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1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, $a = 4$ and $b = 10$]
We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) $(x+8)(x-10)$

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, $a = 8$ and $b = -10$]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) $(3x+4)(3x-5)$

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, $x = 3x$, $a = 4$ and $b = -5$]
We get,

$(3x+4)(3x-5) = (3x)^2+[4+(-5)]3x+4\times(-5)$

$= 9x^2+3x(4-5)-20$

$= 9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x+y)(x-y) = x^2-y^2$

[Here, $x = y^2$ and $y = 3/2$]
We get,

$(y^2+3/2)(y^2-3/2) = (y^2)^2-(3/2)^2$

$= y^4-9/4$

2. Evaluate the following products without multiplying directly:

(i) 103×107

Solution:

$103\times107 = (100+3)\times(100+7)$

Using identity, $[(x+a)(x+b) = x^2+(a+b)x+ab$

Here, $x = 100$

$a = 3$

$b = 7$

We get, $103\times107 = (100+3)\times(100+7)$

$= (100)^2+(3+7)100+(3\times7)$

$= 10000+1000+21$

$= 11021$

(ii) 95×96

Solution:

$95\times96 = (100-5)\times(100-4)$

Using identity, $[(x-a)(x-b) = x^2-(a+b)x+ab$

Here, $x = 100$

$a = -5$

$b = -4$

We get, $95\times96 = (100-5)\times(100-4)$

$= (100)^2+100(-5+(-4))+(-5\times-4)$

$= 10000-900+20$

$= 9120$

(iii) 104×96

Solution:

$104\times96 = (100+4)\times(100-4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a = 100$

$b = 4$

We get, $104\times96 = (100+4)\times(100-4)$

$= (100)^2-(4)^2$

$= 10000-16$

$= 9984$

3. Factorise the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2+2xy+y^2 = (x+y)^2$

Here, $x = 3x$

$y = y$

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1$

Using identity, $x^2 - 2xy + y^2 = (x - y)^2$

Here, $x = 2y$

$y = 1$

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1^2$

$= (2y-1)^2$

$= (2y-1)(2y-1)$

(iii) $x^2-y^2/100$

Solution:

$x^2-y^2/100 = x^2-(y/10)^2$

Using identity, $x^2-y^2 = (x-y)(x+y)$

Here, $x = x$

$y = y/10$

$x^2-y^2/100 = x^2-(y/10)^2$

$= (x-y/10)(x+y/10)$

4. Expand each of the following using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a -7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

(i) $(x+2y+4z)^2$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = x$

$y = 2y$

$z = 4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 2x$

$y = -y$

$z = z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = -2x$

$y = 3y$

$z = 2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

Solution:

Using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 3a$

$y = -7b$

$z = -c$

$(3a-7b-c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a\times -7b)+(2\times -7b\times -c)+(2\times -c\times 3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x+5y-3z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = -2x$

$y = 5y$

$z = -3z$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+(2\times-2x \times 5y)+(2\times 5y\times-3z)+(2\times-3z \times-2x)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12zx$

(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = (1/4)a$

$y = (-1/2)b$

$z = 1$

$$((1/4)a-(1/2)b+1)^2 = (\tfrac{1}{4}a)^2+(-\tfrac{1}{2}b)^2+(1)^2+(2\times\tfrac{1}{4}a\times-\tfrac{1}{2}b)+(2\times-\tfrac{1}{2}b\times1)+(2\times1\times\tfrac{1}{4}a)$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b+\frac{2}{4}a$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

5. Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x)^2+(3y)^2+(-4z)^2+(2\times2x\times3y)+(2\times3y\times-4z)+(2\times-4z\times2x)$

$= (2x+3y-4z)^2$

$= (2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$= (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{2}\times-\sqrt{2}x)$

$= (-\sqrt{2}x+y+2\sqrt{2}z)^2$

$= (-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $((3/2)x+1)^3$

(iv) $(x-(2/3)y)^3$

Solution:

(i) $(2x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$(2x+1)^3= (2x)^3+1^3+(3\times2x\times1)(2x+1)$

$= 8x^3+1+6x(2x+1)$

$= 8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(2a-3b)^3 = (2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$= 8a^3-27b^3-18ab(2a-3b)$

$= 8a^3-27b^3-36a^2b+54ab^2$

(iii) $((3/2)x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$((3/2)x+1)^3=((3/2)x)^3+1^3+(3\times(3/2)x\times1)((3/2)x +1)$

$$= \frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$$
$$= \frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$$
$$= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $(x-(2/3)y)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$(x-\frac{2}{3}y)^3=(x)^3-(\frac{2}{3}y)^3-(3\times x\times\frac{2}{3}y)(x-\frac{2}{3}y)$$
$$=(x)^3-\frac{8}{27}y^3-2xy(x-\frac{2}{3}y)$$
$$=(x)^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(99)^3 = (100-1)^3$

$= (100)^3-1^3-(3\times100\times1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000-1-30000+300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$(100+2)^3 = (100)^3+2^3+(3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(998)^3 = (1000-2)^3$

$=(1000)^3–2^3–(3×1000×2)(1000–2)$

$= 1000000000–8–6000(1000– 2)$

$= 1000000000–8- 6000000+12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$

(ii) $8a^3–b^3–12a^2b+6ab^2$

(iii) $27–125a^3–135a +225a^2$

(iv) $64a^3–27b^3–144a^2b+108ab^2$

(v) $27p^3–(1/216)−(9/2) p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

Here, the identity, $(x +y)^3 = x^3+y^3+3xy(x+y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 = (2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$= (2a–b)^3$

$= (2a–b)(2a–b)(2a–b)$

Here, the identity,$(x–y)^3 = x^3–y^3–3xy(x–y)$ is used.

(iii) $27–125a^3–135a+225a^2$

Solution:

The expression, $27–125a^3–135a +225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2 =$
$3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$= (3–5a)^3$

$= (3–5a)(3–5a)(3–5a)$

Here, the identity, $(x–y)^3 = x^3–y^3-3xy(x–y)$ is used.

(iv) $64a^3–27b^3–144a^2b+108ab^2$

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2=$
$(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3– (1/216)−(9/2) p^2+(1/4)p$

Solution:

The expression, $27p^3–(1/216)−(9/2) p^2+(1/4)p$ can be written as

$(3p)^3–(1/6)^3−(9/2) p^2+(1/4)p = (3p)^3–(1/6)^3−3(3p)(1/6)(3p – 1/6)$

Using $(x – y)^3 = x^3 – y^3 – 3xy (x – y)$

$27p^3–(1/216)−(9/2) p^2+(1/4)p = (3p)^3–(1/6)^3−3(3p)(1/6)(3p – 1/6)$

Taking $x = 3p$ and $y = 1/6$

$= (3p–1/6)^3$

$= (3p–1/6)(3p–1/6)(3p–1/6)$

9. Verify:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$

(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3 = (x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3 = (x+y)[(x+y)^2–3xy]$

Taking $(x+y)$ common $\Rightarrow x^3+y^3 = (x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3 = (x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

We know that, $(x–y)^3 = x^3–y^3–3xy(x–y)$

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)[(x-y)^2 + 3xy]$

Taking (x+y) common $\Rightarrow x^3 - y^3 = (x-y)[(x^2+y^2-2xy)+3xy]$

$\Rightarrow x^3 + y^3 = (x-y)(x^2+y^2+xy)$

10. Factorise each of the following:

(i) $27y^3+125z^3$

(ii) $64m^3-343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 = (3y)^3+(5z)^3$

We know that, $x^3+y^3 = (x+y)(x^2-xy+y^2)$

$27y^3+125z^3 = (3y)^3+(5z)^3$

$= (3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$= (3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 =$
$(4m)^3-(7n)^3$

We know that, $x^3-y^3 = (x-y)(x^2+xy+y^2)$

$64m^3-343n^3 = (4m)^3-(7n)^3$

$= (4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$= (4m-7n)(16m^2+28mn+49n^2)$

11. Factorise: $27x^3+y^3+z^3-9xyz$.

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

$= (3x+y+z)[(3x)^2+y^2+z^2-3xy-yz-3xz]$

$= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify that:

$x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

Solution:

We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

$\Rightarrow x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$

$= (1/2)(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$

$= (1/2)(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$

$= (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

13. If $x+y+z = 0$, show that $x^3+y^3+z^3 = 3xyz$.

Solution:

We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

Now, according to the question, let $(x+y+z) = 0$,

Then, $x^3+y^3+z^3-3xyz = (0)(x^2+y^2+z^2-xy-yz-xz)$

$\Rightarrow x^3+y^3+z^3-3xyz = 0$

$\Rightarrow x^3+y^3+z^3 = 3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

(i) $(-12)^3+(7)^3+(5)^3$

Let $a = -12$

$b = 7$

$c = 5$

We know that if $x+y+z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3\times-12\times7\times5$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a = 28

b = −15

c = −13

We know that if x+y+z = 0, then $x^3+y^3+z^3 = 3xyz$.

Here, x+y+z = 28–15–13 = 0

$(28)^3+(-15)^3+(-13)^3 = 3xyz$

= 0+3(28)(−15)(−13)

= 16380

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2-35a+12$

(ii) Area: $35y^2+13y-12$

Solution:

(i) Area: $25a^2-35a+12$

Using the splitting the middle term method,

We have to find a number whose sum = -35 and product =25×12 = 300

We get -15 and -20 as the numbers [-15+-20=-35 and -15×-20 = 300]

$25a^2-35a+12 = 25a^2-15a-20a+12$

= 5a(5a–3)–4(5a–3)

= (5a–4)(5a–3)

Possible expression for length = 5a–4

Possible expression for breadth = 5a –3

(ii) Area: $35y^2+13y-12$

Using the splitting the middle term method,

We have to find a number whose sum = 13 and product = 35×-12 = 420

We get -15 and 28 as the numbers [-15+28 = 13 and -15×28=420]

$35y^2+13y-12 = 35y^2-15y+28y-12$

= 5y(7y–3)+4(7y–3)

= (5y+4)(7y–3)

Possible expression for length = (5y+4)

Possible expression for breadth = (7y–3)

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2–12x$

(ii) Volume: $12ky^2+8ky–20k$

Solution:

(i) Volume: $3x^2–12x$

$3x^2–12x$ can be written as $3x(x–4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = (x–4)

(ii) Volume:
$12ky^2+8ky–20k$

$12ky^2+8ky–20k$ can be written as $4k(3y^2+2y–5)$ by taking $4k$ out of both the terms.

$12ky^2+8ky–20k = 4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$= 4k(3y^2+5y–3y–5)$

$= 4k[y(3y+5)–1(3y+5)]$

$= 4k(3y+5)(y–1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)