10/04/2023
Morning

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Answers & Solutions

Time : 3 hrs. | for | M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x tf(t)dt$, $f(1) = \frac{2}{3}$. Then 18 $f(3)$ is equal to

(1) 210

(2) 160

(3) 150

(4) 180

Answer (2)

Sol. $x^2 f(x) - x = 4\int_0^x t\,f(t)dt$

Diff. w.r.t. x

$$x^2 f'(x) + 2x f(x) - 1 = 4xf(x)$$

$$\Rightarrow\ x^2 f'(x) - 1 = 2x f(x)$$

$$\Rightarrow\ \frac{dy}{dx} - \frac{2}{x}y = \frac{1}{x^2} \qquad \left(\text{Let } y = f(x)\ \ \frac{dy}{dx} = f'(x)\right)$$

$$\therefore\ \text{I.F} = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$$

$$\therefore\ \frac{y}{x^2} = \int \frac{1}{x^4}dx + C$$

$$\therefore\ \frac{y}{x^2} = -\frac{1}{3x^3} + C \qquad \text{Now, } f(1) = \frac{2}{3}$$

$$\frac{2}{3} = -\frac{1}{3} + C$$

$$\Rightarrow\ \boxed{C = 1}$$

$$\therefore\ f(x) = -\frac{1}{3x} + x^2$$

$$\therefore\ 18\,f(3) = 18\left[-\frac{1}{9} + 9\right] = 160$$

2. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}$, $\overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation

(1) $x^2 + x - 2 = 0$

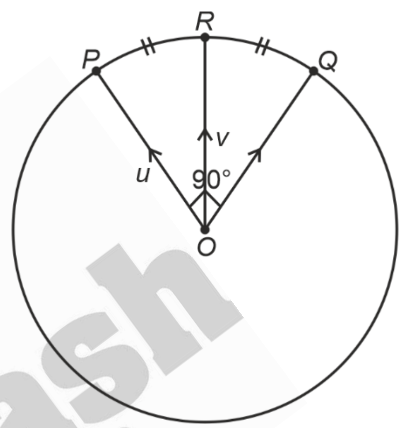
(2) $x^2 - x - 2 = 0$

(3) $3x^2 - 2x - 1 = 0$

(4) $3x^2 + 2x - 1 = 0$

Answer (2)

Sol.



$$\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$$

$$\overrightarrow{OR} = \vec{v}$$

$$\overrightarrow{OP} = \vec{u}$$

$\because$ R will lie on angle bisector of $\overrightarrow{OQ}$ and $\overrightarrow{OP}$

$$\overrightarrow{OQ} \cdot \overrightarrow{OP} = 0$$

$$= \alpha|\vec{v}|^2 + \beta \cdot (\vec{v} \cdot \vec{u}) = 0$$

$$\Rightarrow\ \alpha + \beta \cdot \cos 45° = 0$$

$$\Rightarrow\ \alpha = \frac{-\beta}{\sqrt{2}}$$

$$\overrightarrow{OQ} \cdot \overrightarrow{OR} = \frac{1}{\sqrt{2}}r^2$$

$$\Rightarrow\ (\alpha\vec{v} + \beta\vec{v}) \cdot (\vec{v}) = \frac{r^2}{\sqrt{2}}$$

$$= \alpha \cdot \frac{r^2}{\sqrt{2}} + \beta \cdot r^2 = \frac{r^2}{\sqrt{2}}$$

$$= \frac{\alpha}{\sqrt{2}} + \beta = \frac{1}{\sqrt{2}}$$

$\Rightarrow \quad \beta = \sqrt{2}$

$\therefore \quad \alpha = -1$

$\therefore \quad \alpha = -1, \beta^2 = 2$

$\therefore \quad x^2 - x - 2 = 0$

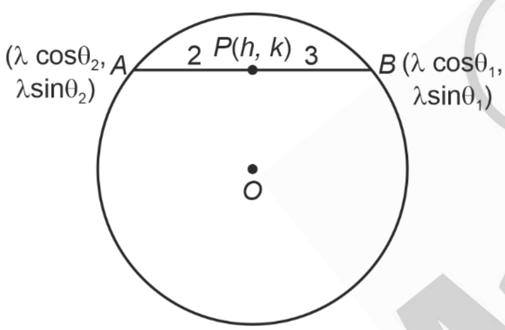
3. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius

(1) $\frac{3}{5}\lambda$ (2) $\frac{2}{3}\lambda$

(3) $\frac{\sqrt{19}}{5}\lambda$ (4) $\frac{\sqrt{19}}{7}\lambda$

Answer (3)

Sol.



Let O be the origin and radius of circle is λ and $AB = \lambda$

$$\Rightarrow \quad 1 = 2 - 2\cos(\theta_1 - \theta_2)$$

$$\Rightarrow \quad \cos(\theta_1 - \theta_2) = \frac{1}{2}$$

$$\therefore \quad h = \frac{2\lambda\cos\theta_1 + 3\lambda\cos\theta_2}{5}$$

$$k = \frac{2\lambda\sin\theta_1 + 3\lambda\sin\theta_2}{5}$$

$$\therefore \quad h^2 + k^2 = \frac{\lambda^2}{25}\left[4 + 9 + 12(\cos(\theta_1 - \theta_2))\right]$$

$$= \frac{\lambda^2}{25} \cdot 19$$

$$\therefore \quad \text{Radius } = \frac{\lambda}{5}\sqrt{19}$$

4. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to:

(1) 11 (2) 44

(3) 22 (4) 33

Answer (3)

Sol. Coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$

$$T_{r+1} = {}^{13}C_r\,(9x)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$

$$\Rightarrow \quad \boxed{r = 2}$$

$$\therefore \quad \text{Coeff. } = {}^{13}C_2\,\frac{a^{11}}{b^2}$$

Similarly coeff. of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$

$$\Rightarrow \quad \boxed{r = 6}$$

$$\therefore \quad \text{Coeff. } = {}^{13}C_6\,\frac{a^7}{b^6}$$

$$\text{Now, } \quad {}^{13}C_2\,\frac{a^{11}}{b^2} = {}^{13}C_6\,\frac{a^7}{b^6}$$

$$\Rightarrow \quad a^4 b^4 = 22$$

5. Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3k$. If the position vectors of the points A, B and C are $-2\hat{i} + \hat{j} - 3k$, $2\hat{i} + 4\hat{j} - 2k$ and $-4\hat{i} + 2\hat{j} - k$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(1) 3 (2) $\frac{8}{3}$

(3) $\frac{7}{3}$ (4) $\frac{10}{3}$

Answer (1)

Sol. $\overrightarrow{OP} = -\hat{i} - 2\hat{j} + 3k$

$\overrightarrow{AB} = 4\hat{i} + 3\hat{j} + k$

$\overrightarrow{AC} = -2\hat{i} + \hat{j} + 2k$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= 5\hat{i} - 10\hat{j} + 10k$$

Projection of $\overrightarrow{OP}$ on

$$\overrightarrow{AB} \times \overrightarrow{AC} = \left| \frac{\overrightarrow{OP} \cdot \left(\overrightarrow{AB} \times \overrightarrow{AC}\right)}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|} \right|$$

$$= \frac{5\left(-\hat{i} - 2\hat{j} + 3k\right)\left(\hat{i} - 2\hat{j} + 2k\right)}{5\sqrt{1+4+4}}$$

= 3

6. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to

(1) 241 (2) 231

(3) 210 (4) 220

Answer (2)

Sol. $a^2 + a^2 r^2 + a^2 r^4 = 33033$

$a^2(1 + r^2 + r^4) = 33033$

$a^2 (1 + r^2 + r^4) = 3 \times 7 \times (11)^2 \times 13$

$\Rightarrow a^2 = (11)^2$

$\boxed{a = 11}$

$\Rightarrow 1 + r^2 + r^4 = 273$

$r^4 + r^2 - 272 = 0$

$\Rightarrow r^2 = 16$

$\boxed{r = 4}$

$a = 11$

$ar = 44$

$ar^2 = 176$

$a + ar + ar^2 = 231$

7. The slope of tangent at any point (x, y) on a curve $y = y(x)$ is $\dfrac{x^2 + y^2}{2xy}, x > 0$. If $y(2) = 0$, then a value of $y(8)$ is

(1) $-4\sqrt{2}$ (2) $2\sqrt{3}$

(3) $-2\sqrt{3}$ (4) $4\sqrt{3}$

Answer (4)

Sol. $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2xy}$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(v + \frac{1}{v}\right)$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1 - v^2}{v}\right)$$

$$\therefore \quad \int \frac{2v}{1 - v^2} dv = \int \frac{dx}{x} \Rightarrow -\log\left|1 - v^2\right| = \ln|x| + \ln c$$

$$\Rightarrow \quad k = x\left(\frac{1 - y^2}{x^2}\right) \Rightarrow k = \frac{x^2 - y^2}{x}$$

$y(2) = 0$

$\Rightarrow k = 2$

$$\Rightarrow \quad 2 = \frac{x^2 - y^2}{x}$$

Put $x = 8$

$$2 = \frac{64 - y^2}{8} \Rightarrow y^2 = 48$$

$$y = 4\sqrt{3}$$

8. The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $(p \vee r) \wedge (\sim q)$

(2) $((\sim p) \vee r)) \wedge (\sim q)$

(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$

(4) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

Answer (2)

Sol. $\sim((p \vee q) \wedge (q \vee (\sim r)))$

$= \sim(p \vee q) \vee \sim(q \vee (\sim r))$

$= (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

$= (\sim p \wedge \sim q) \vee (\sim q) \wedge (\sim p \wedge \sim q) \vee r$

$= \sim q \wedge [(\sim p \vee r) \wedge (\sim q \vee r)]$

$= (\sim q \wedge (\sim p \vee r)) \wedge (\sim q \wedge (\sim q \vee r))$

$= (\sim q \wedge (\sim p \vee r)) \wedge \sim q$

$= ((\sim p) \vee r) \wedge (\sim q)$

9. Let two vertices of a triangle *ABC* be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to

(1) 70 (2) 76

(3) 74 (4) 72

Answer (3)

Sol. Let the vertex '*C*' be (*a*, *b*, *c*)

$$2 = \frac{2+0+a}{3} \Rightarrow a = 4$$

$$1 = \frac{4-2+b}{3} \Rightarrow b = 1$$

$$-1 = \frac{6-5+c}{3} \Rightarrow c = -4$$

$\therefore$ $C(4, 1, -4)$

Image of C in $x + 2y + 4z = 11$

$$\frac{\alpha-4}{1} = \frac{\beta-1}{2} = \frac{\gamma+4}{4} = -2\frac{(4+2-16-11)}{16+4+1} = 2$$

$$\Rightarrow \frac{\alpha-4}{1} = 2 \Rightarrow \alpha = 6$$

$$\frac{\beta-1}{2} = 2 \Rightarrow \beta = 5$$

$$\frac{\gamma+4}{4} = 2 \Rightarrow \gamma = 4$$

$\therefore$ $\alpha\beta + \beta\gamma + \alpha\gamma = 30 + 20 + 24$

$= 74$

10. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then $4m - 3n$ is equal to

(1) 6 (2) 12

(3) 10 (4) 8

Answer (4)

Sol. $2N < N!$

$N = 1$ (not possible) $\rightarrow 0$

$N = 2$ (not possible) $\rightarrow 1$

$N = 3$ (not possible) $\rightarrow 2$

$N = 4$ (possible)

$\therefore$ Required probability $= \frac{36-(1+2)}{36} = \frac{11}{12}$

$\therefore$ $4m - 3n$

$= 44 - 36$

$= 8$

Option (4) is correct

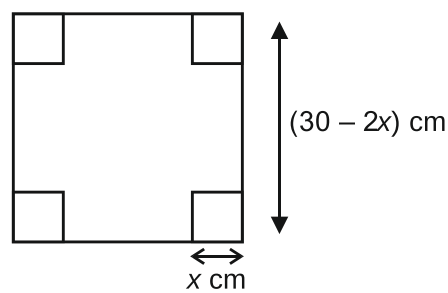
11. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm²) is equal to

(1) 800 (2) 675

(3) 1025 (4) 900

Answer (1)

Sol.



Volume $= (30 - 2x)^2 \cdot x = V(x)$

$$\frac{dV}{dx} = (30-2x)^2 + 2x(30-2x)(-2) = 0$$

$\Rightarrow$ $x = 5$, $x = 15$ (not possible)

$\therefore$ Surface area $= (30 - 2x) \times 4 + (30 - 2x)^2$

$= 800 \text{ cm}^2$ $(x = 5)$

$\therefore$ Option (1) is correct

12. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to

(1) 3 (2) 1

(3) 4 (4) 2

Answer (1)

Sol. $96\frac{\cos\pi}{33}\cdot\frac{\cos 2\pi}{33}\cdot\frac{\cos 4\pi}{33}\cdot\frac{\cos 8\pi}{33}\cdot\frac{\cos 16\pi}{33}$

Let $A=\frac{\pi}{33}$

$96\cos A\cdot\cos 2A\cdot\cos 4A\cdot\cos 8A\cdot\cos 16A$

$$=\frac{96\sin\left(2^5A\right)}{2^5\sin A}=\frac{96\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}}=3$$

13. Let the complex number $z = x + iy$ be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$

(3) $\frac{3}{4}$ (4) $\frac{4}{3}$

Answer (3)

Sol. $\frac{2z-3i}{2z+i}$ is purely imaginary

$$\frac{2(x+iy)-3i}{2(x+iy)+i}=\frac{\left(2x+i(2y-3)\right)}{\left(2x+i(2y+1)\right)}\frac{\left(2x-i(2y+1)\right)}{\left(2x-i(2y+1)\right)}\text{ is PI}$$

$$\frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2}=0$$

$$\Rightarrow 4x^2+4y^2-4y-3=0$$

$$x^2+y^2-y-\frac{3}{4}=0$$

$$\because\ x+y^2=0$$

$$\Rightarrow\ x=-y^2$$

$$y^4+y^2-y-\frac{3}{4}=0$$

$$y^4+y^2-y=\frac{3}{4}$$

14. If $f(x)=\frac{(\tan 1^\circ)x+\log_e(123)}{x\log_e(1234)-(\tan 1^\circ)}$, $x>0$, then the least value of $f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$ is

(1) 0 (2) 8

(3) 2 (4) 4

Answer (3)

Sol. Let $\tan 1^\circ = a$, $\log_e(123) = b$ and $\log_e(1234) = c$

$$\therefore\ f(x)=\frac{ax+b}{cx-a}$$

$$\therefore\ f(f(x))=\frac{a\,f(x)+b}{c\,f(x)-a}=x$$

$$\therefore\ f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)=x+\frac{4}{x}$$

$\because$ $x > 0$, then least value $=\sqrt{x\cdot\frac{4}{x}}=2$

15. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is

(1) 8 (2) 6

(3) 7 (4) 9

Answer (4)

Sol. $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$

$$\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$$

$$SD=\left|\frac{(a_2-a_1)\left(\vec{n_1}\times\vec{n_2}\right)}{\left|\vec{n_1}\times\vec{n_2}\right|}\right|$$

$$\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 1 & 2 & 0\end{vmatrix}$$

$$=-4\hat{i}+2\hat{j}+4\hat{k}$$

$$SD=\left|\frac{\left(16\hat{i}+\hat{j}-8\hat{k}\right)\left(-4\hat{i}+2\hat{j}+4\hat{k}\right)}{\sqrt{16+16+4}}\right|$$

$$=\left|\frac{-24+2-32}{6}\right|=\frac{54}{6}=9$$

16. Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane $x + y + z = 2$. If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and $2q$ are the roots of the equation

(1) $x^2 - 18x - 72 = 0$ (2) $x^2 - 18x + 72 = 0$

(3) $x^2 + 18x + 72 = 0$ (4) $x^2 + 18x - 72 = 0$

Answer (2)

Sol. $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}=\lambda$

$A(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$

$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$

$2\lambda = 6$

$\lambda = 3$

$P(6, 1, -5)$

$$q=\left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right|=\frac{78}{13}=6$$

Equation with roots q and $2q$ is

$x^2 - 3qx + 2q^2 = 0$

$x^2 - 18x + 72 = 0$

17. If A is a 3×3 matrix and $|A| = 2$, then $\left|3\,adj\left(|3A|A^2\right)\right|$ is equal to

(1) $3^{12} \cdot 6^{11}$ (2) $3^{12} \cdot 6^{10}$

(3) $3^{10} \cdot 6^{11}$ (4) $3^{11} \cdot 6^{10}$

Answer (4)

Sol. $\left|3adj\left(|3A|A^2\right)\right| = 3^3\left|adj\left(3^3|A|A^2\right)\right|$

$= 3^3\left|adj\left(2\cdot 3^3 A^2\right)\right|$

$= 3^3\left|2\cdot 3^3 A^2\right|^2$

$= 3^3\left(\left(2\cdot 3^3\right)^3\cdot\left|A^2\right|\right)^2$

$= 3^3\cdot 2^6\cdot 3^{18}\cdot |A|^4$

$= 2^6 \cdot 3^{21} \cdot 2^4 = 2^{10} \cdot 3^{21}$

$= 6^{10} \cdot 3^{11}$

18. Let the ellipse $E : x^2 + 9y^2 = 9$ intersect the positive x- and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then $m - n$ is equal to

(1) 16 (2) 15

(3) 17 (4) 18

Answer (3)

Sol. Ellipse E: $x^2 + 9y^2 = 9$ …(i)

Line L: $\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x=3-3y$ …(ii)

Circle C: $x^2 + y^2 = 9$ …(iii)

Let Q be foot of perpendicular from P upon major axis.

(ii) & (iii) $\Rightarrow (3 - 3y)^2 + y^2 = 9$

$y=\frac{9}{5},\ 0$

$PQ=\frac{9}{5}$

$$\text{Area } =\frac{1}{2}\times OA\times PQ=\frac{1}{2}\times 3\times\frac{9}{5}=\frac{27}{10}$$

$m - n = 17$

19. For the system of linear equations

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$,

which of the following is NOT correct?

(1) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(3) The system in inconsistent for $\alpha = -5$ and $\beta = 8$

(4) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

Answer (2)

Sol. $D = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix}$

$= \quad 2(2\alpha + 5) + (3\alpha + 4) + 3(7)$

$D = 7\alpha + 35$

$D \neq 0 \Rightarrow \alpha \neq -5 \Rightarrow$ unique solution.

$\therefore \quad \alpha = -6$ corresponds to unique solution.

20. If $I(x) = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to

(1) $-\frac{1}{2}e^{\frac{3}{4}}$

(2) $\frac{1}{2}e^{\frac{3}{4}}$

(3) $-e^{\frac{3}{4}}$

(4) $e^{\frac{3}{4}}$

Answer (2)

Sol. $I(x) = \int e^{\sin^2 x}(\cos x \sin 2x - \sin x)dx$

$= \int e^{\sin^2 x}\left(\cos x - \frac{1}{2\cos x}\right)\sin 2x\, dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$

$I(x) = \int e^t\left(\sqrt{1-t} - \frac{1}{2\sqrt{1-t}}\right)dt$

$= \int e^t\left(f(t) + f'(t)\right)dt = e^t f(t) + c$

$\Rightarrow \quad I(x) = e^t\sqrt{1-t} + c = e^{\sin^2 x}\cdot\cos x + c$

$I(0) = 1 \Rightarrow c = 0$

$\Rightarrow \quad I\left(\frac{\pi}{3}\right) = \frac{1}{2}e^{\frac{3}{4}}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The number of permutations, of the digits 1, 2, 3, …, 7 without repetition, which neither contain the string 153 nor the string 2467, is ________.

Answer (4898)

Sol. 1, 2, 3, ….., 7

Total number of permutations = 7! = 5040

Number of permutations containing strong 153 = 5! = 120

Number of permutations containing strong 2467 = 4! = 24

Number of permutations containing strong 153 and 2467 both = 2! = 2

$n(A\cup B) = n(A) + n(B) - n(A\cap B) = 142$

$\Rightarrow \quad n\left(\overline{A\cup B}\right) = 5040 - 142 = 4898$

22. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple played in a match, is 840, then the total numbers of persons, who participated in the tournament, is ________.

Answer (16)

Sol. Let total number of persons = $2n$

$\Rightarrow \quad {}^nC_2 \cdot {}^{n-2}C_2 \cdot 2 = 840$

$\Rightarrow \quad n(n-1)(n-2)(n-3) = 5{\cdot}6{\cdot}7{\cdot}8$

$n = 8$

$\Rightarrow \quad 2n = 16$

23. If the mean of the frequency distribution

Class :	0–10	10–20	20–30	30–40	40–50
Frequency :	2	3	x	5	4

is 28, then its variance is ________.

Answer (151)

Sol. Mean $\left(\bar{x}\right)=\dfrac{\sum f_i x_i}{\sum f_i}$

$$=\frac{2\times5+3\times15+x\times25+5\times35+4\times45}{14+x}=28$$

$\Rightarrow\ x = 6$

Variance $\left(\sigma^2\right)=\dfrac{\sum f_i\left(\bar{x}-x_i\right)^2}{\sum f_i}$

$$\sigma^2=\frac{2\times(23)^2+3\times(13)^2+6\times(3)^2+5\times(7)^2+4\times(17)^2}{20}$$

$\sigma^2 = 151$

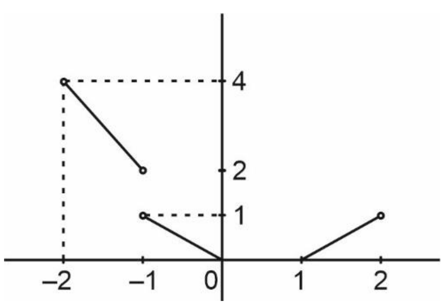
24. Let $f:(-2,\ 2)\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases} x[x] & , \ -2<x<0 \\ (x-1)[x] & , \ 0\le x<2 \end{cases}$$

where $[x]$ denotes the greatest integer function. If m and n respectively are the number of points in $(-2, 2)$ at which $y = |f(x)|$ is not continuous and not differentiable, then $m + n$ is equal to ________.

Answer (04.00)

Sol.



$y = f(x)$ is same as $y = |f(x)|$

$m = 1, n = 3$

25. The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is ________.

Answer (960.00)

Sol. $(1 - x + 2x^3)^{10}$

$$\text{Coefficient of } x^7=\frac{10!}{2!1!7!}(2)^2(-1)^1$$

$$+\frac{10!}{1!4!5!}(2)^1(-1)^4+\frac{10!}{0!7!3!}(2)^0(-1)^7$$

$= 960$

26. The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3, is equal to ________.

Answer (9525.00)

Sol. $T_n = 3 + 5(n - 1) = 5n - 2$

$\Rightarrow\ T_1, T_4, T_7.....$ are divisible by 3

i.e. 3, 18, 33, 48, ..., 363

Sum of numbers divisible by 3.

$$=\frac{25}{2}(3+363)=(25)(183)$$

Sum of all numbers in A.P.

$$=\frac{75}{2}(3+373)=(75)(188)$$

Required sum = 9525

27. Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to ________.

Answer (32.00)

Sol. $y=mx+\dfrac{1}{m}$

$\perp r$ from (4, 0) = 4

$$\left|\frac{4m+\frac{1}{m}}{m^2+1}\right|=4$$

Or $16m^2+\dfrac{1}{m^2}+8=16m^2+16$

Or $m^2=\dfrac{1}{8}$, if $m=\dfrac{1}{2\sqrt{2}}$ then

$$P=\left(\frac{a}{m^2},\frac{2a}{m}\right)=\left(8,\ 4\sqrt{2}\right)$$

Q = foot of $\perp r$ from (4, 0) on tangent

$$=\left(\frac{8}{3},\frac{8\sqrt{2}}{3}\right)$$

$$(PQ)^2=\left(\frac{16}{3}\right)^2+\left(\frac{4\sqrt{2}}{3}\right)^2$$

$$=\frac{256+32}{9}=32$$

28. Let a, b, c be the three distinct positive real numbers such that

$$(2a)^{\log_e a}=(bc)^{\log_e b} \text{ and } b^{\log_e 2}=a^{\log_e c}.$$

Then $6a + 5bc$ is equal to ________.

Answer (8)

Sol. Given equation: $(2a)^{\ln a}=(bc)^{\ln b}$

$\Rightarrow$ $\ln a\cdot(\ln 2a)=\ln b\cdot\ln(bc)$

$\Rightarrow$ $\ln a\cdot(\ln 2+\ln a)=\ln b\,(\ln b+\ln c)$

let $\ln a = x$, $\ln b = y$, $\ln c = z$, $x \neq y \neq z$

$\Rightarrow$ $x(\ln 2 + x) = y(y + z)$

$\Rightarrow$ $x\ln 2 = y^2 - x^2 + yz$ ...(i)

Similarly, from second equation

$\ln 2\cdot \ln b = \ln c\cdot\ln a$

$\Rightarrow$ $y\ln 2 = xz$ ...(ii)

Now, (i) $xy - xx$ (ii)

$\Rightarrow$ $y^3 - yx^2 + y^2z = x^2z$

$\Rightarrow$ $y^2(y + z) - x^2(y + z) = 0$

$\Rightarrow$ $(y^2 - x^2)(y + z) = 0$

$\Rightarrow$ $(x - y)(x + y)(y + z) = 0$

$\because$ $x \neq y \Rightarrow (x + y)(y + z) = 0$

$\Rightarrow$ $y = -z \Rightarrow x = -\ln 2 \Rightarrow a=\frac{1}{2}\ \&\ bc=1$

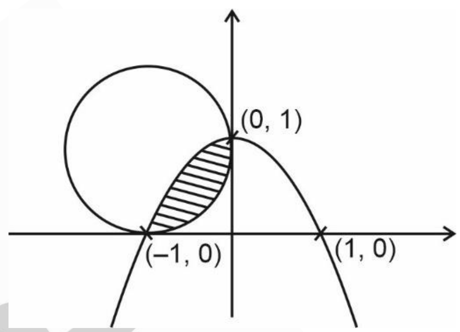
29. Let $y = p(x)$ be the parabola passing through the points (–1, 0), (0, 1) and (1, 0). If the area of the region $\left\{(x,y):(x+1)^2+(y-1)^2\le 1,\ y\le p(x)\right\}$ is A, then $12(\pi - 4A)$ is equal to ________.

Answer (16)

Sol. Given parabola is $y = 1 - x^2$

Required area is as shown

$$\text{Area}=\int_{-1}^{0}\left(-x^2+\sqrt{1-(x+1)^2}\right)dx$$



$$A=\frac{\pi}{4}-\frac{1}{3}$$

Hence, $12(\pi - 4A) = 16$

30. The number of elements in the set

$$\{n\in\mathbb{Z}:|n^2-10n+19|<6\}$$

is ________.

Answer (6)

Sol. $|n^2 - 10n + 19| < 6$

$\Rightarrow$ $-6 < n^2 - 10n + 19 < 6$

$\Rightarrow$ $n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$\Rightarrow$ $(n - 5)^2 > 0$ and

$$(n-(5-2\sqrt{3}))(n-(5+2\sqrt{3}))<0$$

$\Rightarrow$ $n\in\mathbb{Z}-\{5\}$ and $n\in\{2, 3, 4, 5, 6, 7, 8\}$

Hence $n = 2, 3, 4, 6, 7, 8$

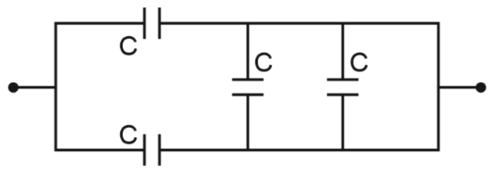
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. The equivalent capacitance of the combination shown is



(1) $2C$

(2) $\frac{5}{3}C$

(3) $\frac{C}{2}$

(4) $4C$

Answer (1)

Sol. $C_{eq} = C + C$

$= 2C$

32. A physical quantity P is given as

$$P = \frac{a^2b^3}{c\sqrt{d}}$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be

(1) 13%

(2) 16%

(3) 12%

(4) 14%

Answer (1)

Sol. $\left(\frac{\Delta P}{P}\right) = \left[\left(\frac{2\Delta a}{a}\right) + \left(\frac{3\Delta b}{b}\right) + \left(\frac{\Delta c}{c}\right) + \left(\frac{1}{2}\frac{\Delta d}{d}\right)\right]$

$\left(\frac{\Delta P}{P}\right) \times 100 = 2\times1 + 3\times2 + 3 + \frac{1}{2}\times4$

$= 13\%$

33. A carrier wave of amplitude 15 V is modulated by a sinusoidal base band signal of amplitude 3 V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is

(1) 2

(2) $\frac{3}{2}$

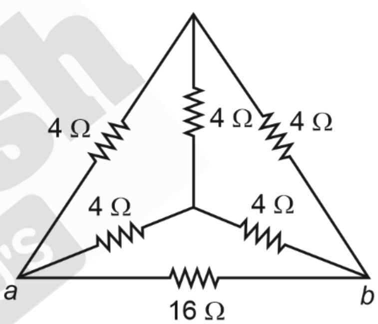
(3) 1

(4) 5

Answer (2)

Sol. $\frac{A_{max}}{A_{min}} = \frac{15+3}{15-3} = \frac{18}{12} = \left(\frac{3}{2}\right)$

34. The equivalent resistance of the circuit shown below between points a and b is :



(1) 16 Ω

(2) 3.2 Ω

(3) 24 Ω

(4) 20 Ω

Answer (2)

Sol. $R_{eq} = \left(\frac{64}{20}\right) = 3.2\ \Omega$

35. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be

(1) Zero

(2) $2L$

(3) L

(4) $\frac{L}{2}$

Answer (3)

Sol. $L = \frac{nh}{2\pi} \Rightarrow L_1 = \frac{h}{2\pi}$

For 2nd orbit

$L_2 = \frac{2h}{2\pi} = 2L$

$\Delta L = 2L - L = L$

36. Two satellites of masses m and $3m$ revolve around the earth in circular orbits of radii r & $3r$ respectively. The ratio of orbital speeds of the satellites respectively is

(1) $\sqrt{3} : 1$

(2) 3 : 1

(3) 9 : 1

(4) 1 : 1

Answer (1)

Sol. $V = \sqrt{\left(\frac{GM}{r}\right)}$

$\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \frac{\sqrt{3}}{1}$

37. Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d = \frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be :

(Given R = radius of earth)

(1) 300 N

(2) 100 N

(3) 400 N

(4) 500 N

Answer (2)

Sol. $g' = g\left(1 - \frac{d}{R}\right)$

$g' = g\left(1 - \frac{1}{2}\right) = \left(\frac{g}{2}\right)$

Weight = $\frac{w}{2} = \frac{200}{2} = 100\text{ N}$

38. The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes

(1) $\frac{1}{2}\lambda_1$

(2) $\sqrt{2}\,\lambda_1$

(3) $\frac{1}{\sqrt{2}}\lambda_1$

(4) $2\lambda_1$

Answer (3)

Sol. $V \propto \sqrt{T}$

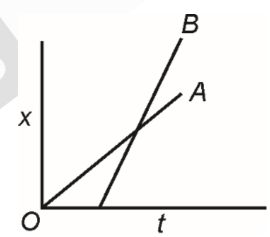
$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{1}{2}}$

$\lambda \propto \frac{1}{V}$

$\frac{\lambda_1}{\lambda_2} = \left(\frac{V_2}{V_1}\right) = \frac{\sqrt{2}}{1}$

$\lambda_2 = \left(\frac{1}{\sqrt{2}}\lambda_1\right)$

39. The position-time graphs for two students A and B returning from the school to their homes are shown in figure.



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below

(1) (A), (C) and (D) only

(2) (A), (C) and (E) only

(3) (B) and (E) only

(4) (A) and (E) only

Answer (4)

Sol. $X_A \propto X_B$

and $V_B > V_A$

40. Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed isothermally to $\frac{1}{8}$ of its original volume while the gas in B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is

(1) 8

(2) 4

(3) $8^{\frac{3}{2}}$

(4) $\frac{1}{8}$

Answer (2)

Sol. Isothermal $(P_f)_A = 8P$

Adiabatic

$$P(V)^{\left(\frac{5}{3}\right)} = P_f\left(\frac{V}{8}\right)^{\frac{5}{3}}$$

$(P_f)_B = (2^5 P) = 32$ P

$$\frac{(P)_B}{(P_A)} = \frac{32P}{8P} = 4$$

41. A particle executes S.H.M. of amplitude A along x-axis. At $t = 0$, the position of the particle is $x = \frac{A}{2}$ and it moves along positive x-axis. The displacement of particle in time t is $x = A\sin(\omega t + \delta)$, then the value δ will be

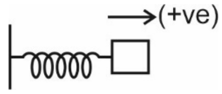
(1) $\frac{\pi}{2}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{4}$

Answer (2)

Sol. 

$t = 0$, $x = \frac{A}{2}$

$x = A\sin(\omega t + \delta)$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \delta) > 0$$

$$x = \frac{A}{2} = A\sin(0 + \delta)$$

$$\sin\delta = \frac{1}{2} \Rightarrow \delta = \left(\frac{\pi}{6}\right)$$

42. The energy of an electromagnetic wave contained in a small volume oscillates with

(1) Double the frequency of the wave

(2) The frequency of the wave

(3) Half the frequency of the wave

(4) Zero frequency

Answer (1)

Sol. $E = \frac{1}{2}\varepsilon_0 E^2 dV$

$E = E_0\sin(\omega t + \phi)$

$$E^2 = E_0^2\sin^2(\omega t + \phi)$$

$$= \frac{E_0}{2}[1 - \cos(2\omega t + \phi)]$$

43. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be

(1) 100 m

(2) $100\sqrt{2}$ m

(3) $50\sqrt{2}$ m

(4) 50 m

Answer (1)

Sol. $R = \frac{u^2\sin 20}{g} = \frac{u^2\sin 30}{g} = 50$

$$\frac{u^2}{g} = 100 \text{ m}$$

$$R = \frac{u^2\sin(90)}{g} = \frac{u^2}{g} = 100 \text{ m}$$

44. Match **List I** with **List II**:

	List I		List II
(A)	3 Translational degrees of freedom	(I)	Monoatomic gases
(B)	3 Translational, 2 rotational degrees of freedoms	(II)	Polyatomic gases
(C)	3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III)	Rigid diatomic gases
(D)	3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV)	Nonrigid diatomic gases

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Answer (1)

Sol. $\rightarrow$ Monoatomic gas has 3 translational degree of freedom

$\rightarrow$ Rigid diatomic gas $\rightarrow$ 3 translation + 2 rotational

$\rightarrow$ Non rigid diatomic gas $\rightarrow$ 3 translational + 2 rotational + 1 vibration

$\rightarrow$ Polyatomic gas $\rightarrow$ 3 translational + 3 rotational and more than one vibrational

45. Given below are two statements:

Statement I: Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs.

Statement II: Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Answer (2)

Sol. P_{max} when $X_L = X_C \rightarrow$ Resonance condition

P_{max} occurs in pure resistance circuit

46. Given below are two statements:

Statement I: If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statement II: Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

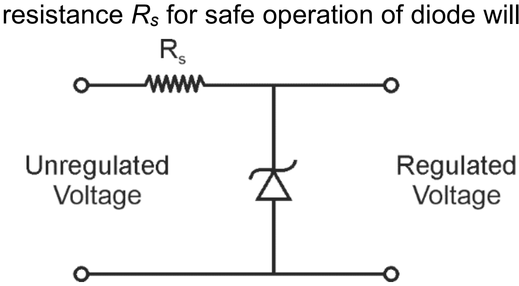
Answer (1)

Sol. $S_i = \frac{nAB}{K}$

$S_v = \left(\frac{nAB}{KR}\right)$

as R increases, $\frac{n}{R}$ ratio remains same.

47. A zener diode of power rating 1.6 W is to be used as voltage regulator. If the zener diode has a breakdown of 8 V and it has to regulate voltage fluctuating between 3 V and 10 V. The value of resistance R_s for safe operation of diode will be



(1) 10 Ω (2) 12 Ω

(3) 13.3 Ω (4) 13 Ω

Answer (1)

Sol. Power rating of zener diode = 1.6 W

$1.6 = 8 \times I$

$$I = \left(\frac{1.6}{8}\right) = 0.2 \text{ A}$$

For maximum voltage of 10 V

So, potential across R is 2 V

$2 \text{ V} = R \times 0.2$

$$R = \frac{2}{0.2} = \frac{20}{2} = 10\ \Omega$$

48. A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

(1) $\frac{v}{3}$

(2) $\frac{v}{4}$

(3) v

(4) $\frac{v}{2}$

Answer (1)

Sol. $mv = (3mv)$

$$v = \left(\frac{v}{3}\right)$$

49. Given below are two statements:

Statement I: Pressure in a reservoir of water is same at all points at the same level of water.

Statement II: The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Answer (4)

Sol. $P = P_0 + \rho gh$

P will be same at same h

and statement (II) is Pascal's law

50. An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be

(1) 4 cm towards mirror

(2) 8 cm towards mirror

(3) 8 cm away from mirror

(4) 2 cm towards mirror

Answer (2)

Sol.

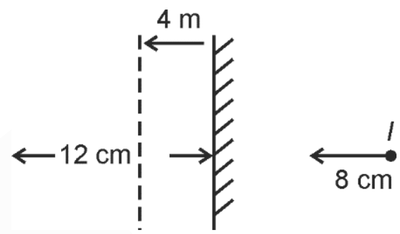


Image moved 8 cm towards the mirror.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. A transverse harmonic wave on a string is given by

$y(x, t) = 5 \sin(6t + 0.003x)$

where x and y are in cm and t in sec. The wave velocity is _________ ms^{-1}.

Answer (20)

Sol. $y = 5\sin(6t + 0.003x)$

$$v = \frac{6}{0.003} = \frac{6000}{3} = 2000 \text{ cm/s}$$

$v = 20$ m/s

52. The decay constant for a radioactive nuclide is $1.5 \times 10^{-5}\ \text{s}^{-1}$. Atomic weight of the substance is 60 g mole^{-1}, ($N_A = 6 \times 10^{23}$). The activity of 1.0 μg of the substance is _________ $\times 10^{10}$ Bq.

Answer (15)

Sol. $\lambda = 1.5 \times 10^{-5}\ s^{-1}$

Total number of atom in 1 μg

$= \frac{1\times10^{-6}}{60}\times6\times10^{23} = 1\times10^{16}$ atoms

Activity = λN_0

$= 1.5 \times 10^{-5} \times 1 \times 10^{16} = 15 \times 10^{10}$

53. If the earth suddenly to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}h$. The value of x is ________

Answer (16)

Sol. L = constant; $I\omega$ = constant

$$\Rightarrow\ r_1^2\omega_1 = r_2^2\omega_2$$

$$\Rightarrow\ \omega_2 = \frac{r^2\omega}{(r/4)^2} = 16\,\omega$$

$$\frac{r_f^3}{(r_i)^3} = \frac{0.1}{64} \Rightarrow r_f = \frac{r}{4}$$

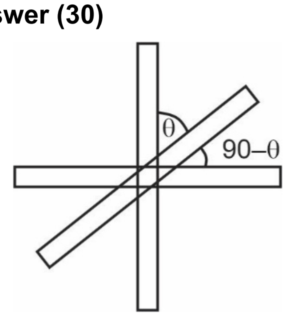
$$T = \frac{2\pi}{\omega} = 24 \text{ hr}$$

$$T = \frac{2\pi}{16\,\omega} = \frac{24}{16} \text{ hr}$$

54. Unpolarised right of intensity 32 Wm^{-2} passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is 3 Wm^{-2}, then the angle between pass axis of first two polaroids is _________°.

Answer (30)

Sol.



$$I_f = \left(\frac{I_0}{2}\cos^2\theta\right)\times\sin^2\theta$$

$$\Rightarrow\ 3 = 16\left(\cos^2\theta\cdot\sin^2\theta\right)$$

$$\Rightarrow\ \cos\theta\cdot\sin\theta = \frac{\sqrt{3}}{4}$$

$$\Rightarrow\ \theta = 30°$$

55. 10 resistors each of resistance 10 Ω can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be________.

Answer (100)

Sol. $R_{max} = 10 \times 10 = 100\ \Omega$

$R_{min} = 10/10 = 1\ \Omega$

Ratio = $R_{max}/R_{min} = 100$

56. The current required to be passed through a solenoid of 15 cm length and 60 turns in order to demagnetise a bar magnet of magnetic intensity 2.4×10^3 Am^{-1} is ________ A.

Answer (6)

Sol. $H = \frac{B}{\mu_0} - M$ for $M = 0$

$$H = \left(\frac{B}{\mu_0}\right) = ni$$

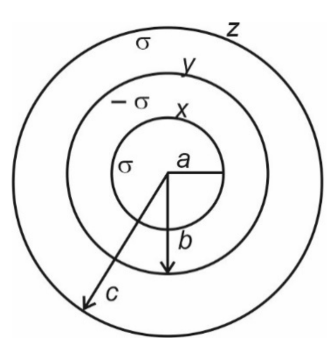
$$\Rightarrow\ 2.4\times10^3 = \frac{60}{0.15}\times i$$

$$i = \frac{2.4\times10^3}{400} = 6 \text{ A}$$

57. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [$a < b < c$] have surface charge densities σ, $-\sigma$ and σ, respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is _____cm.

Answer (5)

Sol.



$V_x = V_z$

$x = 2$ cm

$y = 3$ cm

$V_x = V_z$

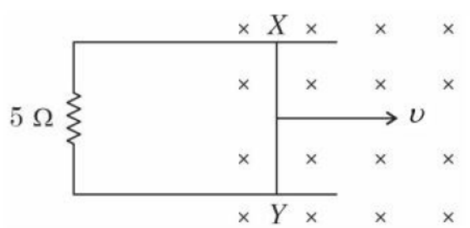
$$\Rightarrow \frac{\sigma a^2}{a} - \frac{\sigma b^2}{b} + \frac{\sigma c^2}{c} = \frac{\sigma a^2 - \sigma b^2 + \sigma c}{c}$$

$$\Rightarrow a - b = \frac{a^2}{c} - \frac{b^2}{c} = \frac{(a+b)(a-b)}{c}$$

$c = a + b$

$c = 2 + 3 = 5$ cm

58. A 1 m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is 5Ω, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be ________ 10^{-3} *N.*



Answer (18)

Sol. $F = i\ell B$

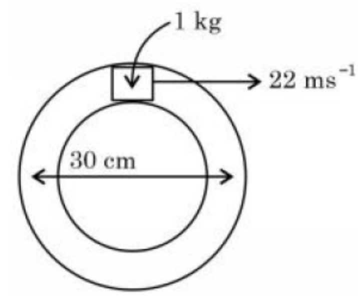
$$= \frac{BV\ell}{R} \times \ell B$$

$$= \frac{B^2\ell^2 V}{R}$$

$$= \frac{(0.15)^2 \times (1)^2 \times 4}{5}$$

$= 18 \times 10^{-3}$ N

59. A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is _________ J. (Given g = 10 m/s^2).



Answer (245)

Sol. Total work done by tube on block is equal to loss in mechanical energy

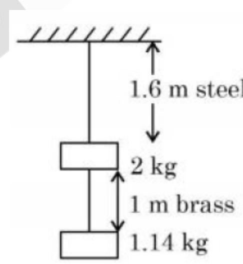
$$w = \frac{1}{2}mv^2 + mgh$$

$$= \frac{1}{2} \times 1 \times (22)^2 + 1 \times 10 \times 0.3$$

$= 3 + 242$

$= 245$ J

60. Two wires each of radius 0.2 cm and negligible mass, one made of steel and the other made of brass are loaded as shown in the figure. The elongation of the steel wire is _______ $10^{-6} m$. [Young's modulus for steel = 2×10^{11} Nm^{-2} and $g = 10$ ms^{-2}]



Answer (20)

Sol. F on steel wire = (2 + 1.14) g

= (3.14) g

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$$

$$\Rightarrow \Delta L = \left(\frac{FL}{AY}\right) = \frac{3.14 \times g \times 1.6}{\pi \times (0.2)^2 \times 2 \times 10^{11} \times 10^{-4}}$$

$$= \frac{16}{.8 \times 10^5}$$

$= 2 \times 10^{-5}$

$= 20 \times 10^{-6}$ m

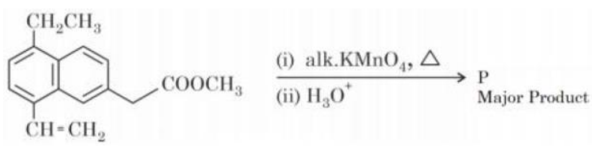
CHEMISTRY

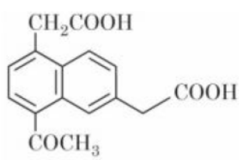
SECTION - A

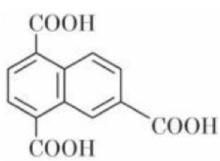
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

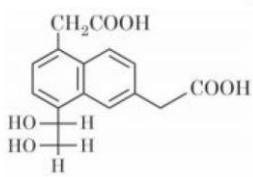
Choose the correct answer:

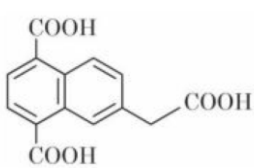
61. The major product 'P' formed in the given reaction is



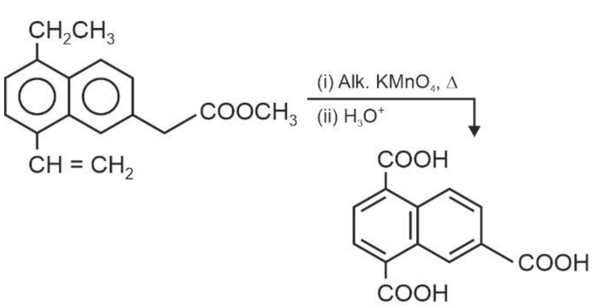
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

(P)
(Major product)

62. Given below are two statements:

Statement I : Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution.

Statement II : $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$.

In the light of the above statements, choose the *correct* answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Answer (3)

Sol. $K_2Cr_2O_7$ is used as a primary standard in redox titration under volumetric analysis.

$Na_2Cr_2O_7$ is more soluble than $K_2Cr_2O_7$.

63. The pair from the following pairs having both compounds with net non-zero dipole moment is

(1) 1,4-Dichlorobenzene, 1,3-Dichlorobenzene
(2) cis-butene, trans-butene
(3) CH_2Cl_2, $CHCl_3$
(4) Benzene, anisidine

Answer (3)

Sol. The CH_2Cl_2 and $CHCl_3$ both have net non-zero dipole moment. The 1,4-dichlorobenzene, trans-butene, benzene have zero dipole moment. The 1,3-dichlorobenzene, cis-butene, anisidine has non-zero dipole moment.

64. The octahedral diamagnetic low spin complex among the following is

(1) $[Co(NH_3)_6]^{3+}$
(2) $[CoF_6]^{3-}$
(3) $[CoCl_6]^{3-}$
(4) $[NiCl_4]^{2}$

Answer (1)

Sol. $[Co(NH_3)_6]^{3+}$ has d^2sp^3 (inner orbital complex) with zero unpaired electrons. Hence, it is octahedral with diamagnetic character and low spin complex.

$[CoF_6]^{3-}$ and $[CoCl_6]^{3-}$ are octahedral but having unpaired electrons.

$[NiCl_4]^{--}$ is not octahedral.

65. The enthalpy change for the adsorption process and micelle formation respectively are

(1) $\Delta H_{ads} > 0$ and $\Delta H_{mic} < 0$

(2) $\Delta H_{ads} > 0$ and $\Delta H_{mic} > 0$

(3) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$

(4) $\Delta H_{ads} < 0$ and $\Delta H_{mic} < 0$

Answer (3)

Sol. The adsorption process ΔH = –ve.

For micelle formation ΔH is +ve and ΔS is +ve.

66. Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to

(1) Prevent hydrolysis

(2) Prevent reduction

(3) Prevent breaking

(4) Prevent oxidation

Answer (4)

Sol. Prolonged heating may oxidise ferrous ions to ferric ions.

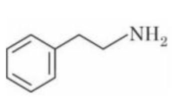
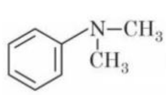
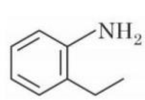
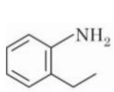
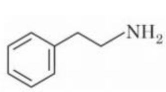
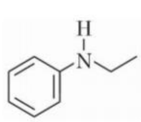
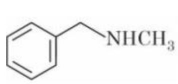
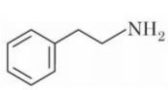
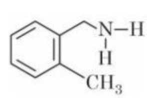
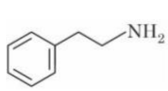
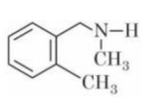
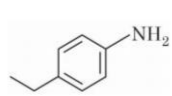
67. Isomeric amines with molecular formula $C_8H_{11}N$ give the following tests

Isomer(P) $\Rightarrow$ Can be prepared by Gabriel phthalimide synthesis

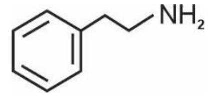
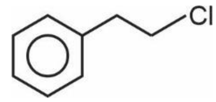
Isomer(Q) $\Rightarrow$ Reacts with Hinsberg's reagent to give solid insoluble in NaOH

Isomer(R) $\Rightarrow$ Reacts with HONO followed by β-naphthol in NaOH to give red dye.

Isomers(P), (Q) and (R) respectively are

	P	Q	R
(1)			
(2)			
(3)			
(4)			

Answer (4)

Sol.  can be prepared by Gabriel phthalimide synthesis because S_N2 is possible in 

2 amine reacts with Hinsberg's reagent to give solid insoluble in NaOH.

Aromatic diazonium are prepared and used in diazocoupling.

68. Match List I with List II

	List I Industry		List II Waste Generated
(A)	Steel plants	(I)	Gypsum
(B)	Thermal power plants	(II)	Fly ash
(C)	Fertilizer Industries	(III)	Slag
(D)	Paper mills	(IV)	Bio-degradable wastes

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Answer (1)

Sol. Slag is waste generated in iron extraction. Fly ash is waste generated in thermal power plants.

Gypsum is the waste of fertilizer industry. Non-cellulosic part of plant is wastes of paper industry and it is biodegradable.

69. Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO_2, a test in which insoluble B is formed. If B is further reacted with CO_2 then soluble compound is formed. 'A' is

(1) Quick lime (2) Slaked lime

(3) White lime (4) Lime water

Answer (2)

Sol. $\underset{\text{(Lime)}}{CaO} \xrightarrow{\text{Limited water}} Ca(OH)_{2(s)}$ the process is k/a slaking of lime and the product is slaked lime. It is an exothermic process.

$Ca(OH)_2$ aq. solution is k/a Lime water and used for test of CO_2.

$Ca(OH)_2$ aq. $\xrightarrow{CO_2}$ Milkiness due to $CaCO_3$

$\downarrow$ CO_2 extra

Milkiness disappears due to formation of $Ca(HCO_3)_2$

70. The one that does not stabilize 2° and 3° structures of proteins is

(1) –S–S–linkage

(2) H-bonding

(3) –O–O–linkage

(4) van der Waals forces

Answer (3)

Sol. The H bonding, van der waals forces as well as –S–S– linkage are the responsible forces for the formation of secondary and tertiary structures of proteins hence they stabilize the structure.

71. Using column chromatography, mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has

(1) low R_f, stronger adsorption

(2) high R_f, weaker adsorption

(3) high R_f, stronger adsorption

(4) low R_f, weaker adsorption

Answer (1)

Sol. That chemical species which is strongly adsorbed will be eluted last, that weakly adsorbed eluted first and will have more R_f so B will have low R_f and stronger adsorption.

72. Which of the following is used as a stabilizer during the concentration of sulphide ores?

(1) Pine oils

(2) Fatty acids

(3) Xanthates

(4) Cresols

Answer (4)

Sol. In froth floatation process the pine oils, fatty acids, xanthates are collectors and froth stabilisers cresols, aniline are used.

Ref: NCERT

73. The compound which does not exist is

(1) NaO_2 (2) BeH_2

(3) $PbEt_4$ (4) $(NH_4)_2BeF_4$

Answer (1)

Sol. Oxide and peroxides of sodium are stable (Na_2O and Na_2O_2) K, Rb, Cs form superoxides

74. Given

(A) $2CO(g)+O_2(g)\rightarrow 2CO_2(g) \quad \Delta H_1^\theta = -x\text{ kJ mol}^{-1}$

(B) $C(\text{graphite})+O_2(g)\rightarrow CO_2(g) \quad \Delta H_2^\theta = -y\text{ kJmol}^{-1}$

The ΔH^θ for the reaction

$C(\text{graphite})+\frac{1}{2}O_2(g)\rightarrow CO(g)$ is

(1) $\frac{2x-y}{2}$ (2) $\frac{x+2y}{2}$

(3) $\frac{x-2y}{2}$ (4) $2y-x$

Answer (3)

Sol. (A) $2CO(g)+O_2(g)\rightarrow 2CO_2(g) \quad \Delta H_1^\theta = -x\text{ kJ mol}^{-1}$

(B) $C(\text{graphite})+O_2(g)\rightarrow CO_2(g) \quad \Delta H_2^\theta = -y\text{ kJmol}^{-1}$

Multiply eg–B by (2) and subtract eg. (A) from it

$2\text{Cgraphite}+O_2(g)\rightarrow 2CO(g) \quad \Delta H^\circ = x-2y$

$\text{Cgraphite}+\frac{1}{2}O_2 \rightarrow CO \quad \Delta H^\circ = \frac{x-2y}{2}$

75. The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively

(1) 7.527×10^{23} and 0.125 mol

(2) 7.527×10^{22} and 0.250 mol

(3) 1.505×10^{23} and 0.250 mol

(4) 7.527×10^{22} and 0.125 mol

Answer (4)

Sol. Moles $=\frac{2.8375}{22.7}=0.125$

Molecules = $0.125 \times 6.022 \times 10^{23}$

$= 7.527 \times 10^{22}$

76. Which of the following statements are correct?

(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese.

(B) The higher oxidation states of first row *d*-block elements get stabilized by oxide ion

(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid

(D) Magnetic moment of V^{2+} is observed between 4.4 – 5.2 BM

Choose the correct answer from the options given below:

(1) (B), (C) only
(2) (A), (B), (D) only
(3) (C), (D) only
(4) (A), (B) only

Answer (1)

Sol. The reduction electrode potential of Mn^{3+}/Mn^{2+} is +1.57 V while that of Fe^{3+}/Fe^{2+} is +0.77 V, hence A is wrong. Higher oxidation state of smaller *d*-block elements is stabilized (or say form compounds) with smaller anion oxide that can be explained by stearic reason hence B is correct.

The oxidation electrode potential of Cr^{2+}/Cr^{3+} is +0.41 V hence it can reduce H^+ and so liberate H_2.

The unpaired electrons in V^{2+} are 3 hence the magnetic moment of V^{2+} will be lesser than 4.4 BM.

Hence, only B and C are correct.

77. Given below are two reactions, involved in the commercial production of dihydrogen (H_2). The two reactions are carried out at temperature "T_1" and "T_2", respectively

$$C(s)+H_2O(g)\xrightarrow{T_1}CO(g)+H_2(g)$$

$$CO(g)+H_2O(g)\xrightarrow[\text{Catalyst}]{T_2}CO_2(g)+H_2(g)$$

The temperatures T_1 and T_2 are correctly related as

(1) $T_1 = T_2$

(2) $T_1 < T_2$

(3) $T_1 = 100$ K, $T_2 = 1270$ K

(4) $T_1 > T_2$

Answer (4)

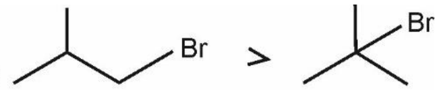
Sol. $C(s)+H_2O(g)\xrightarrow{1270\,K(T_1)}CO(g)+H_2(g)$ is k/a coal gasification reaction.

$CO(g)+H_2O(g)\xrightarrow[\text{Catalyst}]{673\,K(T_2)}CO_2(g)+H_2(g)$ is k/a water gas shift reaction.

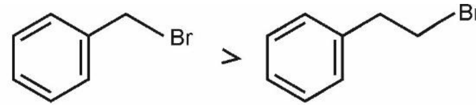
Hence, $T_1 > T_2$

78. Identify the correct order of reactivity for the following pairs towards the respective mechanism

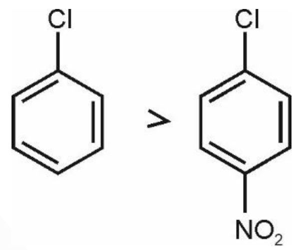
(A) S_N2



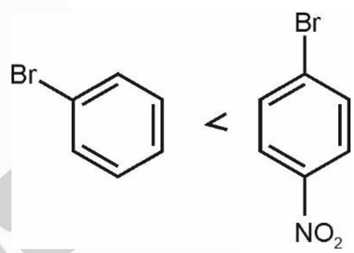
(B) S_N1



(C) Electrophilic substitution



(D) Nucleophilic substitution



Choose the correction answer from the options given below:

(1) (B), (C) and (D) only
(2) (A), (B), (C) and (D)
(3) (A), (B) and (D) only
(4) (A), (C) and (D) only

Answer (2)

Sol. The 2°alkyl halide has lesser stearic hinderance as compared to 3°alkyl halides. Hence, the 2°RX are more reactive for S_N2 as compared to 3°RX.

More stable carbocation formed, higher will be the reactivity of S_N1.

→ NO_2 is strong deactivating group for electrophilic substitution.

→ NO_2 is activating for nucleophilic substitution over benzene ring especially when attached to ortho/para position of leaving group.

Hence A, B, C, D all are correct.

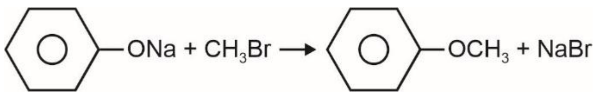
79. Suitable reaction condition for preparation of Methyl phenyl ether is

(1) $PhO^{\ominus}Na^{\oplus}$, MeOH
(2) Benzene, MeBr
(3) $Ph-Br$, $MeO^{\ominus}Na^{\oplus}$
(4) $PhO^{\ominus}Na^{\oplus}$, MeBr

Answer (4)

Sol. PhONa + MeBr $\rightarrow$ PhOMe + NaBr

Williamson's Synthesis



80. Match List-I with List-II.

List-I Polymer		List-II Type/Class	
A.	Nylon-2-Nylon-6	I.	Thermosetting polymer
B.	Buna-N	II.	Biodegradable polymer
C.	Urea-formaldehyde resin	III.	Synthetic rubber
D.	Dacron	IV	Polyester

Choose the correct answer from the options given below:

(1) A $\rightarrow$ IV; B $\rightarrow$ I; C $\rightarrow$ III; D $\rightarrow$ II

(2) A $\rightarrow$ II; B $\rightarrow$ III; C $\rightarrow$ I; D $\rightarrow$ IV

(3) A $\rightarrow$ IV; B $\rightarrow$ III; C $\rightarrow$ I; D $\rightarrow$ II

(4) A $\rightarrow$ II; B $\rightarrow$ I; C $\rightarrow$ IV; D $\rightarrow$ III

Answer (2)

Sol. (A) Nylon-2-Nylon-6 – Biodegradable

(B) Buna N – Synthetic rubber

(C) Urea-formaldehyde resin – Thermosetting (due to cross links)

(D) Dacron – Polyester

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. $FeO_4^{2-} \xrightarrow{+2.2V} Fe^{3+} \xrightarrow{+0.70V} Fe^{2+} \xrightarrow{-0.45V} Fe^0$

$E^{\theta}_{FeO_4^{2-}/Fe^{2+}}$ is $x \times 10^{-3}$ V.

The value of x is ________

Answer (1825)

Sol. $FeO_4^{2-} \xrightarrow{+2.2V} Fe^{3+} \quad \Delta G_1 = -6.6\,F$

$Fe^{3+} \xrightarrow{+0.70V} Fe^{2+} \quad \Delta G_2 = -0.7\,F$

Hence for

$FeO_4^{2-} \longrightarrow Fe^{2+} \quad \Delta G = -7.3\,F$

$= -nEF$

$E^0_{FeO_4^{2-}/Fe^{+2}} = \dfrac{-7.3\,F}{-4\,F} = 1.825,\ n = 4$

$= 1825 \times 10^{-3}\,V$

n = electron exchange of that half cell reaction.

82. In potassium ferrocyanide, there are ______ pairs of electrons in the t_{2g} set of orbitals.

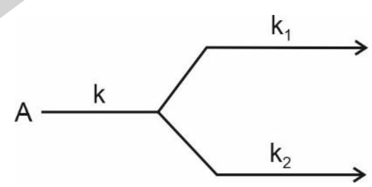
Answer (3)

Sol. $K_4[Fe(CN)_6]$ has Fe at +2 state and forming low spin octahedral complex with CN^-, hence has d^2sp^3 hybridised state. On applying C.F.T. it shows that all the 6 electrons of d subshell are present in the form of three pairs in t_{2g} orbitals. Hence answer is 3.

83. A molecule undergoes two independent first order reactions whose respective half lives are 12 min and 3 min. If both the reactions are occurring then the time taken for the 50% consumption of the reactant is ______ min. (Nearest integer)

Answer (2)

Sol. For parallel reaction



$k = k_1 + k_2$

$$\frac{1}{T_{1/2}} = \frac{1}{12} + \frac{1}{3} = \frac{5}{12}$$

$$\text{Net } T_{1/2} = \frac{12}{5} = 2.4 \text{ min} \simeq 2 \text{ min}$$

Hence time taken for 50% consumption of reactant will be close to 2 min.

84. The number of bent-shaped molecule/s from the following is ______

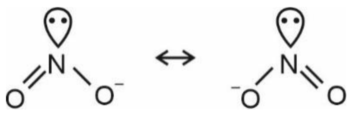
$N_3^-, NO_2^-, I_3^-, O_3, SO_2$

Answer (3)

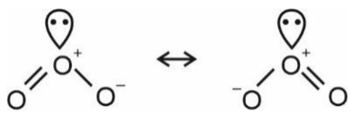
Sol. N_3^- (Azide) : $\overset{-}{N}=\overset{+}{N}=\overset{-}{N} \leftrightarrow \overset{--}{N}-\overset{+}{N}\equiv N$

has sp hybridised central atom. Hence linear

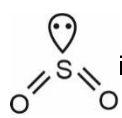
I_3^- (triiodide) :  has linear geometry, sp^3d hybridisation with three lone pairs at central atom.

NO_2^- (nitrite) : 

Is (nonlinear) bent shaped as it has sp^2 hybridisation with one lone pair at central atom.

O_3 (ozone) : 

Is bent with sp^2 hybridisation and one lone pair at central atom.

SO_2 (sulphur dioxide) :  is bent with sp^2 hybridisation and one lone pair at central atom.

85. The number of correct statement/s involving equilibria in physical processes from the following is ________

(A) Equilibrium is possible only in a closed system at a given temperature.

(B) Both the opposing processes occur at the same rate.

(C) When equilibrium is attained at a given temperature, the value of all its parameters became equal

(D) For dissolution of solids in liquids, the solubility is constant at a given temperature.

Answer (3)

Sol. A, B and D statements are correct.

The equilibrium is possible when there is no exchange of matter between system and surroundings.

At equilibrium state the forward and backward processes occur with same rate (speed)

When equilibrium is attained, the value of all its parameters became constant.

The solubility is function of solubility product, the value of which is constant at a given temperature.

86. In the following reaction, the total number of oxygen atoms in X and Y is______

$Na_2O + H_2O \rightarrow 2X$

$Cl_2O_7 + H_2O \rightarrow 2Y$

Answer (5)

Sol. $Na_2O + H_2O \longrightarrow 2NaOH\ (X)$

$Cl_2O_7 + H_2O \longrightarrow 2HClO_4\ (Y)$

X has one O and Y has four O

87. If the degree of dissociation of aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be _____ % higher than the expected/theoretical freezing point. (Nearest integer).

Answer (30)

Sol. $i = 1 + \alpha\ (n - 1)$

$= 1 + 0.3\ (2 - 1)$

$= 1 + 0.3$

$= 1.3$

ΔT_f without considering dissociation = $K_f \times$ molality

ΔT_f without considering dissociation = $iK_f \times$ molality

$= \Delta T_f'$

$\Delta T_f' - \Delta T_f = 0.3\ (k_f \times \text{molality})$

$$\frac{\Delta T_f' - \Delta T_f}{\Delta T_f} = \frac{0.3\ (k_f \times \text{molality})}{k_f \times \text{molality}} = 0.3$$

% value = 30

88. The number of **incorrect** statement/s about the black body from the following is______

(A) Emit or absorb energy in the form of electromagnetic radiation.

(B) Frequency distribution of the emitted radiation depends on temperature.

(C) At a given temperature, intensity vs frequency curve passes through a maximum value.

(D) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature.

Answer (0)

Sol. All the given statements A, B, C & D are correct.

The amount of electromagnetic radiation emitted (intensity of radiation) from a black body and its spectral (frequency) distribution depends only on its temperature.

Fig. 2.8 Wavelength-intensity relationship. (**Ref :** Particle nature of electromagnetic radiation; Planck's quantum theory, ncert chapter structure of atom.)

Shows as the temperature increases, maxima of the curve shifts to shorter wavelength or say higher frequency.

89. At constant temperature, a gas is at a pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is ______ mm Hg. (Nearest integer)

Answer (1567)

Sol. $P_2V_2 = P_1V_1$ Boyle's Law

$P_2 = \dfrac{940.3 \times V_1}{0.6V_1}$ $V_2 = V_1 - 0.4\ V_1 = 0.6\ V_1$

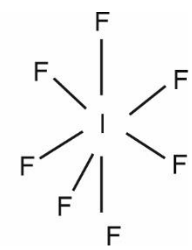
$= 1567.16$

≈ 1567 mm Hg

90. The sum of lone pairs present on the central atom of the interhalogen IF_5 and IF_7 is______

Answer (1)

Sol. IF_5 has one lone pair at central atom and IF_7 has no any lone pair at central atom



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