06/04/2023
Evening

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. $\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$ is equal to

(1) 1
(2) 0
(3) $\sqrt{2}$
(4) $\frac{1}{\sqrt{2}}$

Answer (2)

Sol. $\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$

Since $2^{\frac{1}{2}}-2^{\frac{1}{3}}<1$

$2^{\frac{1}{2}}-2^{\frac{1}{5}}<1$

$2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}<1 \quad \forall n\in\mathbb{N}$

$\therefore \quad \lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$

= 0

2. If gcd $(m, n) = 1$ and $1^2 - 2^2 + 3^2 - 4^2 + \ldots.. + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ then $m^2 - n^2$ is equal to

(1) 240
(2) 200
(3) 220
(4) 180

Answer (1)

Sol. $1^2 - 2^2 + 3^2 - 4^2 + \ldots.. + (2021)^2 - (2022)^2 + (2023)^2$

$= \underbrace{-3-7-11\ldots}_{1011\text{ times}}+(2023)^2$

$= \frac{-1011}{2}[6+(1010)4]+(2023)^2$

= 2023(1012)

$\therefore \quad 2023 = 17^2 \times 7$

$\therefore \quad m = 17, n = 7$

$\therefore \quad m^2 - n^2 = 289 - 49$

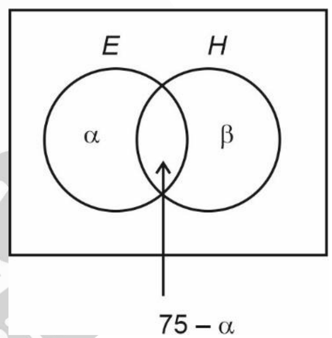
$= 240$

3. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is

(1) $\frac{\sqrt{119}}{12}$
(2) $\frac{\sqrt{117}}{12}$
(3) $\frac{3\sqrt{15}}{12}$
(4) $\frac{\sqrt{129}}{12}$

Answer (1)

Sol.



Now $\beta = 100 - 75 = 25$

$\therefore \quad \alpha = 75 - [40 - 25]$

$= 60$

Now, ellipse $25\left[\frac{x^2}{(60)^2}+\frac{y^2}{25^2}\right]=1$

$\therefore \quad \frac{x^2}{36\times 4}+\frac{y^2}{25}=1$

$\therefore \quad e=\sqrt{1-\frac{25}{36\times 4}}=\frac{\sqrt{119}}{12}$

4. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a}, \vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to

(1) $2V$
(2) $6V$
(3) V
(4) $3V$

Answer (3)

Sol. $[\vec{a}, \vec{b}+\vec{c}, \ \vec{a}+2\vec{b}+3\vec{c}]$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$= [\vec{a}\ \vec{b}\ \vec{c}]$

$= V$

5. If the solution curve $f(x, y) = 0$ of the differential equation $(1+\log_e x)\dfrac{dx}{dy} - x\log_e x = e^y$, $x > 0$, passes through the points (1, 0) and $(\alpha, 2)$, then a^a is equal to

(1) e^{2e^2} (2) e^{e^2}

(3) $e^{\sqrt{2}e^2}$ (4) $e^{2e^{\sqrt{2}}}$

Answer (1)

Sol. $(1+\ln x)\dfrac{dx}{dy} - x\ln x = e^y$

Put $x \ln x = t$

$(1 + \ln x)\, dx = dt$

$\Rightarrow \dfrac{dt}{dy} - t = e^y$

$\text{I.F} = e^{-\int dy}$

$= e^{-y}$

$t \times e^{-y} = \int e^y \times e^{-y} dy + c$

$t \times e^{-y} = y + c$

$x \ln x = y\, e^y + c\, e^y$

Put $x = 1$, $y = 0$

$\Rightarrow c = 0$

Put $x = a$, $y = 2$

$a \ln a = 2e^2$

$\therefore a^a = e^{2e^2}$

6. Let $f(x)$ be a function satisfying $f(x) + f(\pi - x) = \pi^2$, $\forall x \in \mathbb{R}$. Then $\int_0^\pi f(x)\sin x\, dx$ is equal to

(1) $\dfrac{\pi^2}{4}$ (2) $2\pi^2$

(3) π^2 (4) $\dfrac{\pi^2}{2}$

Answer (3)

Sol. $I = \int_0^\pi f(x)\sin x\, dx$

$I = \int_0^\pi f(\pi - x)\sin x\, dx$

$2I = \int_0^\pi \sin x\big(f(x) + f(\pi - x)\big)dx$

$2I = \pi^2 \int_0^\pi \sin x\, dx$

$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x\, dx$

$I = \pi^2$

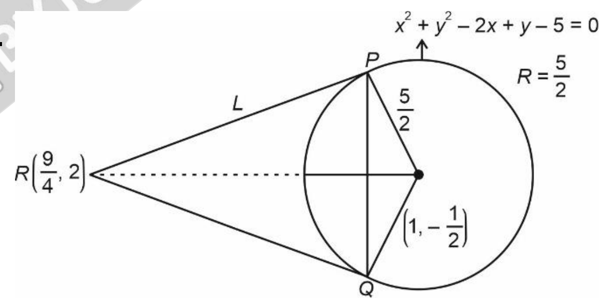
7. If the tangents at the points P and Q on the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R\left(\dfrac{9}{4}, 2\right)$, then the area of the triangle PQR is

(1) $\dfrac{5}{4}$ (2) $\dfrac{13}{8}$

(3) $\dfrac{5}{8}$ (4) $\dfrac{13}{4}$

Answer (3)

Sol.



$L = \sqrt{S_1} = \sqrt{\left(\dfrac{9}{4}\right)^2 + (2)^2 - 2\times\dfrac{9}{4} + 2 - 5}$

$= \dfrac{5}{4}$

$\text{Area} = \dfrac{RL^3}{R^2 + L^2} = \dfrac{\dfrac{5}{2}\times\left(\dfrac{5}{4}\right)^3}{\left(\dfrac{5}{2}\right)^2 + \left(\dfrac{5}{4}\right)^2}$

$= \dfrac{\dfrac{25}{8}}{4+1} = \dfrac{5}{8}$

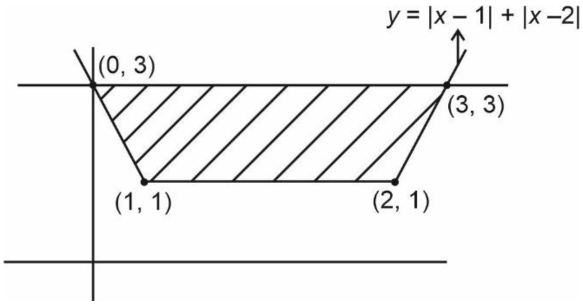
8. The area bounded by the curves $y=|x-1|+|x-2|$ and $y = 3$ is equal to

(1) 4 (2) 6

(3) 3 (4) 5

Answer (1)

Sol.



Area $=\frac{1}{2}[1+3]\times 2$

$= 4$

9. If the coefficients of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$ are equal, then

(1) $729ab = 32$ (2) $32ab = 729$

(3) $64ab = 243$ (4) $243ab = 64$

Answer (1)

Sol. Coefficient of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$$T_{r+1}={}^{11}C_r\left(ax^2\right)^{11-r}\left(\frac{1}{2bx}\right)^r$$

$$={}^{11}C_r\,(a)^{11-r}\left(\frac{1}{2b}\right)^r x^{22-3r}$$

$22-3r=7\Rightarrow r=5$

Coefficient of x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$

$$T_{r+1}={}^{11}C_r\,(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$$

$$={}^{11}C_r\,a^{11-r}\left(-\frac{1}{3b}\right)^r x^{11-3r}$$

$11-3r=-7\Rightarrow r=6$

$\therefore\quad {}^{11}C_5\,(a)^6\left(\frac{1}{2b}\right)^5={}^{11}C_6\,a^5\left(-\frac{1}{3b}\right)^6$

$\Rightarrow\quad 3^6ab=32$

$\Rightarrow\quad 729\,ab=32$

10. Let the sets A and B denote the domain and range respectively of the function $f(x)=\frac{1}{\sqrt{\lceil x\rceil-x}}$, where $\lceil x\rceil$ denotes the smallest integer greater than or equal to x. Then among the statements

$(S1):\ A\cap B=(1,\infty)-\mathbb{N}$ and

$(S2):A\cup B=(1,\infty)$

(1) Only (S2) is true

(2) Only (S1) is true

(3) Neither (S1) nor (S2) is true

(4) Both (S1) and (S2) are true

Answer (3)

Sol. $f(x)=\frac{1}{\sqrt{[x]-x}}$

$$=\frac{1}{\sqrt{-\{x\}}}$$

$\Rightarrow$ Domain = ϕ

11. Let P be a square matrix such that $P^2 = I - P$. For $\alpha,\beta,\gamma,\delta\in\mathbb{N}$, if $P^{\alpha}+P^{\beta}=\gamma I-29P$ and $P^{\alpha}-P^{\beta}=\delta I-13P$, then $\alpha+\beta+\gamma-\delta$ is equal to

(1) 18 (2) 40

(3) 22 (4) 24

Answer (4)

Sol. $P^2 = I - P$

$P^4=(I-P)(I-P)=I+P^2-2P=2I-3P$

$P^6=2I-5P+3P^2=2I-5P+3(IP)=5I-8P$ ...(i)

$P^8=5I-13P+8P^2=13I-21P$ ...(ii)

(ii) + (i)

$P^8+P^6=18I-29P$

(ii) – (i)

$P^8-P^6=8I-13P$

$\alpha = 8,\ \beta = 6,\ \gamma = 18,\ \delta = 8$

$8 + 6 + 18 + 8 = 24$

12. Among the statements

(S1) : $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology

(S2) : $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction

(1) Neither (S1) and (S2) is True

(2) Both (S1) and (S2) are True

(3) Only (S2) is True

(4) Only (S1) is True

Answer (1)

Sol. S-I : $(p \rightarrow q) \vee (\sim p \wedge q)$

$\Rightarrow \quad (p' \vee q) \vee (p' \wedge q)$

$\Rightarrow \quad (p' \vee (p' \wedge q)) \vee q$

$= (p') \vee q$ (not a tautology)

S-2 : $(q \rightarrow p) \rightarrow (p' \wedge q)$

$\Rightarrow \quad (q' \vee p)' \vee (p' \wedge q)$

$\Rightarrow \quad (q \wedge p') \vee (p' \wedge q)$

$= p' \wedge q$ (not a contradiction)

13. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is

(1) 576 (2) 578

(3) 580 (4) 582

Answer (4)

Sol.

5	6	1	4	3	2
P	U	B	L	I	C
4	4	0	2	1	0
5!	4!	3!	2!	1!	0!

Rank = (1 × 1! + 2 × 2! + 4 × 4! + 4 × 5!) + 1

= (1 + 4 + 96 + 480) + 1

= 582

14. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to

(1) 2 (2) 1

(3) 3 (4) 4

Answer (4)

Sol. If numbers are different on all three dice then number of ways

= 6 × 5 × 4 = 120

$$P(E) = \frac{120}{6^3} = \frac{120}{216} = \frac{5}{9} = \frac{p}{q}$$

Now, $q - p = 9 - 5 = 4$

15. Among the statements :

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8.

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

(1) Only (S2) is correct

(2) Only (S1) is correct

(3) Both (S1) and (S2) are correct

(4) Both (S1) and (S2) are incorrect

Answer (2)

Sol. (S1) : $(2023)^{2022} - (1999)^{2022}$ is divisible by 8

We know that $(x - y)$ divides $(x^n - y^n)$ $\forall n \in \mathbb{N}$

$\therefore$ (2023 – 1999) divides $(2023)^{2022} - (1999)^{2022}$

$\Rightarrow$ 24 divides $(2023)^{2022} - (1999)^{2002}$

$\Rightarrow$ 8 will divide $(2023)^{2022} - (1999)^{2002}$

$\therefore$ (S1) is correct.

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for $n \in \mathbb{N}$.

$13(1+12)^n - 11n - 13$

$13\left({}^nC_0 + {}^nC_1 12 + {}^nC_2 12^2 + \ldots + {}^nC_n 12^n\right) - 11n - 13$

$12 \times 13n - 11n + 12^2\lambda$

$145n + 144\lambda$ is not divisible by 144.

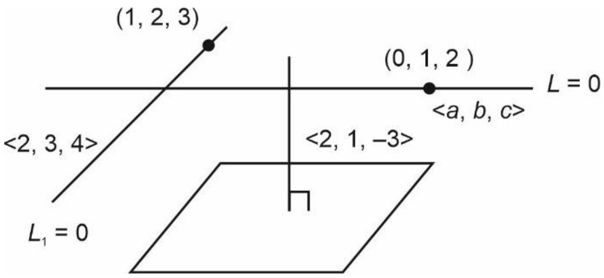
$\therefore$ (S2) is incorrect.

16. Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point $P(1, -9, 2)$ from the line L is

(1) $\sqrt{74}$ (2) $\sqrt{69}$

(3) $\sqrt{54}$ (4) 9

Answer (1)

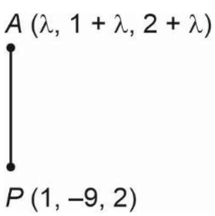
Sol. 

$$\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\left.\begin{aligned} a-2b+c=0 \\ 2a+b-3c=0 \end{aligned}\right\} a=b=c$$

$$\therefore \quad L = \frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \lambda$$

So any point on L can be taken as



$$\overrightarrow{AP} \cdot <1, 1, 1> = 0$$

$$\lambda - 1 + \lambda + 10 + \lambda = 0$$

$$3\lambda + 9 = 0$$

$$\Rightarrow \quad \lambda = -3$$

$$\therefore \quad A(-3, -2, -1) \quad P(1, -9, 2)$$

$$AP = \sqrt{74}$$

17. For the system of equations

$x + y + z = 6$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$, which one of the following is **NOT** true?

(1) System has no solution for $\alpha = 3$, $\beta = 24$

(2) System has a unique solution for $\alpha = -3$, $\beta = 14$

(3) System has infinitely many solutions for $\alpha = 3$, $\beta = 14$

(4) System has a unique solution for $\alpha = 3$, $\beta \neq 14$

Answer (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$

$$= 1(10 - 3\alpha) - (5 - \alpha) + (3 - 2)$$

$$= 6 - 2\alpha$$

$$D \neq 0 \quad \Rightarrow \quad \alpha \neq 3$$

Unique solution $\Rightarrow \alpha \neq 3$

18. The sum of all values of α, for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha + 1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$ are coplanar, is equal to

(1) –2 (2) 2

(3) 6 (4) 4

Answer (2)

Sol. Let the points be A, B, C, D

$$\overrightarrow{AB} = \hat{i} - \hat{j} + \hat{k}$$

$$\overrightarrow{AC} = \alpha\hat{i} + 2\hat{j} - \hat{k}$$

$$\overrightarrow{AD} = 8\hat{i} + (\alpha - 6)\hat{j} + 3\hat{k}$$

So, vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & (\alpha - 6) & 3 \end{vmatrix} = 0$$

$$(6 + \alpha - 6) + (3\alpha + 8) + (\alpha^2 - 6\alpha - 16) = 0$$

$$\alpha^2 - 2\alpha - 14 = 0$$

Sum of values of α = 2

19. A plane P contains the line of intersection of the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is

(1) 620 (2) 155

(3) 310 (4) 1240

Answer (1)

Sol. $P_1 : x + y + z - 6 = 0$

$$P_2 : 2x + 3y + 4z + 5 = 0$$

$$P : (x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$$

Plane P passes through (0, 2, –2)

$$-6+\lambda(3)=0$$

$\lambda = 2$

$$P: 5x+7y+9z+4=0$$

$$\text{Square of distance} = \left(\frac{5(12)+7(12)+9(18)+4}{\sqrt{25+49+81}}\right)^2$$

$$=\frac{310\times 310}{155}=620$$

20. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X=\{z\in C: \mathrm{Re}(az^2+bz)=a \text{ and } \mathrm{Re}(bz^2+az)=b\}$ is equal to

(1) 0 (2) 1

(3) 3 (4) 2

Answer (1*)

Sol. Let $z = x + iy$

$\because$ $\mathrm{Re}(az^2 + bz) = a$

$\Rightarrow$ $\mathrm{Re}(a(x+iy)^2+b(x+iy))=a$

$a(x^2-y^2)+bx=a$ ...(i)

$\because$ $\mathrm{Re}(bz^2+az)=b$

$\Rightarrow$ $b(x^2-y^2)+ax=b$ ...(ii)

from (i) and (ii), (i) – (ii)

$(x^2-y^2)(a-b)-x(a-b)=a-b$

$\Rightarrow$ $x^2-y^2-x=1$ ...(iii)

from (i) and (ii), (i) + (ii)

$\left((x^2-y^2)+x-1\right)(a+b)=0$

(here $a + b \neq 0$ is considered but it is not clear from the question)

$x^2-y^2+x-1=0$ ...(iv)

from (iii) and (iv)

$x = 0$, $y^2 = -1$ (No solution)

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The value of tan9° – tan27° – tan63° + tan81° is ________.

Answer (4)

Sol. $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$=(\cot 81^\circ+\tan 81^\circ)-(\tan 27^\circ+\cot 27^\circ)$

$=(\tan 9^\circ+\cot 9^\circ)-(\tan 27^\circ+\cot 27^\circ)$

$$=\frac{2}{\sin 18^\circ}-\frac{2}{\sin 54^\circ}$$

$$=\left(\frac{2\times 4}{\sqrt{5}-1}-\frac{2\times 4}{\sqrt{5}+1}\right)=4$$

22. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is ________.

Answer (432)

Sol. UNIVERSE

E, E, I, U, (Vowels) + N, R, S, V (Consonants)

Two different vowels + 2 consonants

$$= {}^3C_2 . {}^4C_2 . \underline{|4} = 432$$

23. For $\alpha, \beta, z \in \mathbb{C}$ and $\lambda > 1$, if $\sqrt{\lambda-1}$ is the radius of the circle $|z-\alpha|^2+|z-\beta|^2=2\lambda$, then $|\alpha-\beta|$ is equal to __________.

Answer (2)

Sol. $|z-\alpha|^2+|z-\beta|^2=2\lambda$

$(z-\alpha)(\bar{z}-\bar{\alpha})+(z-\beta)(\bar{z}-\bar{\beta})=2\lambda$

$$z\bar{z}-z\left(\frac{\bar{\alpha}+\bar{\beta}}{2}\right)-\bar{z}\left(\frac{\alpha+\beta}{2}\right)+\frac{\alpha\bar{\alpha}+\beta\bar{\beta}}{2}=\lambda$$

$$\text{Radius}=\sqrt{\left|\frac{\alpha+\beta}{2}\right|^2-\left(\frac{\alpha\bar{\alpha}+\beta\bar{\beta}}{2}-\lambda\right)}=\sqrt{\lambda-1}$$

$\Rightarrow$ $|\alpha+\beta|^2-2(\alpha\bar{\alpha}+\beta\bar{\beta})=-4$

$|\alpha-\beta|^2=4$ $\Rightarrow$ $|\alpha-\beta|=2$

24. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _______.

Answer (02.00)

Sol. $e_H = \sqrt{2}, e_e = \frac{1}{\sqrt{2}}$

Focus of hyperbola = (±1, 0)

Both curves are confocal

$$ae_e = 1 \Rightarrow a = \sqrt{2}$$

$$\frac{2b^2}{a} = 2a\left(1-e_e^2\right)$$

$$= 2\sqrt{2}.\frac{1}{2} = \sqrt{2}$$

25. Let a curve $y = f(x)$, $x\in(0, \infty)$ pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point $S(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to _______.

Answer (05.00)

Sol. $Y - y = m(X - x), m = \frac{dy}{dx}$

Put $X = 0$

$Y = y - mx$

$\Rightarrow$ $x(y - mx) = 3$

or $y - \frac{xdy}{dx} = \frac{3}{x}$

or $\frac{ydx - Xdy}{X^2} = \frac{3dx}{x}.\frac{1}{x^2}$

or $d\left(\frac{-y}{x}\right) = 3d\left(\frac{x^{-2}}{-2}\right)$

$\Rightarrow$ $\frac{y}{x} = \frac{3}{2x^2} + C$

$C = 0$

$\frac{1}{2\alpha} = \frac{3}{2\alpha^2}$

OR $\alpha = 3$

$P\left(1, \frac{3}{2}\right)$ $Q\left(3, \frac{1}{2}\right)$

$(PQ)^2 = 4 + 1 = 5$

26. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is _______.

Answer (25.00)

Sol. Mean $= \frac{8+16+120+80+56+80+6\alpha+12\beta}{40+\alpha+\beta}$

$\Rightarrow$ $360 + 9\alpha + 9\beta = 360 + 6\alpha + 12\beta$

OR

$3\alpha - 3\beta = 0$

$$15.08+81 = \frac{16+64+36\alpha+960+800 +144\alpha+784+1280}{40+2\alpha}$$

$(40 + 2\alpha)(96.08) = 3904 + 180\alpha$

$3843.20 + (192.16)\alpha = 3904 + 180\alpha$

$(12.16)\alpha = 60.80$

$\alpha = 5 = \beta$

27. Let $f(x) = \frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}, x \in \mathbb{R} - \{-1\}, n \in \mathbb{N}, n > 2$.

if $f^n(x)$ = (*fofof*….. upto *n* times) (x), then

$\lim_{n\to\infty} \int_0^1 x^{n-2} (f^n(x))\, dx$ is equal to

Answer (00.00)

Sol. $f^n(x) = \frac{x}{\left(1+n\,x^n\right)^{\frac{1}{n}}}$

$$I = \int_0^1 \frac{x^{n-1}}{(1+nx^n)^{\frac{1}{n}}}dx$$

$1 + nx^n = t^n$

$n^2x^{n-1}\,dx = nt^{n-1}\,dt$

$$I = \int_1^{(1+n)^{\frac{1}{n}}} \frac{\frac{1}{n}t^{n-1}\,dt}{t}$$

$$= \frac{1}{n}\left.\frac{t^{n-1}}{n-1}\right|_1^{(1+n)^{\frac{1}{n}}}$$

$$= \frac{1}{n(n-1)}\left((1+n)^{1-\frac{1}{n}} - 1\right)$$

$$\lim_{n\to\infty} \frac{(1+n)^{1-\frac{1}{n}} - 1}{n(n-1)} = \lim_{n\to\infty} \frac{(1+n)^{1-\frac{1}{n}}}{n(n-1)}$$

$$= 0$$

28. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is ________.

Answer (5)

Sol. $f(x) = x^5 - 20x^3 + 50x + 2$

$f(x)$ is continuous for all $x \in R$

Also, $f(-5) = -873$

$f(-2) = 30$

$f(-1) = -29$

$f(0) = 2$

$f(2) = -26$

$f(5) = 877$

Hence by intermediate value theorem

$f(x) = 0$ for some $x \in (-5, -2)$

Also, for some $x \in (-2, -1)$

Also, for some $x \in (-1, 0)$

Also, for some $x \in (0, 2)$

Also, for some $x \in (2, 5)$

As $f(x)$ is 5th degree polynomial answer is 5.

29. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$, then k is equal to ________.

Answer (400)

Sol. $S = 20^{19} + 2\cdot(20)^{19}\cdot\left(\frac{21}{20}\right)$

$$+3(20)^{19}\left(\frac{21}{20}\right)^2 + \ldots + 20(20)^{19}\cdot\left(\frac{21}{20}\right)^{19}$$

$$= 20^{19}\left(1 + 2\cdot\frac{21}{20} + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19}\right)$$

$$= k\cdot 20^{19}$$

$$\Rightarrow k = 1 + 2\cdot\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19}$$

$$\Rightarrow \frac{21}{20}k = \frac{21}{20} + 2\left(\frac{21}{20}\right)^2$$

$$+ \ldots 19 + \left(\frac{21}{20}\right)^{19} + 20\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k - \frac{21}{20}k = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2$$

$$+ \ldots + \left(\frac{21}{20}\right)^{19} - 20\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow \frac{-k}{20} = \frac{\left(\frac{21}{20}\right)^{20} - 1}{\frac{21}{20} - 1} - 20\left(\frac{21}{20}\right)^{20}$$

$$= \left(\left(\frac{21}{20}\right)^{20} - 1\right)\times 20 - 20\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow \frac{-k}{20} = -20$$

$$\Rightarrow k = 400$$

30. If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is ___________.

Answer (18)

Sol. $L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\alpha} = \lambda$

$L_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{\beta} = \mu$

For point of intersection

$2\lambda + 1 = 5\mu + 4$ …(i)

$3\lambda + 2 = 2\mu + 1$ …(ii)

$\alpha\lambda + 3 = \beta\mu + 0$ …(iii)

From (i) and (ii), $\lambda = \mu = -1$

Now, from (iii) $\alpha - \beta = 3$

Let $E = 8\alpha\beta$

$= 8\alpha(\alpha - 3)$

Minimum value of $E = -18$ at $\alpha = +\frac{3}{2}$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. The work functions of Aluminium and Gold are 4.1 eV and 5.1 eV respectively. The ratio of the slope of the stopping potential versus frequency plot for Gold to that of Aluminium is

(1) 1.24 (2) 2

(3) 1 (4) 1.5

Answer (3)

Sol. $\phi_{Al} = 4.1\,\text{eV}$

$\phi_{Au} = 5.1\,\text{eV}$

Slop of V vs ν graph $= \dfrac{h}{e}$

32. The weight of a body on the surface of the earth is 100 N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is:

(1) 64 N (2) 25 N

(3) 50 N (4) 100 N

Answer (1)

Sol. $W' = 100 \times \dfrac{R^2}{\left(R + \dfrac{R}{4}\right)^2} = 64\,\text{N}$

33. A 2 meter long scale with least count of 0.2 cm is used to measure the locations of objects on an optical bench. While measuring the focal length of a convex lens, the object pin and the convex lens are placed at 80 cm mark and 1 m mark, respectively. The image of the object pin on the other side of lens coincides with image pin that is kept at 180 cm mark. The % error in the estimation of focal length is:

(1) 0.85 (2) 1.70

(3) 1.02 (4) 0.51

Answer (2)

Sol. u = 1 m – 80 cm = 20 cm

v = 1.8 m – 1 m = 80 cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{80} + \frac{1}{20} = \frac{5}{80}$$

f = 16 cm

$$\frac{df}{f^2} = \frac{0.2 \times 2}{6400} + \frac{0.2 \times 2}{400}$$

$$df = \frac{16 \times 16 \times 0.2 \times 6800 \times 2}{6400 \times 400} = 0.136 \times 2$$

$$\frac{df}{f} = 0.0085 \times 2 = 1.70$$

34. A capacitor of capacitance 150.0 μF is connected to an alternating source of emf given by E = 36 sin(120πt) V. The maximum value of current in the circuit is approximately equal to:

(1) 2 A (2) $\sqrt{2}$ A

(3) $2\sqrt{2}$ A (4) $\dfrac{1}{\sqrt{2}}$ A

Answer (1)

Sol. $I_{max} = \dfrac{36 \times 60 \times 150 \times 10^{-6} \times 2\pi}{1}$

= 2.036 A

$\cong$ 2 A

35. The energy density associated with electric field $\vec{E}$ and magnetic field $\vec{B}$ of an electromagnetic wave in free space is given by (ε_0– permittivity of free space, μ_0 – permeability of free space)

(1) $U_E = \dfrac{E^2}{2\varepsilon_0}, U_B = \dfrac{B^2}{2\mu_0}$

(2) $U_E = \dfrac{\varepsilon_0 E^2}{2}, U_B = \dfrac{B^2}{2\mu_0}$

(3) $U_E = \dfrac{\varepsilon_0 E^2}{2}, U_B = \dfrac{\mu_0 B^2}{2}$

(4) $U_E = \dfrac{E^2}{2\varepsilon_0}, U_B = \dfrac{\mu_0 B^2}{2}$

Answer (2)

Sol. Correct option is

$$U_E = \frac{1}{2}\varepsilon_0 E^2 \qquad U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

36. The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

(1) 1 : 2 (2) 4 : 1

(3) 1 : 4 (4) 1 : 1

Answer (2)

Sol. $\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{16}$

$= 4 : 1$

37. For an amplitude modulated wave the minimum amplitude is 3 V, while the modulation index is 60%. The maximum amplitude of the modulated wave is:

(1) 5 V
(2) 15 V
(3) 12 V
(4) 10 V

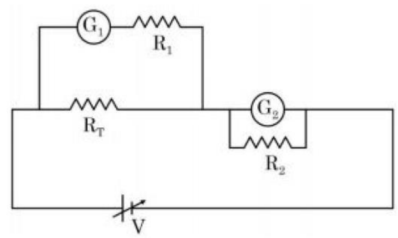
Answer (3)

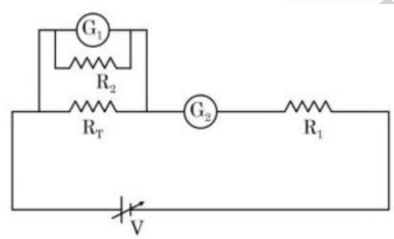
Sol. $\mu = 0.6 = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

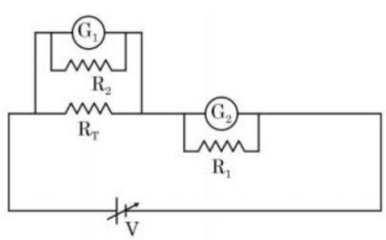
$\Rightarrow 0.6 = \frac{x-3}{x+3}$

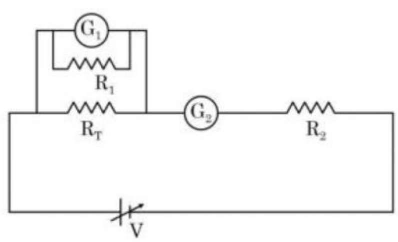
$x = 12$ V

38. A student is provided with a variable voltage source V, a test resistor R_T = 10Ω, two identical galvanometers G_1 and G_2 and two additional resistors, R_1 = 10MΩ and R_2 = 0.001 Ω. For conducting an experiment to verify ohm's law, the most suitable circuit is:

(1)


(2)


(3)


(4)


Answer (1)

Sol. For voltage measurement across R_T

Voltmeter should have very high resistance

$\Rightarrow$ R_1 should be in series with G_1 and Ammeter should be having very less resistance

$\Rightarrow$ R_2 should be in parallel with G_2

39. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The phase difference of two light waves change if they travel through different media having same thickness, but different indices of refraction.

Reason R: The wavelengths of waves are different in different media.

In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(2) **A** is not correct but **R** is correct
(3) **A** is correct but **R** is not correct
(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Answer (4)

Sol. Because of changed wavelength the phase difference changes while the two waves travel the same distance.

40. The temperature of an ideal gas is increased from 200 K to 800 K. If r.m.s. speed of gas at 200 K is v_0. Then, r.m.s. speed of the gas at 800 K will be:

(1) $\frac{v_0}{4}$
(2) v_0
(3) $4v_0$
(4) $2v_0$

Answer (4)

Sol. $\frac{v'}{v_0} = \sqrt{\frac{800}{200}} = 2$

$v' = 2v_0$

41. Choose the incorrect statement from the following:

(1) The speed of satellite in a given circular orbit remains constant
(2) For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant
(3) The linear speed of a planet revolving around the sun remains constant
(4) When a body falls towards earth, the displacement of earth towards the body is negligible

Answer (3)

Sol. Linear speed varies as the planet moves in elliptical orbit.

$$v = \sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$$

42. A child of mass 5 kg is going round a merry-go-round that makes 1 rotation in 3.14 S. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be

(1) 80 N (2) 40 N
(3) 100 N (4) 50 N

Answer (2)

Sol. $\omega = \frac{2\pi}{3.14} = 2\,\text{rad/s}$

$r = 2$ m

$F_r = m\omega^2 r = 5 \times (2)^2 \times 2$

$= 40$ N

43. A dipole comprises of two charged particles of identical magnitude q and opposite in nature. The mass 'm' of the positive charged particle is half of the mass of the negative charged particle. The two charges are separated by a distance 'l'. If the dipole is placed in a uniform electric field '$\overline{E}$'; in such a way that dipole axis makes a very small angle with the electric field, '$\overline{E}$'. The angular frequency of the oscillations of the dipole when released is given by:

(1) $\sqrt{\frac{4qE}{3ml}}$

(2) $\sqrt{\frac{8qE}{ml}}$

(3) $\sqrt{\frac{4qE}{ml}}$

(4) $\sqrt{\frac{8qE}{3ml}}$

Answer (Bonus)

Sol. $Z = pE\theta$

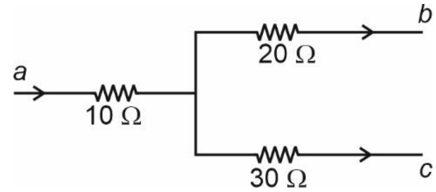
$$I = \left(\frac{m\times 2m}{m+2m}\right)(\ell)^2 = \frac{2m\ell^2}{3}$$

$$\alpha = \frac{3pE}{2m\ell^2}\theta = \frac{3qE}{2m\ell}\theta$$

$$\omega = \sqrt{\frac{3qE}{2m\ell}}$$

$\therefore$ No option is correct.

44. Figure shows a part of an electric circuit. The potentials at points a, b and c are 30 V, 12 V and 2 V respectively. The current through the 20 Ω resistor will be,



(1) 1.0 A (2) 0.4 A
(3) 0.6 A (4) 0.2 A

Answer (2)

Sol. $\frac{30-x}{10}+\frac{12-x}{20}+\frac{2-x}{30}=0$

$$30-x+6-\frac{x}{2}+\frac{2}{3}-\frac{x}{3}=0$$

$x = 20$ V

$$I = \frac{8}{20}\text{A} = 0.4\text{ A}$$

45. A particle starts with an initial velocity of 10.0 ms^{-1} along x-direction and accelerates uniformly at the rate of 2.0 ms^{-2}. The time taken by the particle to reach the velocity of 60.0 ms^{-1} is ___________.

(1) 25 s (2) 3 s
(3) 6 s (4) 30 s

Answer (1)

Sol. 60 = 10 +2t

t = 25 s

46. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Diffusion current in a p-n junction is greater than the drift current in magnitude if the junction is forward biased.

Reason R: Diffusion current in a p-n junction is form the n-side to the p-side if the junction is forward biased.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(2) **A** is correct but **R** is not correct
(3) **A** is not correct but **R** is correct
(4) Both **A** and **R** is correct and **R** is the correct explanation of **A**

Answer (2)

Sol. In forward bias movement of electrons is eased due to external electric field.

$\Rightarrow$ A is correct

R is incorrect as diffusion current in p-n junction is from p side to n-side.

47. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's principle is observed.

Reason R: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) **A** is not correct but **R** is correct

(3) **A** is correct but **R** is not correct

(4) Both **A** and **R** is correct and **R** is the correct explanation of **A**

Answer (4)

Sol. Pascal's law is applicable for an enclosed liquid.

$\Rightarrow$ A is correct

R is correct and explains A.

48. A small particle of mass m moves in such a way that its potential energy $U = \frac{1}{2}m\omega^2 r^2$ where ω is constant and r is the distance of the particle from origin. Assuming Bohr's quantization of momentum and circular orbit, the radius of n^th^ orbit will be proportional to

(1) $\sqrt{n}$ (2) $\frac{1}{n}$

(3) n^2 (4) n

Answer (1)

Sol. $U = \frac{1}{2}m\omega^2 r^2 = cr^2$

$F \propto r$

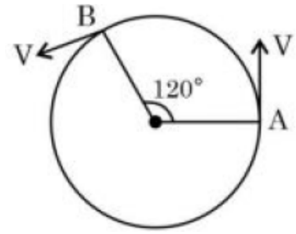
$\Rightarrow \frac{mv^2}{r} = c'r$ and $mvr = \frac{nh}{2\pi}$

$\Rightarrow v \propto r$

$\Rightarrow r^2 \propto n$

$\Rightarrow r \propto \sqrt{n}$

49. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is:



(1) $\sqrt{3}$ m/s

(2) π m/s

(3) $1.5\sqrt{3}$ m/s

(4) $2\sqrt{3}$ m/s

Answer (3)

Sol. $v = \pi$ m/s

$\Delta x = 2R\sin 60^\circ = \sqrt{3}R$

$v_{avg} = \frac{\sqrt{3}R \times v}{\frac{2\pi}{3}R} = \frac{3\sqrt{3}}{2}$

$= 1.5\sqrt{3}$ m/s

50. A body cools in 7 minutes from 60°C to 40°C. The temperature of the surrounding is 10°C. The temperature of the body after the next 7 minutes will be

(1) 30°C

(2) 32°C

(3) 34°C

(4) 28°C

Answer (4)

Sol. $\frac{60-40}{7} = C\left(\frac{60+40}{2} - 10\right)$

$\frac{40-x}{7} = C\left(\frac{40+x}{2} - 10\right)$

$\Rightarrow x = 28$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. A body is dropped on ground from a height 'h_1' and after hitting the ground, it rebounds to a height 'h_2'. If the ratio of velocities of the body just before and after hitting ground is 4, then percentage loss in kinetic energy of the body is $\frac{x}{4}$. The value of x is _____.

Answer (375)

Sol. $\frac{v_1}{v_2} = 4$

$$\frac{v_1^2}{v_2^2} = 16$$

$$\Rightarrow \quad \frac{v_1^2 - v_2^2}{v_1^2} = \frac{15}{16}$$

$$\Rightarrow \quad \frac{x}{4} = \frac{15}{16} \times 100$$

$x = 375$

52. A ring and a solid sphere rotating about an axis passing through their centres have same radii of gyration. The axis of rotation is perpendicular to plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is _____.

Answer (5)

Sol. $mk^2 = mR_r^2 = \frac{2}{5} mR_S^2$

$$\frac{R_r}{R_s} = \sqrt{\frac{2}{5}} \Rightarrow x = 5$$

53. A simple pendulum with length 100 cm and bob of mass 250 g is executing S.H.M of amplitude 10 cm. The maximum tension in the string is found to be $\frac{x}{40}$ N. The value of x is ______.

Answer (99)

Sol. $\omega = \sqrt{\frac{g}{l}} = \sqrt{10}$

$$v_{max} = \sqrt{10} \times 0.1$$

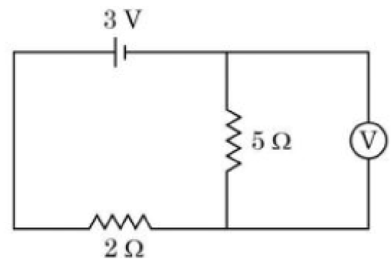
Maximum tension = $mg + \frac{mv^2}{r}$

$$= \frac{1}{4}\left(10 + \frac{10}{100 \times 1}\right)\frac{g}{10}$$

$= 98.98$

$\simeq 99$

54. As shown in the figure the voltmeter reads 2 V across 5 Ω resistor. The resistance of the voltmeter is _______ Ω



Answer (20)

Sol. $V_2 = 3V - 2V = 1V$

$$\Rightarrow \quad \frac{5 \times R_v}{5 + R_v} = 2 \times 2 = 4$$

$R_v = 20$

55. Two concentric circular coils with radii 1 cm and 1000 cm and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be ________ × 10^{-8} H. (Take, $\pi^2 = 10$)

Answer (4)

Sol. $B = \frac{n\mu_0 \times I}{2 \times 10} = \frac{200\mu_0 I}{2 \times 10}$

$$\phi = \pi(0.01)^2 \times \frac{200\,\mu_0 I \times 10}{2 \times 10}$$

$L = 200\,\mu_0\,\pi \times \frac{200\,\mu_0 \pi \times 10^{-4}}{2} = 4 \times 10^{-8}$ H

56. A proton with a kinetic energy of 2.0 eV moves into a region of uniform magnetic field of magnitude $\frac{\pi}{2} \times 10^{-3}$ T. The angle between the direction of magnetic field and velocity of proton is 60°. The pitch of the helical path taken by the proton is ____ cm. (Take, mass of proton = 1.6×10^{-27} kg and charge on proton = 1.6×10^{-19} C).

Answer (40)

Sol. K·E = 2 eV

$B = \frac{\pi}{2} \times 10^{-3}$

$\theta = 60°$

$\text{Pitch} = \frac{2\pi m}{qB} \times v\cos\theta$

$= \frac{2\pi \times \sqrt{2mKE} \times \frac{1}{2} \times 2}{1.6 \times 10^{-19} \times \pi \times 10^{-3}}$

$= \frac{2 \times \sqrt{2 \times 1.6 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19}} \times 10^3}{1.6 \times 10^{-19}}$

$= 2 \times 2 \times 10^{-1} = 0.4$ m

57. Experimentally it is found that 12.8 eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times 10^{-10}$ m.

The value of the x is: _______.

(1 eV = 1.6 × 10^{-19} J, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2/C^2 and electronic charge = 1.6 x 10^{-19} C)

Answer (16)

Sol. $\frac{kq^2}{2r} = 12.8 \times 1.6 \times 10^{-19}$

$r = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{12.8 \times 2}$

$r = \frac{9}{16} \times 10^{-10}$ m

58. A beam of light consisting of two wavelengths 7000 Å and 5500 Å is used to obtain interference pattern in Young's double slit experiment. The distance between the slits is 2.5 mm and the distance between the plane of slits and the screen is 150 cm. The least distance from the central fringe, where the bright fringes due to both the wavelengths coincide, is $n \times 10^{-5}$ m. The value of n is ____.

Answer (462)

Sol. λ_1 = 7000 Å

λ_2 = 5500 Å

$d = 2.5 \times 10^{-3}$ m

$D = 1.5$ m

$n\lambda_1 = m\lambda_2$

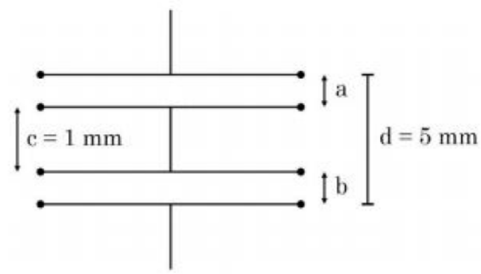
$n7 = 5.5\,m$

$n14 = 11\,m \Rightarrow n = 11\ \&\ m = 14$

$\Rightarrow\ y = \frac{11 \times 7 \times 10^{-7} \times 1.5}{2.5 \times 10^{-3}}$

$= 46.2 \times 10^{-4} = 462 \times 10^{-5}$

59. As shown in the figure, two parallel plate capacitors having equal plate area of 200 cm^2 are joined in such a way that a ≠ b. The equivalent capacitance of the combination is $x\varepsilon_0$ F. The value of x is ____.



Answer (5)

Sol. $C_{eq} = \frac{\varepsilon_0 \times 200 \times 10^{-4}}{4 \times 10^{-3}}$

$= 5\varepsilon_0$ F

60. A metal block of mass m is suspended from a rigid support through a metal wire of diameter 14 mm. The tensile stress developed in the wire under equilibrium state is 7×10^5 Nm^{-2}. The value of mass m is ______ kg.

(Take, $g = 9.8$ ms^{-2} and $\pi = \frac{22}{7}$)

Answer (11)

Sol. $mg = 7 \times 10^5 \times \frac{22}{7} \times 7^2 \times 10^{-6}$

$mg = \frac{49 \times 22}{10}$

$m = \frac{49 \times 22}{98} = 11$

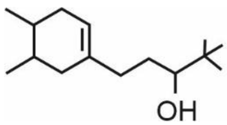
CHEMISTRY

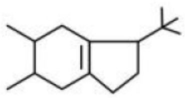
SECTION - A

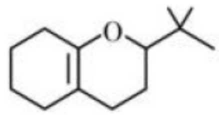
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

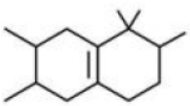
Choose the correct answer:

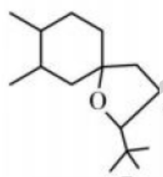
61. In the following reaction, 'B' is



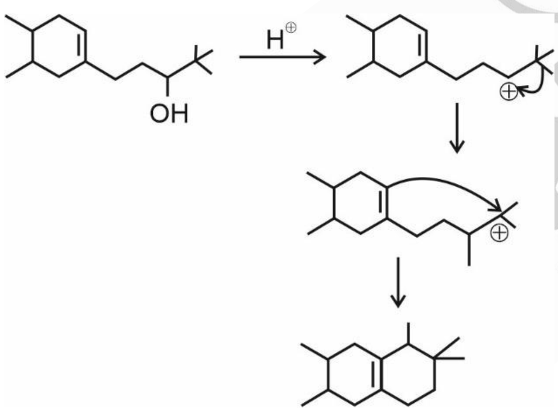
(1) 

(2) 

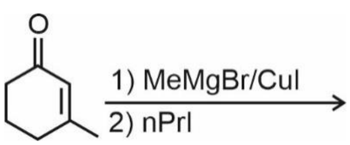
(3) 

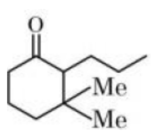
(4) 

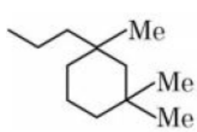
Answer (3)

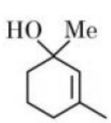
Sol. 

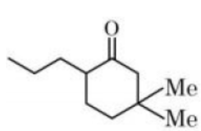
62. Find out the major product from the following reaction.



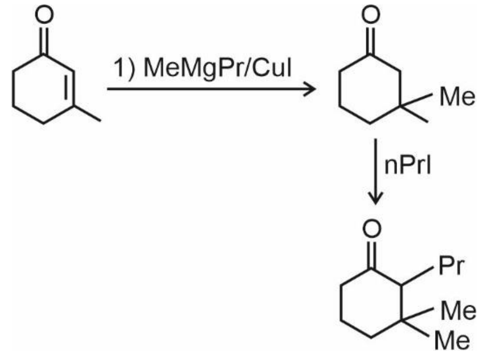
(1) 

(2) 

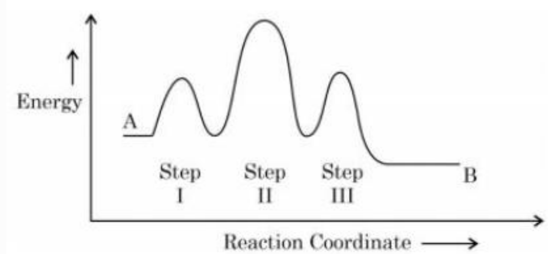
(3) 

(4) 

Answer (1)

Sol. 

63. Consider the following reaction that goes from A to B in three steps as shown below:



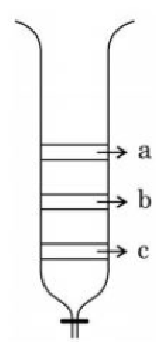
Choose the correct option

	Number of Intermediates	Number of Activated Complexes	Rate determining step
(1)	2	3	I
(2)	2	3	III
(3)	2	3	II
(4)	3	2	II

Answer (3)

Sol. As the reaction profile, clearly the number of intermediates are 2 and the number of activated complexes/transition state is 3. Rate determining step is II.

64. From the figure of column chromatography given below, identify incorrect statements.



A. Compound 'c' is more polar than 'a' and 'b'

B. Compound 'a' is least polar

C. Comound 'b' comes out of the column before 'c' and after 'a'

D. Compound 'a' spends more time in the column

Choose the correct answer from the options given below

(1) A, B and D only (2) A, B and C only

(3) B and D only (4) B, C and D only

Answer (2)

Sol. As the chromatogram, degree of polarity

$\rightarrow$ a > b > c.

$\therefore$ Statements A, B are incorrect as b comes out before 'C' the statement C is also incorrect.

As a is most polar, it spends most time. Hence, A, B & C are incorrect statements.

65. The group of chemicals used as pesticide is

(1) Aldrin, Sodium Chlorate, Sodium arsinite

(2) DDT, Aldrin

(3) Sodium chlorate, DDT, PAN

(4) Dieldrin, Sodium arsinite, Tetrachloroethene

Answer (2)

Sol. DDT, Aldrin and Dialdrin are pesticides.

66. The IUPAC name of $K_3[Co(C_2O_4)_3]$ is:

(1) Potassium tris(oxalato)cobaltate(III)

(2) Potassium tris(oxalato)cobalt(III)

(3) Potassium trioxalatocobalt(III)

(4) Potassium trioxalatocobaltate(III)

Answer (4)

Sol. IUPAC name is potassium tri(oxalato)cobaltate(III).

67. Which one of the following elements will remain as liquid inside pure boiling water?

(1) Ga (2) Br

(3) Li (4) Cs

Answer (1)

Sol. As Br_2, Li and Cs can react with H_2O, Ga remains as liquid inside boiling water.

68. During the reaction of permanganate with thiosulphate, the change in oxidation of manganese occurs by value of 3. Identify which of the below medium will favour the reaction.

(1) Both aqueous acidic and neutral

(2) Aqueous neutral

(3) Both aqueous acidic and faintly alkaline

(4) Aqueous acidic

Answer (2)

Sol. $MnO_4^{\ominus} + S_2O_3^{-2} \longrightarrow MnO_2 + SO_4^{-2}$

This ionic mechanism is favoured in neutral aqueous medium.

69. Group-13 elements react with O_2 in amorphous form to form oxides of type M_2O_3 (M = element). Which among the following is the most basic oxide?

(1) Al_2O_3 (2) B_2O_3

(3) Tl_2O_3 (4) Ga_2O_3

Answer (3)

Sol. Most basic oxide is Tl_2O_3

Basic character $\rightarrow$ $Tl_2O_3 > Ga_2O_3 > Al_2O_3 > B_2O_3$

70. The volume of 0.02 M aqueous HBr required to neutralize 10.0 mL of 0.01 M aqueous $Ba(OH)_2$ is (Assume complete neutralization)

(1) 2.5 mL (2) 5.0 mL

(3) 10.0 mL (4) 7.5 mL

Answer (3)

Sol. Meq of $Ba(OH)_2$ = Meq of HBr

$0.1 \times 2 = 0.02 \times V$

$$V = \frac{0.2}{0.02} = 10 \text{ mL}$$

71. The product, which is not obtained during the electrolysis of brine solution is

(1) H_2 (2) HCl

(3) NaOH (4) Cl_2

Answer (2)

Sol. Anode: $H_2O \longrightarrow \frac{1}{2} O_2 + 2H^{\oplus} + 2e^{\ominus}$

Cathode: $H_2O + e^{\ominus} \longrightarrow \frac{1}{2} H_2 + OH^{\ominus}$

Electrolyte: NaOH

Hence, HCl is not obtained

72. Given below are two statements: one is labelled as "**Assertion A**" and the other is labelled as "**Reason R**"

Assertion A : In the complex $Ni(CO)_4$ and $Fe(CO)_5$, the metals have zero oxidation state.

Reason R : Low oxidation states are found when a complex has ligands capable of π-donor character in addition to the σ-bonding.

In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) **A** is correct but **R** is not correct

(2) **A** is not correct but **R** is correct

(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Answer (1)

Sol. Statement 1 is correct as metals have zero oxidation state in both $Ni(CO)_4$ and $Fe(CO)_5$.

Statement 2 is incorrect as ligands have σ-donor and π-acceptor character or have both π-donor and π-acceptor character for the metal to show low oxidation state.

73. Ion having highest hydration enthalpy among the given alkaline earth metal ions is:

(1) Be^{2+} (2) Sr^{2+}

(3) Ba^{2+} (4) Ca^{2+}

Answer (1)

Sol. Ion having highest hydration enthalpy among alkaline earth metal ions is Be^{+2}.

74. Element not present in Nessler's reagent is

(1) N (2) Hg

(3) I (4) K

Answer (1)

Sol. Nessler's Reagent is K_2HgI_4.

So, N is not present

75. If the radius of the first orbit of hydrogen atom is α_0, then de Broglie's wavelength of electron in 3rd orbit is

(1) $\frac{\pi a_0}{6}$ (2) $\frac{\pi a_0}{3}$

(3) $6\pi a_0$ (4) $3\pi a_0$

Answer (3)

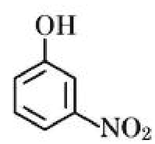
Sol. $mvr = \frac{nh}{2\pi}$

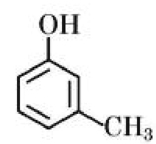
$mvr = \frac{3h}{2\pi}$

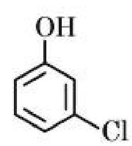
$\frac{2\pi r}{3} = \frac{h}{mv}$

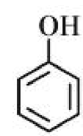
$\therefore \quad \lambda = \frac{2\pi 9a_0}{3} = 6\pi a_0$

76. The strongest acid from the following is

(1) 

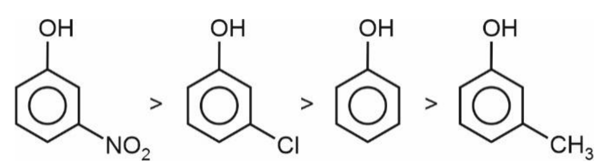
(2) 

(3) 

(4) 

Answer (1)

Sol. Acidic strength order is :



77. Given below are two statements:

Statement I: Morphine is a narcotic analgesic. It helps in relieving pain without producing sleep.

Statement II: Morphine and its derivatives are obtained from opium poppy.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Answer (4)

Sol. Morphine is a narcotic analgesic which produces sleep.

Hence, Statement I is incorrect.

Morphine narcotics are obtained from opium poppy.

Hence, Statement II is correct.

78. Formation of which complex, among the following, is not a confirmatory test of Pb^{2+} ions

(1) Lead sulphate (2) Lead nitrate

(3) Lead chromate (4) Lead iodide

Answer (2)

Sol. As lead nitrate is water soluble, it cannot be a confirmatory test.

Also, it is colourless.

79. Structures of $BeCl_2$ in solid state, vapour phase and at very high temperature respectively are:

(1) Monomeric, Dimeric, Polymeric

(2) Dimeric, Polymeric, Monomeric

(3) Polymeric, Monomeric, Dimeric

(4) Polymeric, Dimeric, Monomeric

Answer (4)

Sol. $BeCl_2$ is dimeric in vapour phase.

$BeCl_2$ is monomeric at high temperature.

$BeCl_2$ is polymeric in solid state.

80. Match List-I with List-II.

	List-I **Natural Amino acid**		**List-II** **One Letter Code**
(A)	Arginine	(I)	D
(B)	Aspartic acid	(II)	N
(C)	Asparagine	(III)	A
(D)	Alanine	(IV)	R

Choose the correct answer from the options given below:

(1) (A)–(IV, B–I, (C)–II, (D)–III

(2) (A)–I, B–III, (C)–IV, (D)–II

(3) (A)–III, B–I, (C)–II, (D)–IV

(4) (A)–IV, B–I, (C)–III, (D)–II

Answer (1)

Sol.

Natural Amino acid	**One Letter Code**
(A) Arginine	(IV) R
(B) Aspartic acid	(I) D
(C) Asparagine	(II) N
(D) Alanine	(III) A

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Number of crystal systems from the following where body centred unit cell can be found, is ________.

Cubic, tetragonal, orthorhombic, hexagonal, rhombohedral, monoclinic, triclinic

Answer (3)

Sol. Crystal systems where body centred unit cell can be found

Cubic, orthorhombic and tetragonal

Hence, correct answer is 3

82. The number of species having a square planar shape from the following is

$XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, [Cu(NH_3)_4]^{2+}, [FeCl_4]^{2-}, [PtCl_4]^{2-}$

Answer (4)

Sol. $XeF_4 \rightarrow$ Square planar

$SF_4 \rightarrow$ See saw

$SiF_4 \rightarrow$ Tetrahedral

$BF_4^- \rightarrow$ Tetrahedral

$[Cu(NH_3)_4]^{2+} \rightarrow$ Square planar

$[FeCl_4]^{2-} \rightarrow$ Tetrahedral

$[PtCl_4]^{2-} \rightarrow$ Square planar

$BrF_4^- \rightarrow$ Square planar

So, 4 square planer shape compounds are present.

83. The equilibrium composition for the reaction $PCl_3 + Cl_2 \rightleftharpoons PCl_5$ at 298 K is given below:

$[PCl_3]_{eq} = 0.2$ mol L^{-1}, $[Cl_2]_{eq} = 0.1$ mol L^{-1}, $[PCl_5]_{eq} = 0.40$ mol L^{-1}

If 0.2 mol of Cl_2 is added at the same temperature, the equilibrium concentrations of PCl_5 is _______ × 10^{-2} mol L^{-1}

Given: K_c for the reaction at 298 K is 20

Answer (49)

Sol. $\underset{0.2}{PCl_3(g)} + \underset{0.1}{Cl_2(g)} \rightleftharpoons \underset{0.4}{PCl_5(g)}$

$$K_c = \frac{0.4}{0.2 \times 0.1} = 20$$

If 0.2 moles of Cl_2 is added

$$20 = K_c = \frac{0.4 + X}{(0.3 - X)(0.2 - X)}$$

$\Rightarrow \quad (0.4 + X) = 20(0.3 - X)(0.2 - X)$

$\therefore \quad 0.4 + X = 20(0.06 + X^2 - 0.5 X)$

$0.4 + X = 1.2 + 20 X^2 - 10 X$

$20 X^2 - 11 X + 0.8 = 0$

$$X = \frac{11 \pm \sqrt{121 - 64}}{40}$$

$$= \frac{11 - 7.55}{40} \simeq 0.08625$$

$\therefore (PCl_5) = 0.48625 \simeq 48.625 \times 10^{-2}$ or 49×10^{-2}

84. Consider the following pairs of solution which will be isotonic at the same temperature. The number of pairs of solutions is/ are _____

A. 1 M aq. NaCl and 2 M aq. urea

B. 1 M aq. $CaCl_2$ and 1.5 M aq. KCl

C. 1.5 M aq. $AlCl_3$ and 2 M aq. Na_2SO_4

D. 2.5 M aq. KCl and 1 M aq. $Al_2(SO_4)_3$

Answer (4)

Sol. A. isotonic (product of $i \times C$ is same)

B. isotonic

C. isotonic

D. isotonic

So, number of isotonic pairs = 4

85. The number of colloidal systems from the following, which will have 'liquid' as the dispersion medium, is ______.

Gem stones, paints, smoke, cheese, milk, hair cream, insecticide sprays, froth, soap lather

Answer (5)

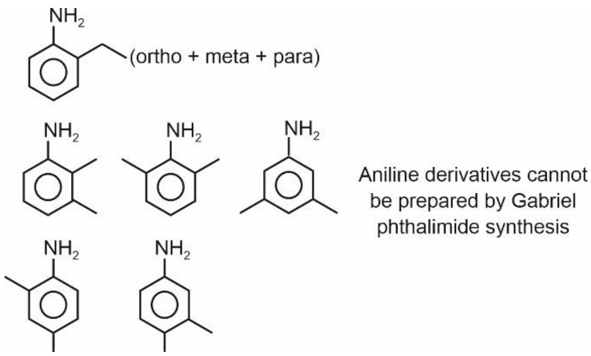
Sol. Paints, milk, froth, soap lather and hair cream have liquid as dispersion medium.

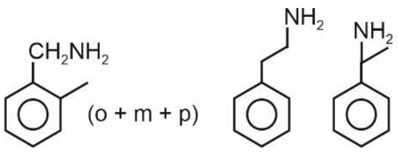
86. Number of isomeric aromatic amines with molecular formula $C_8H_{11}N$, which can be synthesized by Gabriel Phthalimide synthesis is ________.

Answer (5)

Sol. $C_8H_{11}N$

Degree of unsaturation = 4





So, number of aromatic amines = 5

87. Consider the following data

Heat of combustion of $H_2(g)$ = –241.8 kJ mol^{-1}

Heat of combustion of C(s) = –393.5 kJ mol^{-1}

Heat of combustion of = –1234.7 kJ mol^{-1}

$C_2H_5OH(l)$

The heat of formation of $C_2H_5OH(l)$ is (–) _____ kJ mol^{-1} (Nearest integer).

Answer (278)

Sol. $\underset{+3O_2(g)}{C_2H_5OH(l)} \rightarrow 2CO_2(g) + 3H_2O(l)$

$$\Delta H_C = \left[2\Delta H_f^o(CO_2) + 3\Delta H_f^o(H_2O)\right] - \left[\Delta H_f^o(C_2H_5OH(l))\right]$$

$$-1234.7 = [2 \times (-393.5) + 3 \times (-241.8)] - \left[\Delta H_f^o(C_2H_5OH(l))\right]$$

$$\Delta H_f^o C_2H_5OH = -277.7 \text{ kJ/mol}$$

$$\simeq -278 \text{ kJ/mol.}$$

88. The standard reduction potentials at 295 K for the following half cells are given below:

$NO_3^- + 4H^+ + 3e^- \rightarrow NO(g) + 2H_2O$	E^θ = 0.97 V
$V^{2+}(aq) + 2e^- \rightarrow V$	E^θ = –1.19 V
$Fe^{3+}(aq) + 3e^- \rightarrow Fe$	E^θ = –0.04 V
$Ag^+(aq) + e^- \rightarrow Ag(s)$	E^θ = 0.80 V
$Au^{3+}(aq) + 3e^- \rightarrow Au(s)$	E^θ = 1.40 V

The number of metal(s) which will be oxidized by NO_3^- in aqueous solution is _______.

Answer (3)

Sol. For feasibility, check $E^o_{cell} > 0$

For electrodes having oxidation potential greater than –0.97V,

$E^o_{cell} > 0$.

$\therefore$ Ag, Fe & V can be oxidised

89. In an ice crystal, each water molecule is hydrogen bonded to_____ neighbouring molecules.

Answer (4)

Sol. Each water molecule is H-bonded to 4 neighbouring molecules.

90. Among the following the number of compounds which will give positive iodoform reaction is ______.

(a) 1-Phenylbutan-2-one

(b) 2-Methylbutan-2-ol

(c) 3-Methylbutan-2-ol

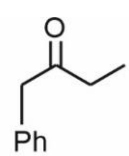
(d) 1-Phenylethanol

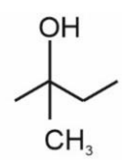
(e) 3,3-dimethylbutan-2-one

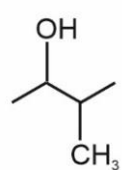
(f) 1-Phenylpropan-2-ol

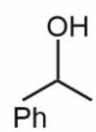
Answer (4)

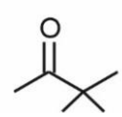
Sol.

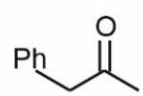
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

(c), (d), (e) & (f) give iodoform reaction

❑ ❑ ❑