)

BD = DC (D is mid-point)

∠BPD = ∠CQD = 90^{o}

By RHS Criterion: △BDP ≅ △CDQ

BP = CQ … (i) (By CPCT)

In △APD and △AQD

PD = QD (given)

AD = AD (common)

APD = AQD = 90°

By RHS Criterion: △APD ≅ △AQD

So, PA = QA … (ii) (By CPCT)

Adding (i) and (ii)

BP + PA = CQ + QA

AB = AC

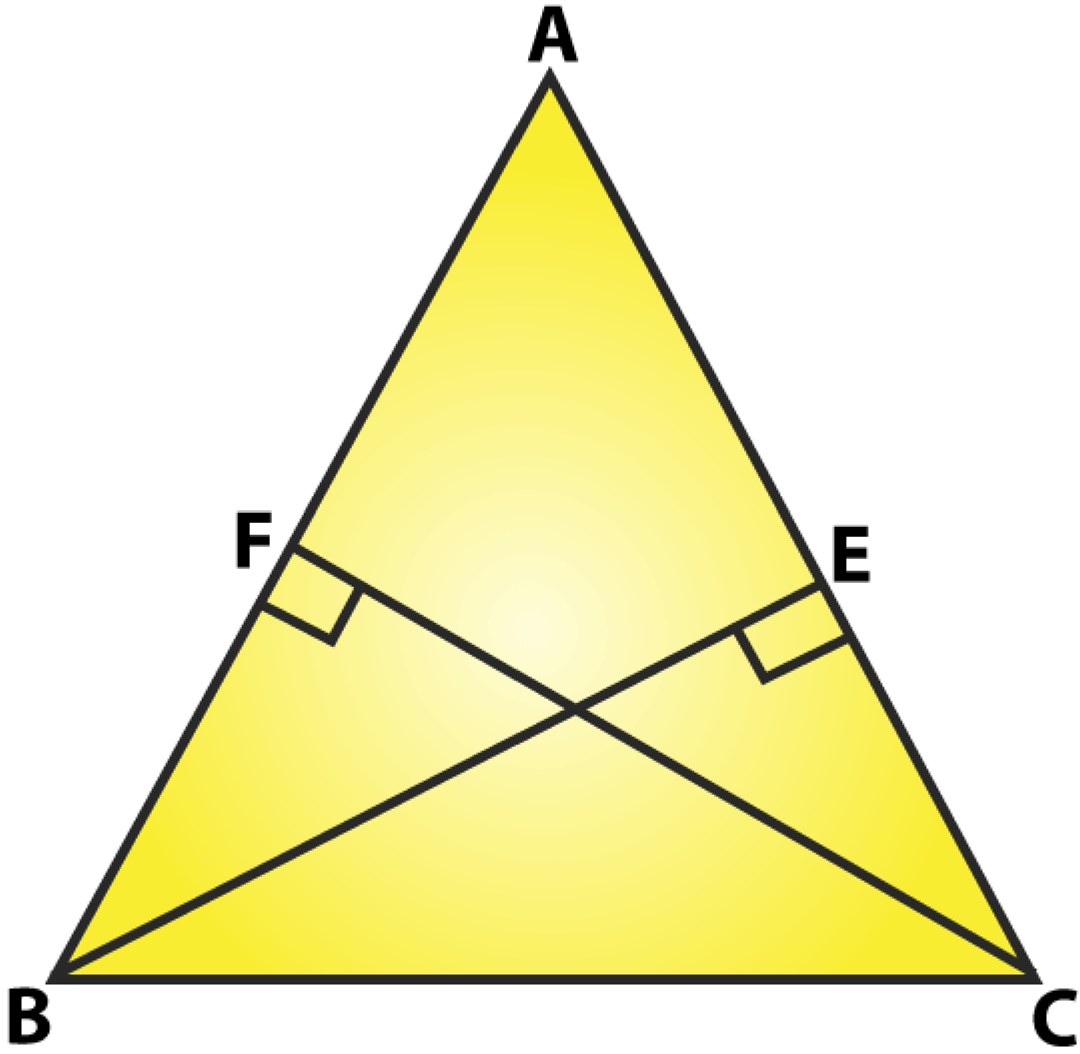
Two sides of the triangle are equal, so ABC is an isosceles.

Question 2: ABC is a triangle in which BE and CF ar**e, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that Δ ABC is isosceles**

Solution:

ABC is a triangle in which BE and CF are perpendicular to the sides AC and AB, respectively, s.t. BE = CF.

To prove: Δ ABC is isosceles



In Δ BCF and Δ CBE,

∠ BFC = CEB = 90° [Given]

BC = CB [Common side]

And CF = BE [Given]

By RHS congruence criterion: ΔBFC ≅ ΔCEB

So, ∠ FBC = ∠ EBC [By CPCT]

=>∠ ABC = ∠ ACB

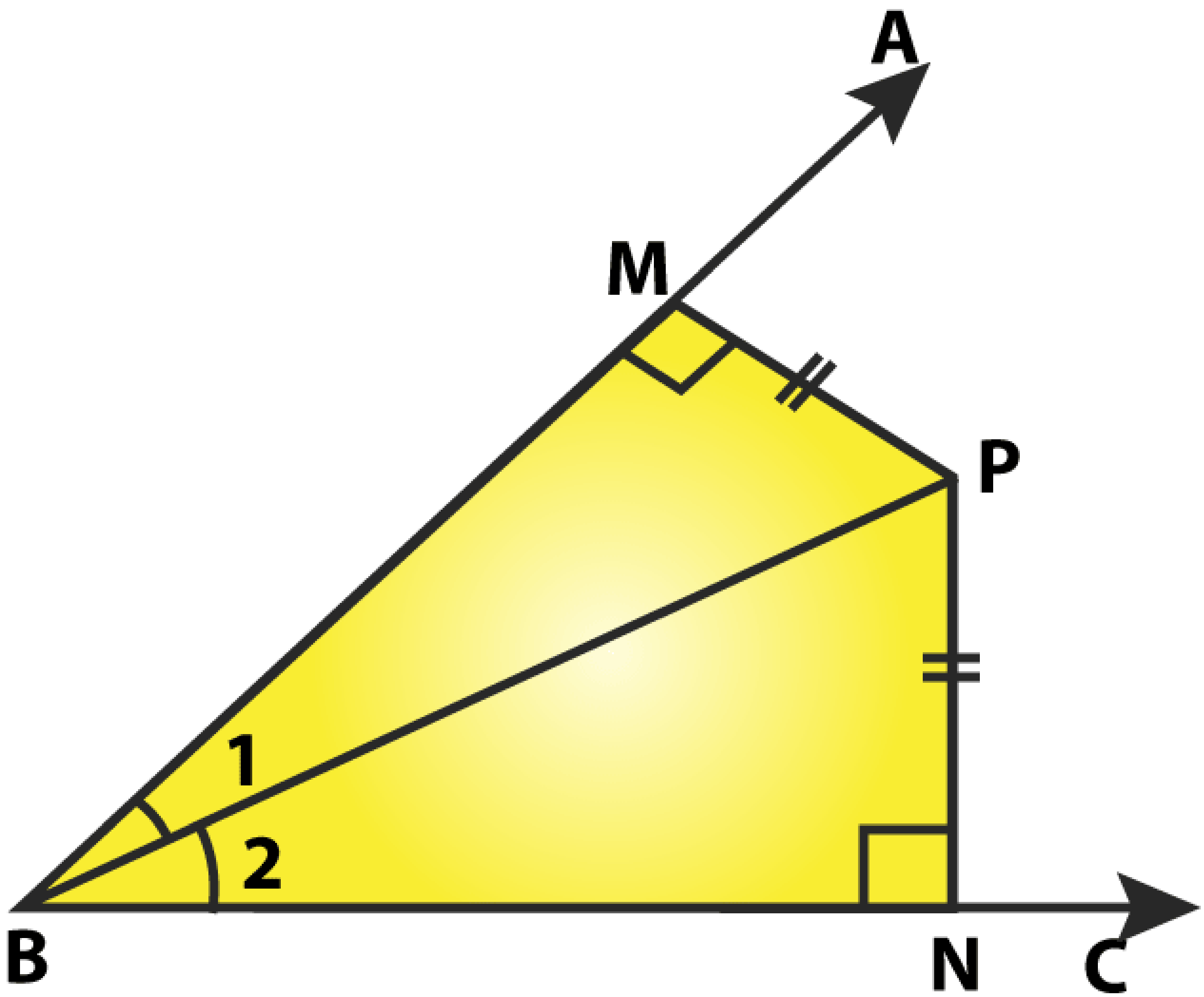
AC = AB [Opposite sides to equal angles are equal in a triangle]

Two sides of triangle ABC are equal.

Therefore, Δ ABC is isosceles. Hence Proved.

Question 3: If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

Solution:



Consider an angle ABC and BP be one of the arms within the angle.

Draw perpendiculars PN and PM on the arms BC and BA.

In Δ BPM and Δ BPN,

$\angle$ BMP = $\angle$ BNP = 90° [given]

BP = BP [Common side]

MP = NP [given]

By RHS congruence criterion: $\Delta BPM \cong \Delta BPN$

So, ∠ MBP = ∠ NBP [By CPCT]

BP is the angular bisector of ∠ABC.

Hence proved

EXERCISE 10.6 PAGE NO: 10.66

Question 1: In Δ ABC, if ∠ A = 40° and ∠ B = 60°. Determine the longest and shortest sides of the triangle.

Solution: In Δ ABC, ∠ A = 40° and ∠ B = 60°

We know the sum of angles in a triangle = 180°

∠ A + ∠ B + ∠ C = 180°

40° + 60° + ∠ C = 180°

∠ C = 180° – 100° = 80°

∠ C = 80°

Now, 40° < 60° < 80°

=> ∠ A < ∠ B < ∠ C

=> ∠ C is a greater angle and ∠ A is a smaller angle.

Now, ∠ A < ∠ B < ∠ C

We know the side opposite to a greater angle is larger, and the side opposite to a smaller angle is smaller.

Therefore, BC < AC < AB

AB is the longest and BC is the shortest side.

Question 2: In a Δ ABC, if ∠ B = ∠ C = 45°, which is the longest side?

Solution: In Δ ABC, ∠ B = ∠ C = 45°

The sum of angles in a triangle = 180°

∠ A + ∠ B + ∠ C = 180°

∠ A + 45° + 45° = 180°

∠ A = 180° – (45° + 45°) = 180° – 90° = 90°

∠ A = 90°

=> ∠ B = ∠ C < ∠ A

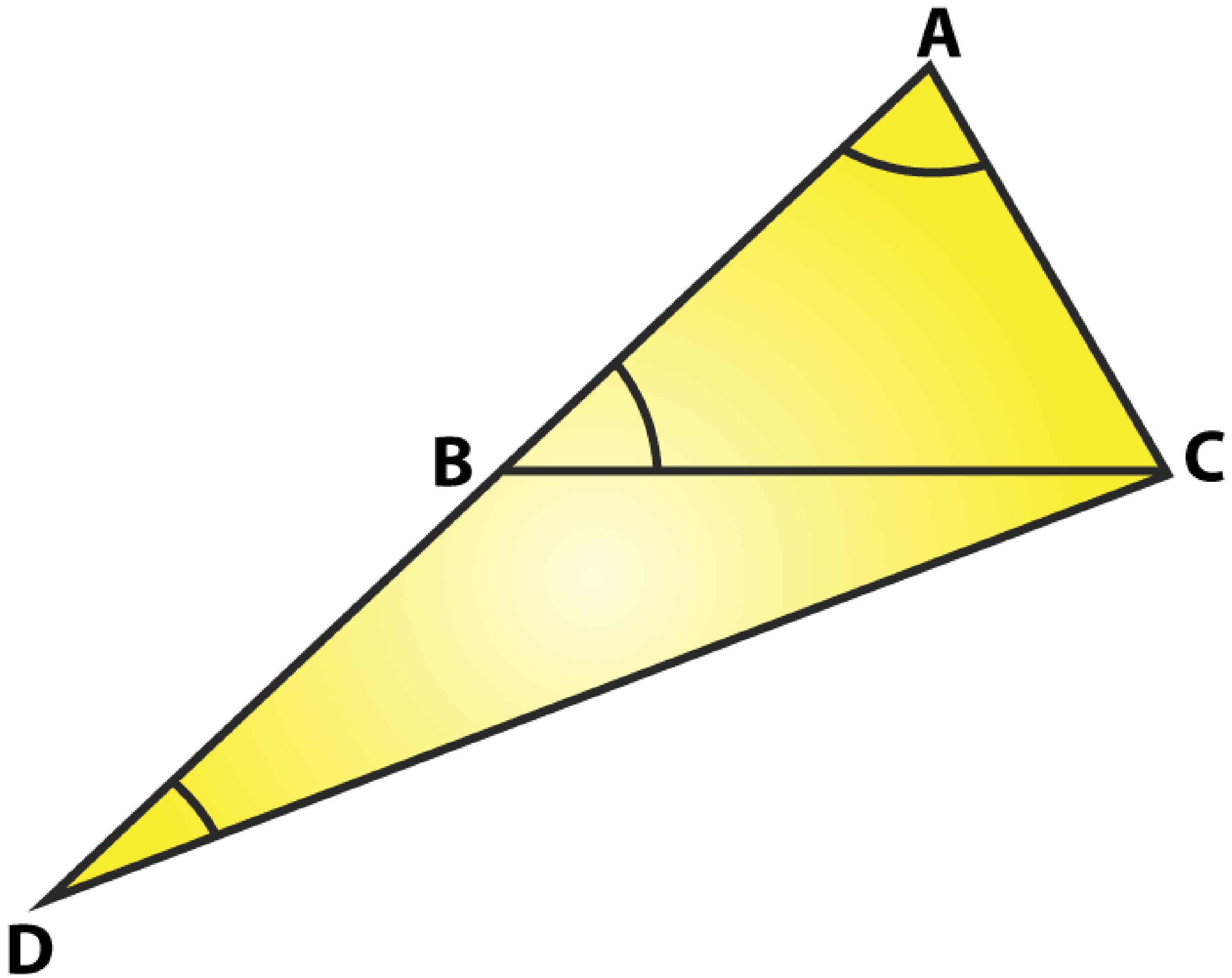
Therefore, BC is the longest side.

Question 3: In Δ ABC, side AB is produced to D so that BD = BC. If ∠ B = 60° and ∠ A = 70°.

Prove that: (i) AD > CD (ii) AD > AC

Solution: In Δ ABC, side AB is produced to D so that BD = BC.

∠ B = 60°, and ∠ A = 70°



To prove: (i) AD > CD (ii) AD > AC

Construction: Join C and D

We know the sum of angles in a triangle = 180°

$\angle A + \angle B + \angle C = 180°$

$70° + 60° + \angle C = 180°$

$\angle C = 180° – (130°) = 50°$

$\angle C = 50°$

$\angle ACB = 50°$ ……(i)

And also in Δ BDC

$\angle DBC = 180° – \angle ABC = 180 – 60° = 120°$

[$\angle$DBA is a straight line]

and BD = BC [given]

∠ BCD = ∠ BDC [Angles opposite to equal sides are equal]

The sum of angles in a triangle =180°

∠ DBC + ∠ BCD + ∠ BDC = 180°

120° + ∠ BCD + ∠ BCD = 180°

120° + 2∠ BCD = 180°

2∠ BCD = 180° – 120° = 60°

∠ BCD = 30°

∠ BCD = ∠ BDC = 30° ….(ii)

Now, consider Δ ADC.

∠ DAC = 70° [given]

∠ ADC = 30° [From (ii)]

∠ ACD = ∠ ACB+ ∠ BCD = 50° + 30° = 80° [From (i) and (ii)]

Now, ∠ ADC < ∠ DAC < ∠ ACD

AC < DC < AD

[Side opposite to the greater angle is longer, and the smaller angle is smaller]
AD > CD and AD > AC

Hence proved.

Question 4: Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Solution:

Lengths of sides are 2 cm, 3 cm and 7 cm.

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

2 + 3 ≯ 7 or 2 + 3 < 7

2 + 7 > 3

and 3 + 7 > 2

Here 2 + 3 ≯ 7

So, the triangle does not exist.

EXERCISE VSAQS PAGE NO. 10.69

Question 1: In two congruent triangles, ABC and DEF, if AB = DE and BC = EF. Name the pairs of equal angles.

Solution:

In two congruent triangles ABC and DEF, if AB = DE and BC = EF, then

∠A = ∠D, ∠B = ∠ E and ∠ C = ∠F

Question 2: In two triangles, ABC and DEF, it is given that ∠A = ∠D, ∠B = ∠ E and ∠ C = ∠F. Are the two triangles necessarily congruent?

Solution: No.

Reason: Two triangles are not necessarily congruent because we know only the angle-angle-angle (AAA) criterion. This criterion can produce similar but not congruent triangles.

Question 3: If ABC and DEF are two triangles such that AC = 2.5 cm, BC = 5 cm, C = 75°, DE = 2.5 cm, DF = 5 cm and D = 75°. Are two triangles congruent?

Solution: Yes.

Reason: Given triangles are congruent as AC = DE = 2.5 cm, BC = DF = 5 cm and

∠D = ∠C = 75°.

By the SAS theorem, triangle ABC is congruent to triangle EDF.

Question 4: In two triangles, ABC and ADC, if AB = AD and BC = CD. Are they congruent?

Solution: Yes.

Reason: Given triangles are congruent as

AB = AD

BC = CD and

AC [common side]

By the SSS theorem, triangle ABC is congruent to triangle ADC.

Question 5: In triangles ABC and CDE, if AC = CE, BC = CD, ∠A = 60°, ∠C = 30° and ∠D = 90°. Are two triangles congruent?

Solution: Yes.

Reason: Given triangles are congruent

Here AC = CE

BC = CD

∠B = ∠D = 90°

By SSA criteria, triangle ABC is congruent to triangle CDE.

Question 6: ABC is an isosceles triangle in which AB = AC. BE and CF are its two medians. Show that BE = CF.

Solution: ABC is an isosceles triangle (given)

AB = AC (given)

BE and CF are two medians (given)

To prove: BE = CF

In △CFB and △BEC

CE = BF (Since, AC = AB = AC/2 = AB/2 = CE = BF)

BC = BC (Common)

∠ECB = ∠FBC (Angle opposite to equal sides are equal)

By SAS theorem: △CFB ≅ △BEC

So, BE = CF (By c.p.c.t)