Exercise 2.1 Page: 32

1. Which of the following expressions are polynomials in one variable, and which are not? State reasons for your answer.

(i) $4x^2–3x+7$

Solution:

The equation $4x^2–3x+7$ can be written as $4x^2–3x^1+7x^0$

Since x is the only variable in the given equation and the powers of x (i.e. 2, 1 and 0) are whole numbers, we can say that the expression $4x^2–3x+7$ is a polynomial in one variable.

(ii) $y^2+\sqrt{2}$

Solution:

The equation $y^2+\sqrt{2}$ can be written as $y^2+\sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2+\sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t}+t\sqrt{2}$

Solution:

The equation $3\sqrt{t}+t\sqrt{2}$ can be written as $3t^{1/2}+\sqrt{2}t$

Though t is the only variable in the given equation, the power of t (i.e., 1/2) is not a whole number. Hence, we can say that the expression $3\sqrt{t}+t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) y+2/y

Solution:

The equation y+2/y can be written as $y+2y^{-1}$

Though y is the only variable in the given equation, the power of y (i.e., -1) is not a whole number. Hence, we can say that the expression y+2/y is **not** a polynomial in one variable.

(v) $x^{10}+y^3+t^{50}$

Solution:

Here, in the equation $x^{10}+y^3+t^{50}$

Though the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10}+y^3+t^{50}$. Hence, it is **not** a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

Solution:

The equation $2+x^2+x$ can be written as $2+(1)x^2+x$

We know that the coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

Hence, the coefficient of x^2 in $2+x^2+x$ is 1.

(ii) $2–x^2+x^3$

Solution:

The equation $2–x^2+x^3$ can be written as $2+(–1)x^2+x^3$

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

Hence, the coefficient of x^2 in $2–x^2+x^3$ is -1.

(iii) $(\pi/2)x^2+x$

Solution:

The equation $(\pi/2)x^2+x$ can be written as $(\pi/2)x^2 + x$

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\pi/2$.

Hence, the coefficient of x^2 in $(\pi/2)x^2+x$ is $\pi/2$.

(iii) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2+\sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

Hence, the coefficient of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35.

For example, $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100.

For example, $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3+4x^2+7x = 5x^3+4x^2+7x^1$

The powers of the variable x are: 3, 2, 1

The degree of $5x^3+4x^2+7x$ is 3, as 3 is the highest power of x in the equation.

(ii) $4–y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4–y^2$,

The power of the variable y is 2

The degree of $4–y^2$ is 2, as 2 is the highest power of y in the equation.

(iii) $5t–\sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t–\sqrt{7}$

The power of the variable t is: 1

The degree of $5t–\sqrt{7}$ is 1, as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3 = 3\times1 = 3\times x^0$

The power of the variable here is: 0

Hence, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

(i) x^2+x

Solution:

The highest power of x^2+x is 2

The degree is 2

Hence, x^2+x is a quadratic polynomial

(ii) $x–x^3$

Solution:

The highest power of $x–x^3$ is 3

The degree is 3

Hence, $x–x^3$ is a cubic polynomial

(iii) $y+y^2+4$

Solution:

The highest power of $y+y^2+4$ is 2

The degree is 2

Hence, $y+y^2+4$ is a quadratic polynomial

(iv) 1+x

Solution:

The highest power of 1+x is 1

The degree is 1

Hence, 1+x is a linear polynomial.

(v) 3t

Solution:

The highest power of 3t is 1

The degree is 1

Hence, 3t is a linear polynomial.

(vi) r^2

Solution:

The highest power of r^2 is 2

The degree is 2

Hence, $r^2$is a quadratic polynomial.

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

The degree is 3

Hence, $7x^3$ is a cubic polynomial.

Exercise 2.2 Page: 34

1. Find the value of the polynomial (x)=$5x-4x^2+3$.

(i) x = 0

(ii) x = – 1

(iii) x = 2

Solution:

Let $f(x) = 5x-4x^2+3$

(i) When x = 0

$f(0) = 5(0)-4(0)^2+3$

= 3

(ii) When x = -1

$f(x) = 5x-4x^2+3$

$f(-1) = 5(-1)-4(-1)^2+3$

$= -5-4+3$

$= -6$

(iii) When x = 2

$f(x) = 5x-4x^2+3$

$f(2) = 5(2)-4(2)^2+3$

$= 10-16+3$

$= -3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y) = y^2-y+1$

$\therefore p(0) = (0)^2-(0)+1 = 1$

$p(1) = (1)^2-(1)+1 = 1$

$p(2) = (2)^2-(2)+1 = 3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t) = 2+t+2t^2-t^3$

$\therefore p(0) = 2+0+2(0)^2-(0)^3 = 2$

$p(1) = 2+1+2(1)^2–(1)^3=2+1+2–1 = 4$

$p(2) = 2+2+2(2)^2–(2)^3=2+2+8–8 = 4$

(iii) $p(x)=x^3$

Solution:

$p(x) = x^3$

$\therefore p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

(iv) $p(x) = (x−1)(x+1)$

Solution:

$p(x) = (x–1)(x+1)$

$\therefore p(0) = (0–1)(0+1) = (−1)(1) = –1$

$p(1) = (1–1)(1+1) = 0(2) = 0$

$p(2) = (2–1)(2+1) = 1(3) = 3$

3. Verify whether the following are zeroes of the polynomial indicated against them.

(i) $p(x)=3x+1$, $x = −1/3$

Solution:

For, $x = -1/3$, $p(x) = 3x+1$

$\therefore p(−1/3) = 3(-1/3)+1 = −1+1 = 0$

$\therefore$ -1/3 is a zero of p(x).

(ii) $p(x) = 5x–\pi$, $x = 4/5$

Solution:

For, $x = 4/5$, $p(x) = 5x–\pi$

$\therefore p(4/5) = 5(4/5)- \pi = 4-\pi$

$\therefore$ 4/5 is not a zero of p(x).

(iii) $p(x) = x^2−1$, $x = 1, −1$

Solution:

For, $x = 1, −1$;

$p(x) = x^2−1$

$\therefore p(1)=1^2−1=1−1 = 0$

$p(−1)=(-1)^2−1 = 1−1 = 0$

$\therefore$ 1, −1 are zeros of p(x).

(iv) $p(x) = (x+1)(x-2)$, $x = -1, 2$

Solution:

For, $x = -1, 2$;

$p(x) = (x+1)(x-2)$

$\therefore p(-1) = (-1+1)(-1-2)$

$= (0)(-3) = 0$

$p(2) = (2+1)(2-2) = (3)(0) = 0$

$\therefore -1, 2$ are zeros of $p(x)$.

(v) $p(x) = x^2$, $x = 0$

Solution:

For, $x = 0$ $p(x) = x^2$

$p(0) = 0^2 = 0$

$\therefore$ 0 is a zero of $p(x)$.

(vi) $p(x) = lx+m$, $x = -m/l$

Solution:

For, $x = -m/l$; $p(x) = lx+m$

$\therefore p(-m/l) = l(-m/l)+m = -m+m = 0$

$\therefore -m/l$ is a zero of $p(x)$.

(vii) $p(x) = 3x^2-1$, $x = -1/\sqrt{3}$, $2/\sqrt{3}$

Solution:

For, $x = -1/\sqrt{3}$, $2/\sqrt{3}$; $p(x) = 3x^2-1$

$\therefore p(-1/\sqrt{3}) = 3(-1/\sqrt{3})^2-1 = 3(1/3)-1 = 1-1 = 0$

$\therefore p(2/\sqrt{3}) = 3(2/\sqrt{3})^2-1 = 3(4/3)-1 = 4-1 = 3 \neq 0$

$\therefore -1/\sqrt{3}$ is a zero of $p(x)$, but $2/\sqrt{3}$ is not a zero of $p(x)$.

(viii) $p(x) = 2x+1$, $x = 1/2$

Solution:

For, $x = 1/2$ $p(x) = 2x+1$

$\therefore p(1/2) = 2(1/2)+1 = 1+1 = 2 \neq 0$

$\therefore 1/2$ is not a zero of $p(x)$.

4. Find the zero of the polynomials in each of the following cases:

(i) $p(x) = x+5$

Solution:

$p(x) = x+5$

$\Rightarrow x+5 = 0$

$\Rightarrow x = -5$

∴ -5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x–5

Solution:

$p(x) = x-5$

$\Rightarrow x-5 = 0$

$\Rightarrow x = 5$

∴ 5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x+5

Solution:

$p(x) = 2x+5$

$\Rightarrow 2x+5 = 0$

$\Rightarrow 2x = -5$

$\Rightarrow x = -5/2$

∴x = -5/2 is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x–2

Solution:

$p(x) = 3x-2$

$\Rightarrow 3x-2 = 0$

$\Rightarrow 3x = 2$

$\Rightarrow x = 2/3$

∴ x = 2/3 is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x) = 3x$

$\Rightarrow 3x = 0$

$\Rightarrow x = 0$

∴ 0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a≠0

Solution:

p(x) = ax

⇒ ax = 0

⇒ x = 0

∴ x = 0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx+d, c ≠ 0, c, d are real numbers.

Solution:

p(x) = cx + d

⇒ cx+d =0

⇒ x = -d/c

∴ x = -d/c is a zero polynomial of the polynomial p(x).

Exercise 2.3

Page: 40

1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) x+1

Solution:

$x+1= 0$

$\Rightarrow x = -1$

∴ Remainder:

$p(-1) = (-1)^3+3(-1)^2+3(-1)+1$

$= -1+3-3+1$

$= 0$

(ii) x−1/2

Solution:

$x-1/2 = 0$

$\Rightarrow x = 1/2$

∴ Remainder:

$p(1/2) = (1/2)^3+3(1/2)^2+3(1/2)+1$

$= (1/8)+(3/4)+(3/2)+1$

$= 27/8$

(iii) x

Solution:

$x = 0$

∴ Remainder:

$p(0) = (0)^3+3(0)^2+3(0)+1$

$= 1$

(iv) x+π

Solution:

$x+\pi = 0$

$\Rightarrow x = -\pi$

∴ Remainder:

$p(0) = (-\pi)^3 +3(-\pi)^2+3(-\pi)+1$

$= -\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x = 0$

$\Rightarrow 2x = -5$

$\Rightarrow x = -5/2$

$\therefore$ Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1 = (-125/8)+(75/4)-(15/2)+1$

$= -27/8$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x) = x^3-ax^2+6x-a$

$x-a = 0$

$\therefore x = a$

Remainder:

$p(a) = (a)^3-a(a^2)+6(a)-a$

$= a^3-a^3+6a-a = 5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x = 0$

$\Rightarrow 3x = -7$

$\Rightarrow x = -7/3$

$\therefore$ Remainder:

$3(-7/3)^3+7(-7/3) = -(343/9)+(-49/3)$

$= (-343-(49)3)/9$

$= (-343-147)/9$

$= -490/9 \neq 0$

$\therefore$ 7+3x is not a factor of $3x^3+7x$

Exercise 2.4

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x) = x^3+x^2+x+1$

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴ By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let $p(x)= x^4+x^3+x^2+x+1$

The zero of x+1 is -1. [x+1= 0 means x = -1]

$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let $p(x)= x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$=1-3+3-1+1$

$=1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1) = (-1)^3-(-1)^2-(2+\sqrt{2})(-1) + \sqrt{2} = -1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2} \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^3–x^2–(2+\sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3+x^2–2x–1$, $g(x) = x+1$

Solution:

$p(x) = 2x^3+x^2–2x–1$, $g(x) = x+1$

$g(x) = 0$

$\Rightarrow x+1 = 0$

$\Rightarrow x = -1$

∴ Zero of g(x) is -1.

Now,

$p(-1) = 2(-1)^3+(-1)^2–2(-1)–1$

$= -2+1+2-1$

$= 0$

∴ By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x+2$

Solution:

$p(x) = x^3+3x^2+3x+1$, $g(x) = x+2$

$g(x) = 0$

$\Rightarrow x+2 = 0$

$\Rightarrow x = -2$

∴ Zero of g(x) is -2.

Now,

$p(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+12-6+1$

$= -1 \neq 0$

∴ By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3–4x^2+x+6$, $g(x) = x–3$

Solution:

$p(x) = x^3–4x^2+x+6$, $g(x) = x -3$

$g(x) = 0$

$\Rightarrow x-3 = 0$

⇒ $x = 3$

∴ Zero of g(x) is 3.

Now,

$p(3) = (3)^3-4(3)^2+(3)+6$

$= 27-36+3+6$

$= 0$

∴ By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

⇒ $(1)^2+(1)+k = 0$

⇒ $1+1+k = 0$

⇒ $2+k = 0$

⇒ $k = -2$

(ii) $p(x) = 2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

⇒ $2(1)^2+k(1)+\sqrt{2} = 0$

⇒ $2+k+\sqrt{2} = 0$

⇒ $k = -(2+\sqrt{2})$

(iii) $p(x) = kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

⇒ $k(1)^2-\sqrt{2}(1)+1=0$

⇒ $k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

⇒ $k(1)^2-3(1)+k = 0$

⇒ $k-3+k = 0$

⇒ $2k-3 = 0$

⇒ $k= 3/2$

4. Factorise:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product =1×12 = 12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4 = 12]

$12x^2-7x+1= 12x^2-4x-3x+1$

$= 4x(3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product = 2×3 = 6

We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$= 2x (x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product = 6×-6 = -36

We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

$6x^2+5x-6 = 6x^2+9x-4x-6$

$= 3x(2x+3)-2(2x+3)$

$= (2x+3)(3x-2)$

(iv) $3x^2-x-4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -1 and product = 3×-4 = -12

We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

$3x^2–x–4 = 3x^2–4x+3x–4$

$= x(3x–4)+1(3x–4)$

$= (3x–4)(x+1)$

5. Factorise:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x) = x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

Now,

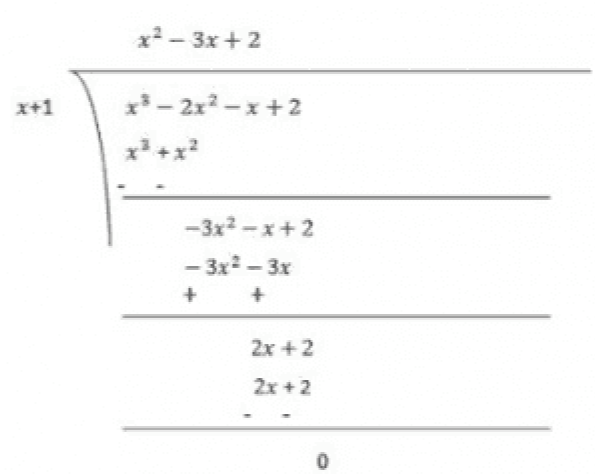
$p(x) = x^3–2x^2–x+2$

$p(−1) = (−1)^3–2(−1)^2–(−1)+2$

$= −1−2+1+2$

$= 0$

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) = (x+1)(x^2–x–2x+2)$

$= (x+1)(x(x−1)−2(x−1))$

$= (x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let p(x) = $x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By the trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

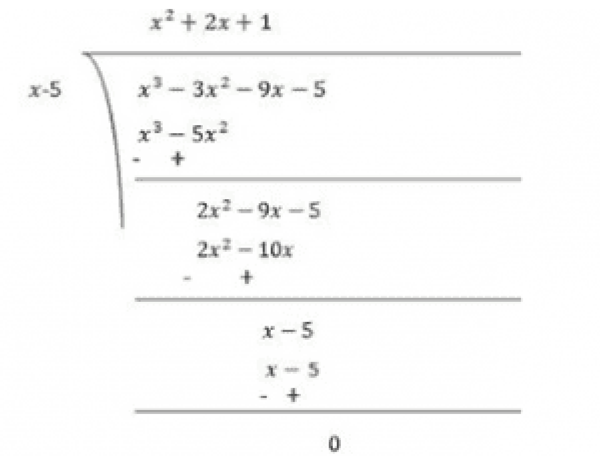
p(x) = $x^3–3x^2–9x–5$

p(5) = $(5)^3–3(5)^2–9(5)–5$

= 125−75−45−5

= 0

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x−5)(x^2+2x+1) = (x−5)(x^2+x+x+1)$

$= (x−5)(x(x+1)+1(x+1))$

$= (x−5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let p(x) = $x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By the trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

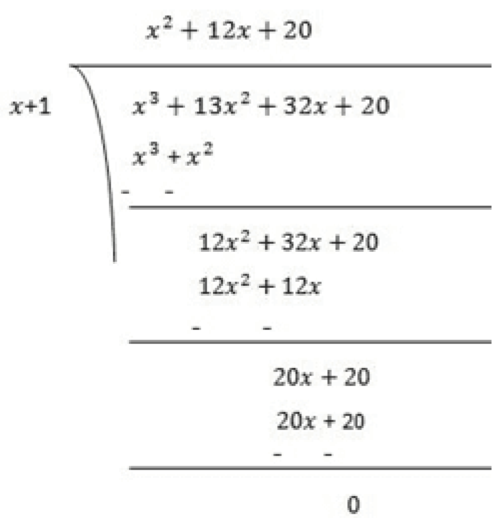
p(x)= $x^3+13x^2+32x+20$

p(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

= −1+13−32+20

= 0

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient +Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$

= (x+1)x(x+2)+10(x+2)

= (x+1)(x+2)(x+10)

(iv) $2y^3+y^2–2y–1$

Solution:

Let p(y) = $2y^3+y^2–2y–1$

Factors = 2×(−1)= -2 are ±1 and ±2

By the trial method, we find that

p(1) = 0

So, (y-1) is factor of p(y)

Now,

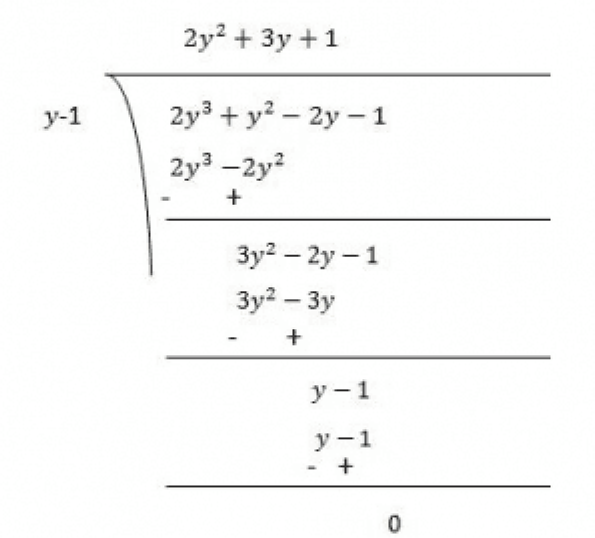
p(y) = $2y^3+y^2–2y–1$

p(1) = $2(1)^3+(1)^2–2(1)–1$

= 2+1−2

= 0

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1)$

$= (y-1)(2y(y+1)+1(y+1))$

$= (y-1)(2y+1)(y+1)$

Exercise 2.5

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1. Use suitable identities to find the following products:

(i) (x+4)(x +10)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, a = 4 and b = 10]
We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x+8)(x –10)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, a = 8 and b = −10]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x+4)(3x–5)

Solution:

Using the identity, $(x+a)(x+b) = x^2+(a+b)x+ab$

[Here, x = 3x, a = 4 and b = −5]
We get,

$(3x+4)(3x-5) = (3x)^2+[4+(-5)]3x+4\times(-5)$

$= 9x^2+3x(4-5)-20$

$= 9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x+y)(x-y) = x^2-y^2$

[Here, $x = y^2$ and $y = 3/2$]
We get,

$(y^2+3/2)(y^2-3/2) = (y^2)^2-(3/2)^2$

$= y^4-9/4$

2. Evaluate the following products without multiplying directly:

(i) 103×107

Solution:

103×107= (100+3)×(100+7)

Using identity, $[(x+a)(x+b) = x^2+(a+b)x+ab$

Here, x = 100

a = 3

b = 7

We get, 103×107 = (100+3)×(100+7)

$= (100)^2+(3+7)100+(3\times7)$

= 10000+1000+21

= 11021

(ii) 95×96

Solution:

95×96 = (100-5)×(100-4)

Using identity, $[(x-a)(x-b) = x^2-(a+b)x+ab$

Here, x = 100

a = -5

b = -4

We get, 95×96 = (100-5)×(100-4)

$= (100)^2+100(-5+(-4))+(-5\times-4)$

= 10000-900+20

= 9120

(iii) 104×96

Solution:

104×96 = (100+4)×(100–4)

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, a = 100

b = 4

We get, 104×96 = (100+4)×(100–4)

$= (100)^2–(4)^2$

= 10000–16

= 9984

3. Factorise the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2+2xy+y^2 = (x+y)^2$

Here, $x = 3x$

$y = y$

$9x^2+6xy+y^2 = (3x)^2+(2\times3x\times y)+y^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1$

Using identity, $x^2 - 2xy + y^2 = (x - y)^2$

Here, $x = 2y$

$y = 1$

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1^2$

$= (2y-1)^2$

$= (2y-1)(2y-1)$

(iii) $x^2-y^2/100$

Solution:

$x^2-y^2/100 = x^2-(y/10)^2$

Using identity, $x^2-y^2 = (x-y)(x+y)$

Here, $x = x$

$y = y/10$

$x^2-y^2/100 = x^2-(y/10)^2$

$= (x-y/10)(x+y/10)$

4. Expand each of the following using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

(i) $(x+2y+4z)^2$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = x$

$y = 2y$

$z = 4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 2x$

$y = -y$

$z = z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = -2x$

$y = 3y$

$z = 2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

Solution:

Using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = 3a$

$y = -7b$

$z = -c$

$(3a-7b-c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a\times -7b)+(2\times -7b\times -c)+(2\times -c\times 3a)$

$= 9a^2 + 49b^2 + c^2 – 42ab+14bc–6ca$

(v) $(–2x+5y–3z)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = –2x$

$y = 5y$

$z = – 3z$

$(–2x+5y–3z)^2 = (–2x)^2+(5y)^2+(–3z)^2+(2×–2x × 5y)+(2× 5y×– 3z)+(2×–3z ×–2x)$

$= 4x^2+25y^2 +9z^2– 20xy–30yz+12zx$

(vi) $((1/4)a-(1/2)b+1)^2$

Solution:

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here, $x = (1/4)a$

$y = (-1/2)b$

$z = 1$

$$((1/4)a-(1/2)b+1)^2 = (\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+(1)^2+(2\times\frac{1}{4}a\times-\frac{1}{2}b)+(2\times-\frac{1}{2}b\times1)+(2\times1\times\frac{1}{4}a)$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1^2-\frac{2}{8}ab-\frac{2}{2}b+\frac{2}{4}a$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

5. Factorise:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Solution:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$4x^2+9y^2+16z^2+12xy–24yz–16xz = (2x)^2+(3y)^2+(−4z)^2+(2×2x×3y)+(2×3y×−4z)+(2×−4z×2x)$

$= (2x+3y–4z)^2$

$= (2x+3y–4z)(2x+3y–4z)$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Using identity, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that, $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$= (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{2}\times-\sqrt{2}x)$

$= (-\sqrt{2}x+y+2\sqrt{2}z)^2$

$= (-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $((3/2)x+1)^3$

(iv) $(x-(2/3)y)^3$

Solution:

(i) $(2x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$(2x+1)^3= (2x)^3+1^3+(3\times2x\times1)(2x+1)$

$= 8x^3+1+6x(2x+1)$

$= 8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(2a-3b)^3 = (2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$= 8a^3-27b^3-18ab(2a-3b)$

$= 8a^3-27b^3-36a^2b+54ab^2$

(iii) $((3/2)x+1)^3$

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$((3/2)x+1)^3=((3/2)x)^3+1^3+(3\times(3/2)x\times1)((3/2)x+1)$

$$= \frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$$
$$= \frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$$
$$= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $(x-(2/3)y)^3$

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$(x-\frac{2}{3}y)^3=(x)^3-(\frac{2}{3}y)^3-(3\times x\times\frac{2}{3}y)(x-\frac{2}{3}y)$$
$$=(x)^3-\frac{8}{27}y^3-2xy(x-\frac{2}{3}y)$$
$$=(x)^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(99)^3 = (100-1)^3$

$= (100)^3-1^3-(3\times100\times1)(100-1)$

$= 1000000 -1-300(100 - 1)$

$= 1000000-1-30000+300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$(100+2)^3 =(100)^3+2^3+(3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$(998)^3 =(1000-2)^3$

$=(1000)^3-2^3-(3\times1000\times2)(1000-2)$

$= 1000000000-8-6000(1000-2)$

$= 1000000000-8-6000000+12000$

$= 994011992$

8. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$

(ii) $8a^3-b^3-12a^2b+6ab^2$

(iii) $27-125a^3-135a+225a^2$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

(v) $27p^3-(1/216)-(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

Here, the identity, $(x+y)^3 = x^3+y^3+3xy(x+y)$ is used.

(ii) $8a^3-b^3-12a^2b+6ab^2$

Solution:

The expression, $8a^3-b^3-12a^2b+6ab^2$ can be written as $(2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$

$8a^3-b^3-12a^2b+6ab^2 = (2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$

$= (2a-b)^3$

$= (2a-b)(2a-b)(2a-b)$

Here, the identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$ is used.

(iii) $27-125a^3-135a+225a^2$

Solution:

The expression, $27-125a^3-135a+225a^2$ can be written as $3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$27-125a^3-135a+225a^2 =$
$3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$= (3–5a)^3$

$= (3–5a)(3–5a)(3–5a)$

Here, the identity, $(x–y)^3 = x^3–y^3-3xy(x–y)$ is used.

(iv) $64a^3–27b^3–144a^2b+108ab^2$

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2=$
$(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3– (1/216)−(9/2) p^2+(1/4)p$

Solution:

The expression, $27p^3–(1/216)−(9/2) p^2+(1/4)p$ can be written as

$(3p)^3–(1/6)^3−(9/2) p^2+(1/4)p = (3p)^3–(1/6)^3−3(3p)(1/6)(3p – 1/6)$

Using $(x – y)^3 = x^3 – y^3 – 3xy (x – y)$

$27p^3–(1/216)−(9/2) p^2+(1/4)p = (3p)^3–(1/6)^3−3(3p)(1/6)(3p – 1/6)$

Taking $x = 3p$ and $y = 1/6$

$= (3p–1/6)^3$

$= (3p–1/6)(3p–1/6)(3p–1/6)$

9. Verify:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$

(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3 = (x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3 = (x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3 = (x+y)[(x+y)^2–3xy]$

Taking $(x+y)$ common $\Rightarrow x^3+y^3 = (x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3 = (x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3 = (x–y)(x^2+xy+y^2)$

We know that, $(x–y)^3 = x^3–y^3–3xy(x–y)$

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)[(x-y)^2 + 3xy]$

Taking (x+y) common $\Rightarrow x^3 - y^3 = (x-y)[(x^2+y^2-2xy)+3xy]$

$\Rightarrow x^3 + y^3 = (x-y)(x^2+y^2+xy)$

10. Factorise each of the following:

(i) $27y^3+125z^3$

(ii) $64m^3-343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 = (3y)^3+(5z)^3$

We know that, $x^3+y^3 = (x+y)(x^2-xy+y^2)$

$27y^3+125z^3 = (3y)^3+(5z)^3$

$= (3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$= (3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$ can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 =$
$(4m)^3-(7n)^3$

We know that, $x^3-y^3 = (x-y)(x^2+xy+y^2)$

$64m^3-343n^3 = (4m)^3-(7n)^3$

$= (4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$= (4m-7n)(16m^2+28mn+49n^2)$

11. Factorise: $27x^3+y^3+z^3-9xyz$.

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

$= (3x+y+z)[(3x)^2+y^2+z^2-3xy-yz-3xz]$

$= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

12. Verify that:

$x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

Solution:

We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

$\Rightarrow x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[2(x^2+y^2+z^2-xy-yz-xz)]$

$= (1/2)(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz)$

$= (1/2)(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)]$

$= (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

13. If $x+y+z = 0$, show that $x^3+y^3+z^3 = 3xyz$.

Solution:

We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

Now, according to the question, let $(x+y+z) = 0$,

Then, $x^3+y^3+z^3-3xyz = (0)(x^2+y^2+z^2-xy-yz-xz)$

$\Rightarrow x^3+y^3+z^3-3xyz = 0$

$\Rightarrow x^3+y^3+z^3 = 3xyz$

Hence Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

(i) $(-12)^3+(7)^3+(5)^3$

Let $a = -12$

$b = 7$

$c = 5$

We know that if $x+y+z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3 = 3xyz$

$= 3\times-12\times7\times5$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a = 28

b = −15

c = −13

We know that if x+y+z = 0, then $x^3+y^3+z^3 = 3xyz$.

Here, x+y+z = 28–15–13 = 0

$(28)^3+(-15)^3+(-13)^3 = 3xyz$

= 0+3(28)(−15)(−13)

= 16380

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2–35a+12$

(ii) Area: $35y^2+13y–12$

Solution:

(i) Area: $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum = -35 and product =25×12 = 300

We get -15 and -20 as the numbers [-15+-20=-35 and -15×-20 = 300]

$25a^2–35a+12 = 25a^2–15a−20a+12$

= 5a(5a–3)–4(5a–3)

= (5a–4)(5a–3)

Possible expression for length = 5a–4

Possible expression for breadth = 5a –3

(ii) Area: $35y^2+13y–12$

Using the splitting the middle term method,

We have to find a number whose sum = 13 and product = 35×-12 = 420

We get -15 and 28 as the numbers [-15+28 = 13 and -15×28=420]

$35y^2+13y–12 = 35y^2–15y+28y–12$

= 5y(7y–3)+4(7y–3)

= (5y+4)(7y–3)

Possible expression for length = (5y+4)

Possible expression for breadth = (7y–3)

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2–12x$

(ii) Volume: $12ky^2+8ky–20k$

Solution:

(i) Volume: $3x^2–12x$

$3x^2–12x$ can be written as $3x(x–4)$ by taking 3x out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = (x–4)

(ii) Volume:
$12ky^2+8ky–20k$

$12ky^2+8ky–20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.

$12ky^2+8ky–20k = 4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$= 4k(3y^2+5y–3y–5)$

$= 4k[y(3y+5)–1(3y+5)]$

$= 4k(3y+5)(y–1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)