Exercise 2.4 Page: 43

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x) = x^3+x^2+x+1$

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴ By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let $p(x)= x^4+x^3+x^2+x+1$

The zero of x+1 is -1. [x+1= 0 means x = -1]

$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let $p(x)= x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$=1-3+3-1+1$

$=1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x) = x^3-x^2-(2+\sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1) = (-1)^3-(-1)^2-(2+\sqrt{2})(-1) + \sqrt{2} = -1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2} \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^3–x^2–(2+\sqrt{2})x +\sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3+x^2–2x–1$, $g(x) = x+1$

Solution:

$p(x) = 2x^3+x^2–2x–1$, $g(x) = x+1$

$g(x) = 0$

$\Rightarrow x+1 = 0$

$\Rightarrow x = -1$

∴ Zero of g(x) is -1.

Now,

$p(-1) = 2(-1)^3+(-1)^2–2(-1)–1$

$= -2+1+2-1$

$= 0$

∴ By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x+2$

Solution:

$p(x) = x^3+3x^2+3x+1$, $g(x) = x+2$

$g(x) = 0$

$\Rightarrow x+2 = 0$

$\Rightarrow x = -2$

∴ Zero of g(x) is -2.

Now,

$p(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+12-6+1$

$= -1 \neq 0$

∴ By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3–4x^2+x+6$, $g(x) = x–3$

Solution:

$p(x) = x^3–4x^2+x+6$, $g(x) = x -3$

$g(x) = 0$

$\Rightarrow x-3 = 0$

$\Rightarrow x = 3$

$\therefore$ Zero of g(x) is 3.

Now,

$p(3) = (3)^3-4(3)^2+(3)+6$

$= 27-36+3+6$

$= 0$

$\therefore$ By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow (1)^2+(1)+k = 0$

$\Rightarrow 1+1+k = 0$

$\Rightarrow 2+k = 0$

$\Rightarrow k = -2$

(ii) $p(x) = 2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2} = 0$

$\Rightarrow 2+k+\sqrt{2} = 0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x) = kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k = 0$

$\Rightarrow k-3+k = 0$

$\Rightarrow 2k-3 = 0$

$\Rightarrow k= 3/2$

4. Factorise:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product =1×12 = 12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4 = 12]

$12x^2-7x+1= 12x^2-4x-3x+1$

$= 4x(3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product = 2×3 = 6

We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$= 2x\ (x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product = 6×-6 = -36

We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

$6x^2+5x-6 = 6x^2+9x-4x-6$

$= 3x(2x+3)-2(2x+3)$

$= (2x+3)(3x-2)$

(iv) $3x^2-x-4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -1 and product = 3×-4 = -12

We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

$3x^2–x–4 = 3x^2–4x+3x–4$

$= x(3x–4)+1(3x–4)$

$= (3x–4)(x+1)$

5. Factorise:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x) = x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

Now,

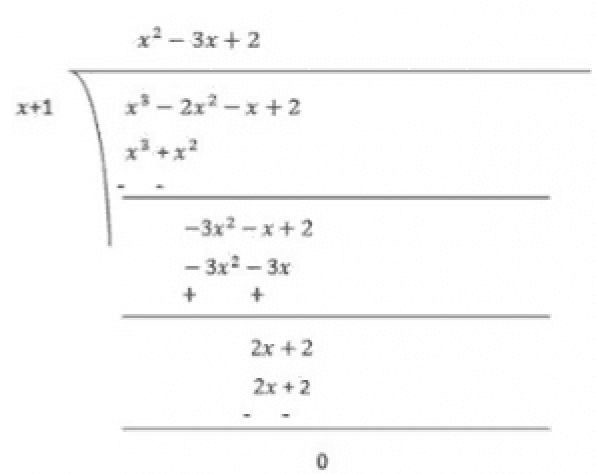
$p(x) = x^3–2x^2–x+2$

$p(−1) = (−1)^3–2(−1)^2–(−1)+2$

$= −1−2+1+2$

$= 0$

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) = (x+1)(x^2–x–2x+2)$

$= (x+1)(x(x−1)−2(x−1))$

$= (x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By the trial method, we find that

$p(5) = 0$

So, (x-5) is factor of p(x)

Now,

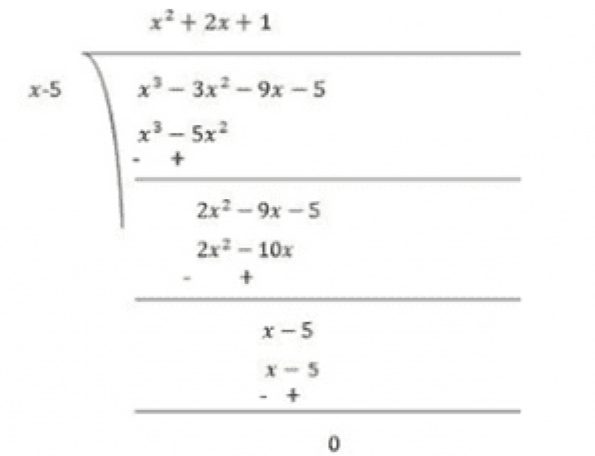
$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$= 125−75−45−5$

$= 0$

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x−5)(x^2+2x+1) = (x−5)(x^2+x+x+1)$

$= (x−5)(x(x+1)+1(x+1))$

$= (x−5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By the trial method, we find that

$p(-1) = 0$

So, (x+1) is factor of p(x)

Now,

p(x)= $x^3+13x^2+32x+20$

p(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

= $-1+13-32+20$

= 0

Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2+32x+20 \\
 & \quad 12x^2+12x \\
 & \quad - \quad - \\
\hline
 & \qquad 20x+20 \\
 & \qquad 20x+20 \\
 & \qquad - \quad - \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient +Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$

= $(x+1)x(x+2)+10(x+2)$

= $(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let p(y) = $2y^3+y^2-2y-1$

Factors = $2\times(-1)$= -2 are ±1 and ±2

By the trial method, we find that

p(1) = 0

So, (y-1) is factor of p(y)

Now,

p(y) = $2y^3+y^2-2y-1$

p(1) = $2(1)^3+(1)^2-2(1)-1$

= 2+1−2

= 0

Therefore, (y-1) is the factor of p(y)

$$\begin{array}{r|l}
 & 2y^2+3y+1 \\
\hline
y\text{-}1 & 2y^3+y^2-2y-1 \\
 & 2y^3-2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2-2y-1 \\
 & \quad 3y^2-3y \\
 & \quad - \quad + \\
\hline
 & \qquad y-1 \\
 & \qquad y-1 \\
 & \qquad - \; + \\
\hline
 & \qquad\quad 0
\end{array}$$

Now, Dividend = Divisor × Quotient + Remainder

$(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1)$

$= (y-1)(2y(y+1)+1(y+1))$

$= (y-1)(2y+1)(y+1)$