12/04/2023

Morning

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Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The number of five-digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 132
(2) 120
(3) 72
(4) 96

Answer (2)

Sol. Case I : Numbers start with 5

$\underline{5}\ \underset{\text{4 ways}}{\underline{\ \ }}\ \underset{\text{3 ways}}{\underline{\ \ }}\ \underset{\text{2 ways}}{\underline{\ \ }}\ \underline{0} = 4 \times 3 \times 2 = 24$

Case II :

$\underline{7}\ \underset{4}{\underline{\ \ }}\ \underset{3}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }} = 48$

Case III :

$\underline{9}\ \underset{4}{\underline{\ \ }}\ \underset{3}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }} = 48$

Total numbers = 120

2. Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is equal to

(1) 2
(2) –1
(3) –2
(4) 1

Answer (4)

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$\Rightarrow (a-1)[c(b-1)-(1-c)] + 1[(1-b)(1-c)] = 0$

$\Rightarrow c(a-1)(b-1)-(a-1)(1-c)+(1-b)(1-c) = 0$

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c} = 1$

3. Let $<a_n>$ be a sequence such that $a_1+a_2+...+a_n = \frac{n^2+3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1\,p_2\,p_3 \ldots p_m$, where p_1, p_2,…p_m are the first m prime numbers, then m is equal to

(1) 5
(2) 8
(3) 6
(4) 7

Answer (3)

Sol. $a_n = \frac{n^2+3n}{(n+1)(n+2)} - \frac{(n-1)^2+3(n-1)}{n(n+1)}$

$= \frac{4}{n(n+1)(n+2)}$

$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{4}\sum_{k=1}^{10}k(k+1)(k+2)$

$= \frac{1}{16}\sum_{k=1}^{10}k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2)$

$\frac{1}{16}\Big[(1\cdot2\cdot3\cdot4.-0)+(2\cdot3\cdot4\cdot5-1\cdot2\cdot3\cdot4)$

$(10\cdot11\cdot12\cdot13-9\cdot10\cdot11\cdot12)\Big]$

$= \frac{1}{16}[10\cdot11\cdot12\cdot13] =$

$\therefore\ 28\sum_{k=1}^{10}\frac{1}{a_k} = \frac{28\times5\times11\times3\times13}{2}$

$= 2\cdot3\cdot5\cdot7\cdot11\cdot13$

$\therefore\ \boxed{m=6}$

4. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(1) 75

(2) 125

(3) 50 (4) 100

Answer (4)

Sol. $\because \overset{(M)}{\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}} \overset{(N)}{\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad MN = I = NM$

$B = MAN$

$B^n = (MAN)^n = (MAN)\,(MAN) \ldots$

$= (MA^2N)\, MAN$

$= MA^nN$

Now $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = I + E$

$E^2 = 0$

$\therefore\ A^n = (I+E)^n = I + nE + \underbrace{{}^nC_2E^2}_{0}$

$= I + nE$

$= \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

So, $B^n = MA^nN = \begin{bmatrix} 1+\frac{n}{51} & \frac{n}{51} \\ \frac{-n}{51} & 1-\frac{n}{51} \end{bmatrix}$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50+\frac{50\cdot 51}{2\cdot 51} & \frac{50\cdot 51}{2\cdot 51} \\ \frac{-50\cdot 51}{2\cdot 51} & 50-\frac{50\cdot 51}{2\cdot 51} \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

$\therefore \quad$ Sum = 100

5. If $\frac{1}{n+1}\,{}^nC_n + \frac{1}{n}\,{}^nC_{n-1} + \ldots + \frac{1}{2}\,{}^nC_1 + {}^nC_0 = \frac{1023}{10}$ then n is equal to

(1) 9 (2) 8

(3) 7 (4) 6

Answer (1)

Sol. $\sum_{r=0}^{n} \frac{1}{r+1}\,{}^nC_r = \frac{1023}{10} \qquad \left(\because\ {}^{n+1}C_{r+1} = \frac{n+1}{r+1}\,{}^nC_r\right)$

$\Rightarrow \sum_{r=0}^{n} \frac{1}{n+1}\,{}^{n+1}C_{r+1} = \frac{1023}{10}$

$\Rightarrow \frac{1}{n+1}\left[{}^{n+1}C_1 + {}^{n+1}C_2 + \ldots + {}^{n+1}C_{n+1}\right] = \frac{1023}{10}$

$\Rightarrow \frac{2^{n+1}-1}{n+1} = \frac{1023}{10} = \frac{2^{10}-1}{10}$

$\Rightarrow \quad n + 1 = 10$

$\Rightarrow \quad n = 9$

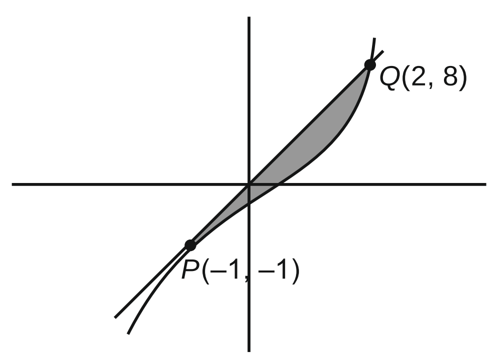
6. The area of the region enclosed by the curve $y = x^3$ and its tangent at the point (–1, –1) is

(1) $\frac{19}{4}$ (2) $\frac{23}{4}$

(3) $\frac{31}{4}$ (4) $\frac{27}{4}$

Answer (4)

Sol.



Given $y = x^3$ …(i)

$\Rightarrow \frac{dy}{dx} = 3x^2$

$\left(\frac{dy}{dx}\right)_{(-1,\,-1)} = 3$

Equation of tangent at (–1, –1)

$(y + 1) = 3(x + 1)$

$y = 3x + 2$ ...(ii)

Solving (i) and (ii)

$x^3 = 3x + 2$

$x^3 - 3x - 2 = 0$ → -1, -1, $\alpha = 2$

$Q(2, 8)$

$$\text{Required area } = \int_{-1}^{2}\left(3x + 2 - x^3\right)dx$$

$$= \frac{3}{2}(4-1) + 2(2+1) - \frac{1}{4}(16-1)$$

$$= \frac{27}{4}$$

7. If the total maximum value of the function $f(x) = \left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

(1) $e^3 + e^6 + e^{11}$

(2) $e^5 + e^6 + e^{11}$

(3) $e^3 + e^6 + e^{10}$

(4) $e^3 + e^5 + e^{11}$

Answer (1)

Sol. $f(x) = \left(\frac{\sqrt{3}e}{2\sin x}\right)^{\sin^2 x}$

$$f'(x) = f(x)\left[2\sin x\cos x \times \ln\left(\frac{\sqrt{3}e}{2\sin x}\right) + \sin^2 x \frac{1 \times 2\sin x}{\sqrt{3}e} \times \frac{\sqrt{3}e}{2} - \frac{1}{\sin^2 x} \times \cos x\right]$$

$$= f(x)\left[\sin 2x \ln\left(\frac{\sqrt{3}e}{2\sin x}\right) - \sin x\cos x\right] = 0$$

Sin $2x = 0$ (not possible)

$$\ln\left(\frac{\sqrt{3}e}{2\sin x}\right) = +\frac{1}{2}$$

$$\frac{\sqrt{3}e}{2\sin x} = e + \frac{1}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$f_{\max} = (e)^{\frac{3}{8}} = \frac{c^{\frac{11}{8}}}{e} \Rightarrow k = e^{\frac{17}{8}}$$

$$\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

8. Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to

(1) 81

(2) 9

(3) 72

(4) 729

Answer (2)

Sol. $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$

Let $a_n = \alpha^n + \beta^n$

$$= \frac{a_{23} + a_{14}}{a_{15} + a_{10}}$$

$x^2 + \sqrt{6}x + 3 = 0$ → α, β

$$x = \frac{-\sqrt{6} \pm \sqrt{-6}}{2}$$

$$= \sqrt{6}\left(\frac{-1 \pm i}{2}\right)$$

$$= \sqrt{3}\left(\frac{-1 \pm i}{\sqrt{2}}\right)$$

$$\alpha = \sqrt{3}\, e^{\frac{i3\pi}{4}}$$

$$\beta = \sqrt{3}\, e^{\frac{i5\pi}{4}}$$

$$\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$$

$$= 9\left[\frac{\left(\sqrt{3}\right)^{9}\left(e^{i23\times\frac{3\pi}{4}}+e^{i23\times\frac{5\pi}{4}}\right)+\left(e^{i14\times\frac{3\pi}{4}}+e^{i14\times\frac{5\pi}{4}}\right)}{\left(\sqrt{3}\right)^{5}\left(e^{i15\times\frac{3\pi}{4}}+e^{i15\times\frac{5\pi}{4}}\right)+\left(e^{i10\times\frac{3\pi}{4}}+e^{i\times10\times\frac{5\pi}{4}}\right)}\right]$$

$$= \left(\frac{\left(\sqrt{3}\right)^{9}\left(\frac{1+i-1+i}{\sqrt{2}}\right)+0}{\left(\sqrt{3}\right)^{5}\left(\frac{1+i-1+i}{\sqrt{2}}\right)+0}\right)\times 9$$

= 81

9. Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a|+|b|+|c|$ is equal to

(1) 12 (2) 14

(3) 8 (4) 10

Answer (4)

Sol. for $P_2 : \overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$

$$= \hat{i}(7) - \hat{j}(5) + k(-11)$$

Let R lies on l_2

Let $z = 0$

$\therefore$ $3x + 2y = 2$

$x - 3y = 13$

$\therefore$ $11y = -39 + 2 = -37$

$\therefore$ $y = \frac{-37}{11}$

$\therefore$ $x = 13 + 3y$

$$= 13 - \frac{37\times 3}{11}$$

$$= \frac{143-111}{11} = \frac{32}{11}$$

$\therefore$ $R\left(\frac{32}{11}, \frac{-37}{11}, 0\right)$

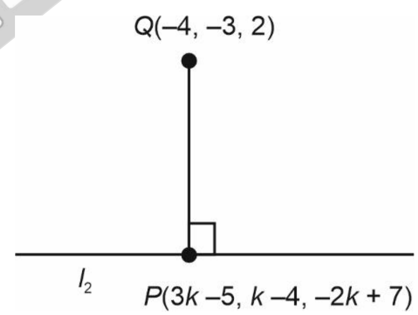
l_1 & l_2 are coplanar

$$\therefore \begin{pmatrix} -5-\frac{32}{11} & -4+\frac{37}{11} & \alpha \\ 3 & 1 & -2 \\ 7 & -5 & -11 \end{pmatrix} = 0$$

$$= \begin{vmatrix} \frac{-87}{11} & \frac{-7}{11} & \alpha \\ 3 & 1 & -2 \\ 7 & -5 & -11 \end{vmatrix} = 0$$

$\Rightarrow$ $\alpha = 7$

$\therefore$ $P(3k - 5, k - 4, -2k + 7)$ is nearest to $(-4, -3, 2)$



$$\overrightarrow{QP}\cdot\overrightarrow{n_1} = 0$$

$$\left((3k-1)\hat{i}+(k-1)\hat{j}+(-2k+5)k\right)\cdot\left(3\hat{i}+\hat{j}-2k\right) = 0$$

$\Rightarrow$ $9k - 3 + k - 1 + 4k - 10 = 0$

$=$ $14k = 14 \Rightarrow k = 1$

$\therefore$ $P(-2, -3, 5) \equiv (a, b, c)$

$|a| + |b| + |c| = 2 + 3 + 5 = 10$

$\therefore$ option (4) is correct

10. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then $p + q$ is equal to

(1) 147

(2) 143

(3) 137

(4) 157

Answer (4)

Sol. $OP = r_1 = \sqrt{\left(\frac{2\sqrt{3}}{\sqrt{7}}\right)^2 + \left(\frac{6}{\sqrt{7}}\right)^2} = \sqrt{\frac{48}{7}}$

Let P be $(r_1 \cos\theta, r_1 \sin\theta)$

P lies on ellipse

$$\frac{r_1^2\cos^2\theta}{4} + \frac{r_1^2\sin^2\theta}{9} = 1$$

$$\Rightarrow \frac{\cos^2\theta}{4} + \frac{\sin^2\theta}{9} = \frac{7}{48} \quad \ldots\text{(i)}$$

Let R be $(-r_2 \sin\theta, r_2 \cos\theta)$

$$\frac{r_2^2\sin^2\theta}{4} + \frac{r_2^2\cos^2\theta}{9} = 1$$

$$\Rightarrow \frac{\sin^2\theta}{4} + \frac{\cos^2\theta}{9} = \frac{1}{r_2^2}$$

From (i)

$$\frac{1}{r_2^2} = \frac{1}{4} + \frac{1}{9} - \frac{7}{48} = \frac{31}{144}$$

$$\frac{1}{PQ^2} + \frac{1}{RS^2} = \frac{1}{4}\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right)$$

$$= \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right) = \frac{13}{144} = \frac{p}{m}$$

$\therefore \quad p + m = 157$

Option (4) is correct.

11. Two dice A and B are rolled. Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha - \beta$ is $\frac{p}{q}$, where p and q are co-prime, then the sum of the positive divisors of p is equal to

(1) 72 (2) 36

(3) 48 (4) 31

Answer (3)

Sol. $\alpha \in \{1, 2, 3, 4, 5, 6\}$

$\beta \in \{1, 2, 3, 4, 5, 6\}$

$(\alpha - \beta) = 0$ (6 case)

$(\alpha - \beta) = -1$ (5 case)

$(\alpha - \beta) = -2$ (4 case)

$(\alpha - \beta) = -3$ (3 case)

$(\alpha - \beta) = -4$ (2 case)

$(\alpha - \beta) = -5$ (1 case)

$(\alpha - \beta) = 1$ (5 case)

$(\alpha - \beta) = 2$ (4 case)

$(\alpha - \beta) = 3$ (3 case)

$(\alpha - \beta) = 4$ (2 case)

$(\alpha - \beta) = 5$ (1 case)

Mean = 0

$$\text{Volume} = \sigma^2 = \frac{0^2 \times 6 + 2\times 1^2 \times 5 + 2 \times 2^2 \times 4 + 2\times 3^2 \times 3 + 2 \times 4^2 \times 2 \times 5^2 \times 1}{36}$$

$$= \frac{2}{36} \times (5 + 16 + 27 + 32 + 25) = \frac{105}{18} = \frac{35}{6}$$

$\therefore \quad p = 35$

Sum of divisors of $p = 1 + 5 + 7 + 35 = 48$

Option (3) is correct.

12. Let the plane $P : 4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x + y - z = 4$. If α is the distance of the point $(2, 3, -4)$ from the new position of the plane P, then 35α is equal to

(1) 85 (2) 105

(3) 126 (4) 90

Answer (3)

Sol. Equation of plane after rotation :

$(4x - y + z - 10) + \lambda(x + y - z - y) = 0$

$\Rightarrow (4 + \lambda)x + y(\lambda - 1) + z(1 - \lambda) - 4\lambda - 10 = 0$

$\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$

$\Rightarrow (4 + \lambda)4 + (\lambda - 1)(-1) + (1 - \lambda)1 = 0$

$\Rightarrow 16 + 4\lambda - \lambda + 1 + 1 - \lambda = 0$

$\Rightarrow 2\lambda = -18$

$\Rightarrow \lambda = -9$

$\therefore$ equation of plane : $-5x - 10y + 10z + 26 = 0$

Distance of plane from (2, 3, –4)

$$= \left|\frac{-10-30-40+26}{\sqrt{100+100+26}}\right| = \frac{54}{15} = \alpha$$

$$35\alpha = 35 \cdot \frac{54}{15} = 7 \times \frac{54}{3} = 7 \times 18 = 126$$

Option (3) is correct.

13. Let $\lambda \in \mathbb{Z}$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $\left(\vec{a}+\vec{b}+\vec{c}\right)\times\vec{c} = \vec{0}$, $\vec{a}\cdot\vec{c} = -17$ and $\vec{b}\cdot\vec{c} = -20$.

Then $\left|\vec{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)\right|^2$ is equal to

(1) 46

(2) 53

(3) 62

(4) 49

Answer (1)

Sol. $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

$k\left(\vec{a}+\vec{b}\right) = \vec{c}$

$\vec{a}\cdot\vec{c} = -17$

$\vec{b}\cdot\vec{c} = -20$

$k(\lambda^2 + 3\lambda - 1) = -17$

$k(3\lambda + 11) = -20$

$\lambda = 3, \dfrac{-69}{20}$

For $\lambda = 3$, $k = -1$

$\vec{c} = -1(\vec{a}+\vec{b})$

$= -\left((\lambda+3)\hat{i}+\hat{j}\right) = -6\hat{i} - \hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 0 \end{vmatrix} = \hat{i} - 3\hat{j} + 6\hat{k}$$

$\left|\vec{c}\times\left(\lambda\hat{i}+\hat{j}+\hat{k}\right)\right|^2 = 46$

14. Let D be the domain of the function $f\ x\ = \sin^{-1}\left(\log_{3x}\left(\dfrac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g : D \to \mathbb{R}$ defined by $g(x) = x - [x]$, ([x] is the greatest integer function), is (α, β), then $\alpha^2 + \dfrac{5}{\beta}$ is equal to

(1) 135 (2) 45

(3) 46 (4) 136

Answer (*)

Sol. $f(x) = \sin^{-1}\left(\log_{3x}\left(\dfrac{6+2\log_3 x}{-5x}\right)\right)$

$$\underset{-(1)}{\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\in[-1,1]},\ \underset{-(2)}{3x>0},\ \underset{-(3)}{3x\neq 1},\ \underset{-(4)}{\frac{6+2\log_3 x}{-5x}>0}$$

$\Rightarrow$ From (2), $x > 0$, from (3), $x \neq \dfrac{1}{3}$

From (4), $6 + 2\log_3 x < 0$ $(\because x > 0)$

$\log_3 x < -3$

$\Rightarrow x < 3^{-3}$

$\Rightarrow \boxed{x < \dfrac{1}{27}}$

$\Rightarrow$ From (2), (3), (4), $\boxed{0 < x < \dfrac{1}{27}}$ ...(5)

From (1), $-1 \le \log_{3x}\left(\dfrac{6+2\log_3 x}{-5x}\right) \le 1$

$\because$ $3x \in \left(0, \dfrac{1}{9}\right)$

$\Rightarrow \quad \dfrac{1}{3x} \geq \dfrac{6+2\log_3 x}{-5x} \geq 3x$

$\Rightarrow \quad -\dfrac{5}{3} \leq 6+2\log_3 x \leq -15x^2$

$\Rightarrow \quad -\dfrac{23}{6} \leq \log_3 x \leq \dfrac{-15x^2-6}{2}$

$\Rightarrow \quad 3^{-\frac{23}{6}} \leq x \leq 3^{\frac{-15x^2-6}{2}}$

$\Rightarrow \quad \boxed{\dfrac{1}{3^{\frac{23}{6}}} \leq x \leq \dfrac{1}{3^{\frac{15x^2+6}{2}}}}$ ...(6)

From (5) & (6), $0 < x < \dfrac{1}{27}$, $[x] = 0$

$\Rightarrow \quad g(x) = x$

$\Rightarrow$ Range $g(x)$ is domain of $f(x)$

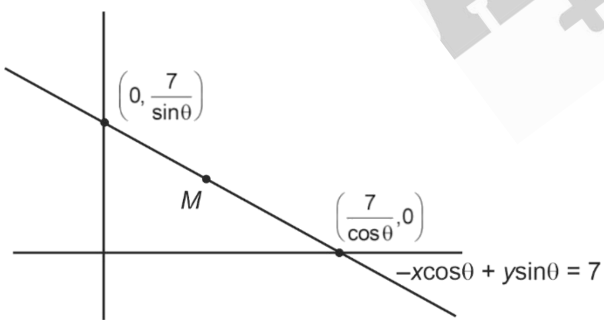
15. If the point $\left(\alpha, \dfrac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7,\ \theta \in \left(0, \dfrac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) -7 (2) $-7\sqrt{3}$

(3) $7\sqrt{3}$ (4) 7

Answer (4)

Sol.



$M\left(\dfrac{7}{2\cos\theta}, \dfrac{7}{2\sin\theta}\right) \equiv (h, k)$

$\therefore \quad \cos\theta = \dfrac{7}{2h} \qquad \sin\theta = \dfrac{7}{2k}$

$\because \ \theta \in \left(0, \dfrac{\pi}{2}\right)$

$\dfrac{49}{4h^2} + \dfrac{49}{4k^2} = 1 \qquad \therefore\ x$ & y are positive

$\therefore$ Locus $= \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{49}$

$\left(\alpha, \dfrac{7\sqrt{3}}{3}\right)$ lies on curve

$\dfrac{1}{\alpha^2} + \dfrac{9}{147} = \dfrac{4}{49}$

$\dfrac{1}{\alpha^2} = \dfrac{1}{49}$

$\therefore \quad \alpha = \pm 7$

$\therefore \quad \alpha = 7$

16. Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)\, dy = y(x - y)\, dx$. If $y(0) = 1$ and $y\left(2\sqrt{2}\right) = \beta$, then

(1) $e^{3\beta^{-1}} = e\left(3+2\sqrt{2}\right)$

(2) $e^{3\beta^{-1}} = e\left(5+\sqrt{2}\right)$

(3) $e^{\beta^{-1}} = e^{-2}\left(3+2\sqrt{2}\right)$

(4) $e^{\beta^{-1}} = e^{-2}\left(5+\sqrt{2}\right)$

Answer (1)

Sol. $\left(1+x^2\right)dy = y(x-y)dx$

$\dfrac{dy}{dx} = \dfrac{x}{1+x^2}y - \dfrac{y^2}{1+x^2}$

$\dfrac{-1}{y^2}\dfrac{dy}{dx} + \dfrac{x}{1+x^2}\cdot\dfrac{1}{y} = \dfrac{1}{1+x^2}$

Let $\dfrac{1}{y} = t$

$\dfrac{-1}{y^2}y' = \dfrac{dt}{dx}$

$\therefore \quad \dfrac{dt}{dx} + \dfrac{x}{1+x^2}t = \dfrac{1}{1+x^2}$

$\text{IF} = e^{\int \frac{x}{1+x^2}dx} = \sqrt{1+x^2}$

$t\sqrt{1+x^2} = \int \dfrac{\sqrt{1+x^2}}{1+x^2}dx$

$$\frac{1}{y}\sqrt{1+x^2} = \int \frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{1}{y}\sqrt{1+x^2} = \ln\left(x+\sqrt{x^2+1}\right)+C$$

$$\because \quad y(0)=1 \Rightarrow C=1$$

$$\frac{1}{y}\sqrt{1+x^2} = \ln\left(x+\sqrt{x^2+1}\right)+1$$

For $y = 2\sqrt{2}$

$$\frac{3}{y} = \ln\left|2\sqrt{2}+3\right|+1$$

$$y = \beta = \frac{3}{1+\ln\left|2\sqrt{2}+3\right|}$$

$$\Rightarrow \quad 3\beta^{-1} = 1+\ln\left|2\sqrt{2}+3\right|$$

$$e^{3\beta^{-1}} = e^{\left|2\sqrt{2}+3\right|}$$

17. In a triangle ABC, if cos A + 2 cos B + cos C = 2 and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then cos A – cos C is equal to

(1) $\frac{9}{7}$ (2) $\frac{10}{7}$

(3) $\frac{5}{7}$ (4) $\frac{3}{7}$

Answer (2)

Sol. $\cos A + 2\cos B + \cos C = 2$

$$\cos A + \cos C = 2(1-\cos B)$$

$$2\cos\frac{A+C}{2}\cos\left(\frac{A-C}{2}\right) = 2\times 2\sin^2\frac{B}{2}$$

$$\cos\frac{A-C}{2} = 2\sin\frac{B}{2}$$

$$2\cos\frac{B}{2}\cos\frac{A-C}{2} = 4\sin\frac{B}{2}\cos\frac{B}{2}$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 2\sin B$$

$$\sin A + \sin C = 2\sin B$$

$a + c = 2b$

$\Rightarrow \quad b = 5 \qquad (\because a = 3, c = 7)$

$$\cos A - \cos C = \frac{25+49-9}{70} - \frac{9+25-49}{30} = \frac{10}{7}$$

18. Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1+3i)$ and radius $r = 1$. Let $z_1 = 1+i$ and the complex number z_2 be outside circle C such that $|z_1-z_0||z_2-z_0| = 1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$

(3) $\frac{13}{2}$ (4) $\frac{3}{2}$

Answer (1)

Sol. $z_0 = \frac{1+3i}{2}$, $z_1 = (1+i)$

$$|z_1-z_0||z_2-z_0| = 1$$

$$\frac{1}{\sqrt{2}}|z_2-z_0| = 1$$

$$\Rightarrow \quad |z_2-z_0| = \sqrt{2}$$

$$\frac{z_2-z_0}{z_1-z_0} = \frac{|z_2-z_0|}{|z_1-z_0|}(\pm 1) = \pm 2$$

$$z_2 = z_0 \pm 2(z_1-z_0)$$

$$z_2 = 2z_1 - z_0 = \frac{3}{2}+\frac{1}{2}i \Rightarrow |z_2|^2 = \frac{5}{2}$$

OR

$$z_2 = 3z_0 - 2z_1 = \frac{-1}{2}+\frac{5}{2}i \Rightarrow |z_2|^2 = \frac{13}{2}$$

19. Among the two statements

(S_1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S_2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology

(1) only (S_2) is true

(2) only (S_1) is true

(3) both are false

(4) both are true

Answer (4)

Sol. $S_1 : (p \Rightarrow q) \wedge (p \wedge \sim q)$

$$\equiv (\sim p \vee q) \wedge (p \wedge \sim q)$$

$$\equiv (\sim p \wedge p \wedge \sim q) \vee (q \wedge p \wedge \sim q)$$

$$\equiv (f \wedge \sim q) \vee (f \wedge p)$$

$$\equiv f \vee f \equiv f$$

$S_2 : (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$$\equiv \left((p \vee \sim p) \wedge q\right) \vee \left((p \vee \sim p) \wedge \sim q\right)$$

$$\equiv (t \wedge q) \vee (t \wedge \sim q) \equiv q \vee \sim q \equiv t$$

20. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1 - x)^{100}$, is equal to

(1) ${}^{101}C_{50}$ (2) ${}^{99}C_{49}$

(3) $-{}^{101}C_{50}$ (4) $-{}^{99}C_{49}$

Answer (4)

Sol. $$\left({}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - .... - {}^{100}C_{49}\right) + {}^{100}C_{50} + \left(-{}^{100}C_{51} + {}^{100}C_{52} - .... + {}^{100}C_{100}\right) = 0$$

$$\lambda_1 + {}^{100}C_{50} + \lambda_2 = 0$$

$$\lambda_1 = -\frac{1}{2}\,{}^{100}C_{50} \quad (\because\ \lambda_1 = \lambda_2)$$

$$= -{}^{99}C_{49}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1) = \alpha + 7\log_e\left(1 + 2\sqrt{2}\right)$, then α^4 is equal to ________.

Answer (64)

Sol. $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$

$$\frac{x+7}{x} = t^2 \Rightarrow -\frac{7}{x^2}dx = 2t\,dt$$

$$dx = \frac{-14t}{(t^2-1)^2}dt$$

$$I(x) = -14\int \frac{t^2}{(t^2-1)^2}dt$$

$$= -14\int \frac{dt}{\left(t^2 + \frac{1}{t^2} - 2\right)} = \frac{-14}{2}\int \left[\frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right)^2 - 4} + \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2}\right]dt$$

$$I(x) = -7\left(\frac{1}{4}\ln\left|\frac{t + \frac{1}{t} - 2}{t + \frac{1}{t} + 2}\right| - \frac{1}{t - \frac{1}{t}}\right) + c$$

when $x = 9,\ t = \frac{4}{3}$

$$\Rightarrow\ 12 + 7\ln 7 = \frac{-7}{4}\ln\left(\frac{1}{7}\right)^2 + 7 \times \frac{12}{7} + c$$

$$\Rightarrow\ c = \frac{7}{2}\ln 7$$

when $x = 1,\ t = 2\sqrt{2}$

$$\Rightarrow\ I(1) = +\frac{7}{4}\ln\left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1}\right)^2 + 7 \times \frac{2\sqrt{2}}{7} + \frac{7}{2}\ln 7$$

$$= \frac{7}{2}\ln\left(\frac{(2\sqrt{2}+1)^2}{7}\right) + 2\sqrt{2} + \frac{7}{2}\ln 7$$

$$= 7\ln(2\sqrt{2}+1) - \frac{7}{2}\ln 7 + 2\sqrt{2} + \frac{7}{2}\ln 7$$

$$\Rightarrow\ \alpha = 2\sqrt{2} \Rightarrow \boxed{\alpha^4 = 64}$$

22. If $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx = \frac{k}{3000}$, then k is equal to ________.

Answer (575)

Sol. $\int_{-0.15}^{0.15} |100x^2 - 1| dx = 2\int_{0}^{0.15} |100x^2 - 1| dx$

$$= 2\int_{0}^{0.1} (-100x^2 + 1)dx + 2\int_{0.1}^{0.15} (100x^2 - 1)dx$$

$$= 2.\left(\frac{-100x^3}{3} + x\right)_0^{0.1} + 2\left(\frac{100x^3}{3} - x\right)_{0.1}^{0.15}$$

$$= 4\left(-\frac{1}{30} + \frac{1}{10}\right) + 2\left(\frac{9}{80} - \frac{3}{20}\right)$$

$$= \frac{8}{30} - \frac{3}{40} = \frac{575}{3000} \Rightarrow k = 575$$

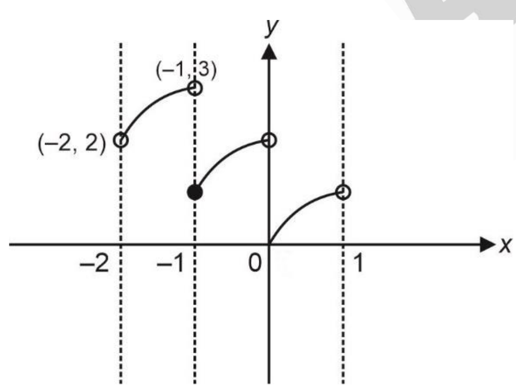
23. Let $[x]$ be the greatest integer $\le x$. Then the number of points in the interval (–2, 1) where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous, is ________.

Answer (2)

Sol. $f(x) = |[x]| + \sqrt{x - [x]}$

$x - [x] \ge 0 \Rightarrow x \in R$

$$f(x) = \begin{cases} 2 + \sqrt{x+2}, & -2 < x < -1 \\ 1 + \sqrt{x+1}, & -1 \le x < 0 \\ \sqrt{x}, & 0 \le x < 1 \end{cases}$$



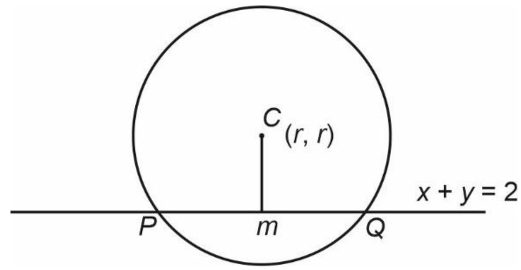
$f(x)$ is discontinuous at two points $x = \{-1, 0\}$

24. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then $r_1^2 + r_2^2 - r_1 r_2$ is equal to ________.

Answer (07.00)

Sol. $(x - r)^2 + (y - r)^2 = r^2$

$$PQ = 2\sqrt{r^2 - \left(\frac{2r - 2}{\sqrt{2}}\right)^2} = 2$$



$$r^2 - \frac{4(r-1)^2}{2} = 1$$

$$r^2 - 1 = 2(r - 1)^2$$

$$r^2 - 1 = 2r^2 - 4r + 2$$

or $r^2 - 4r + 3 = 0$

$r = 1, 3$

$r_1^2 + r_2^2 - r_1 r_2 = 7$

25. Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{10}$ and $a_3 + a_4 + a_5 = 14$, then $m + n$ is equal to ________.

Answer (211.00)

Sol. $a + ar + ar^2 + ar^3 + ar^4 = \frac{31}{10} \cdot 5 = \frac{31}{2}$

$$\frac{a(r^5 - 1)}{r - 1} = \frac{31}{2} \qquad \text{...(i)}$$

$$\frac{1}{a}\left(1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4}\right) = \frac{31}{40} \cdot 5 = \frac{31}{8}$$

$$\frac{1}{a}\left(\frac{1 - \left(\frac{1}{r}\right)^5}{1 - \frac{1}{r}}\right) = \frac{31}{8}$$

or $\frac{1}{a}\left(\frac{r^5 - 1}{r - 1}\right)\frac{1}{r^4} = \frac{31}{8} \qquad \text{...(ii)}$

From (i) and (ii)

$$\frac{1}{a}\cdot\frac{31}{2a}\cdot\frac{1}{r^4}=\frac{31}{8}$$

$$\boxed{ar^2=2}$$

From (i)

$$\frac{2}{r^2}\left(\frac{r^5-1}{r-1}\right)=\frac{31}{2}$$

$$\frac{1+r+r^2+r^3+r^4}{r^2}=\frac{31}{4}$$

$$\left(r^2+\frac{1}{r^2}\right)+\left(r+\frac{1}{r}\right)=\frac{27}{4}$$

$$t^2-2+t=\frac{27}{4}$$

$$4t^2+4t-35=0$$

$$4t^2+14t-10t-35=0$$

$$(2t-5)(2t+7)=0$$

$$t=\frac{5}{2},\frac{-7}{2}\Rightarrow\boxed{r=2}$$

$$r=2,\ a=\frac{1}{2}$$

Variance of data set $\left\{\frac{1}{2},1,2,4,8\right\}$

$$=\frac{\left(\frac{341}{4}\right)}{5}-\left(\frac{31}{10}\right)^2$$

$$=\frac{341}{20}-\frac{961}{100}=\frac{1705-961}{100}$$

$$=\frac{744}{100}=\frac{186}{25}$$

26. Let $D_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+2 & n^2\\ n & n^2+n & n^2+n+2\end{vmatrix}$. If $\sum_{k=1}^{n}D_k=96$, then n is equal to ______.

Answer (06.00)

Sol. $\sum_{k=1}^{n}D_k=\begin{vmatrix}\sum 1 & 2\sum k & 2\sum k-\sum 1\\ n & n^2+n+2 & n^2\\ n & n^2+n & n^2+n+2\end{vmatrix}$

$$=\begin{vmatrix}n & n(n+1) & n^2\\ n & n^2+n+2 & n^2\\ n & n^2+n & n^2+n+2\end{vmatrix}$$

$$=\begin{vmatrix}0 & -2 & 0\\ 0 & 2 & -n-2\\ n & n^2+n & n^2+n+2\end{vmatrix}$$

$= 2((-n)(-n-2)) = 96$

$n^2 + 2n = 48$

$n = 6, -8$

$\boxed{n=6}$

27. A fair n ($n > 1$) faces die is rolled repeatedly until a number less than n appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to ________.

Answer (10)

Sol.

x_i	1	2	3	4	...
p_i	$\frac{n-1}{n}$	$\frac{1}{n}\cdot\left(\frac{n-1}{n}\right)$	$\frac{1}{n^2}\cdot\frac{n-1}{n}$	$\frac{1}{n^3}\cdot\left(\frac{n-1}{n}\right)$	...

$$\text{Mean}=\sum_{i=1}^{\infty}p_ix_i=1\cdot\frac{n-1}{n}+\frac{2}{n}\cdot\left(\frac{n-1}{n}\right)+\frac{3}{n^2}\left(\frac{n-1}{n}\right)+\ldots$$

$$\frac{n}{9}=\left(1-\frac{1}{n}\right)S \qquad \ldots(i)$$

where $S=1+\frac{2}{n}+\frac{3}{n^2}+\frac{4}{n^3}+\ldots$

$$\frac{1}{n}S=\frac{1}{n}+\frac{2}{n^2}+\frac{3}{n^3}+\ldots$$

$$\left(1-\frac{1}{n}\right)S=1+\frac{1}{n}+\frac{1}{n^2}+\frac{1}{n^3}+\ldots$$

$$\left(1-\frac{1}{n}\right)S=\frac{1}{1-\frac{1}{n}}$$

$$\Rightarrow\ \frac{n}{9}=\left(1-\frac{1}{n}\right)\times\frac{1}{\left(1-\frac{1}{n}\right)^2}=\frac{n}{n-1}$$

$$\Rightarrow\ n=10$$

28. The number of relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is __________.

Answer (4)

Sol. $(1, 1), (2, 2), (3, 3) \in R$

Since $(1, 2), (2, 3) \in R$, $(1, 3)$ must $\in R$

Possible cases :

Case-1 : All of $(2, 1), (3, 2), (3, 1) \notin R \rightarrow 1$ relation.

Case-2 : Only one of $(2, 1), (3, 2), (3, 1) \in R \rightarrow 3$ relations.

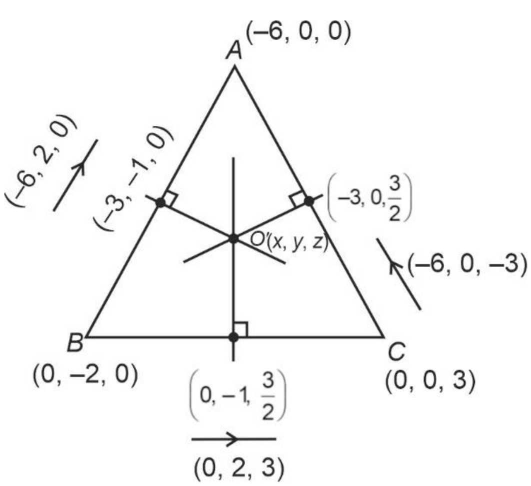
Note that exactly two of $(2, 1), (3, 2), (3, 1) \in R$ is not possible because if two of these $\in R$, third must $\in R$ to make relation transitive.

Total number of relations = 4

29. Let the plane $x+3y-2z+6=0$ meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha, \beta, \frac{6}{7}\right)$, then $98(\alpha+\beta)^2$ is equal to ________.

Answer (288)

Sol.



$$(x-0)\times 0+(y+1)\times 2+\left(z-\frac{3}{2}\right)\times 3=0$$

$\Rightarrow\ 4y + 6z - 5 = 0$ ...(i)

$$(x+3)\times(-6)+(y-0)\times 0+\left(z-\frac{3}{2}\right)(-3)=0$$

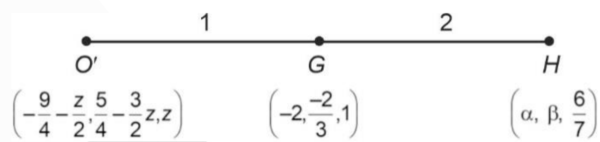
$\Rightarrow\ 4x + 2z + 9 = 0$ ...(ii)

$\Rightarrow\ (x + 3) \times (-6) + (y + 1) \times 2 + (z - 0) \times 0 = 0$

$\Rightarrow\ 3x - y + 8 = 0$ ...(iii)

From (i), (ii), (iii),

$$x=\frac{-9}{4}-\frac{z}{2},\ y=\frac{5}{4}-\frac{3}{2}z,\ z=z$$



$$\Rightarrow\ G\equiv\left(\frac{\alpha-\frac{9}{2}-z}{3},\frac{\beta+\frac{5}{2}-3z}{3},\frac{\frac{6}{7}+2z}{3}\right)$$

$$\Rightarrow\ z=\frac{15}{14},\ \alpha=-\frac{3}{7},\ \beta=-\frac{9}{7}$$

$$\Rightarrow\ 98(\alpha+\beta)^2=98\times\frac{144}{49}=288$$

30. Let the digits a, b, c be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

Answer (1260)

Sol. Digits are in A.P. at least once

Possible cases $\boxed{a\,|\,b\,|\,c}$ or $\boxed{c\,|\,b\,|\,a}$

Total ways of selecting 3 consecutive places out of 9 places is 7.

$$\text{Total ways }=\frac{{}^7C_1\times 2\times 6!}{2!\times 2!\times 2!}=1260$$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. A proton and an α-particle are accelerated from rest by 2 V and 4 V potentials, respectively. The ratio of their de-Broglie wavelength is :

(1) 8 : 1 (2) 2 : 1

(3) 4 : 1 (4) 16 : 1

Answer (3)

Sol. $\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha}}{\sqrt{m_p}} \times \sqrt{\frac{q_\alpha 4}{q_p 2}}$

= 2 × 2

32. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

Reason R : Infrared EM waves are more energetic than microwaves, (used in Radar)

In the light of given statements, choose the ***correct*** answer from the options given below.

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Answer (2)

Sol. Assertion is false

as $\lambda_{\text{optical}} < \lambda_{\text{microwave}}$

33. If the r.m.s speed of chlorine molecule is 490 m/s at 27°C, the r.m.s speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9 u, molecular mass of chlorine = 70.9 u)

(1) 551.7 m/s (2) 651.7 m/s

(3) 451.7 m/s (4) 751.7 m/s

Answer (2)

Sol. $\frac{v_{\text{Ar}}}{v_{\text{Cl}}} = \sqrt{\frac{M_{\text{Cl}}}{M_{\text{Ar}}}}$

$\Rightarrow \quad v_{\text{Ar}} = \sqrt{\frac{71}{40}} \times 490$

$\cong 651.7$

34. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be

(1) Speed (2) Kinetic energy

(3) Total energy (4) Potential energy

Answer (1)

Sol. Speed will be independent of mass of satellite.

35. A particle is executing simple harmonic motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be

(1) 1 : 1 (2) 1 : 3

(3) 2 : 1 (4) 1 : 4

Answer (2)

Sol. $\frac{\text{P.E.}}{\text{K.E.}} = \frac{x^2}{A^2 - x^2} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

36. Given below are two statements:

Statement I : The diamagnetic property depends on temperature.

Statement II : The induced magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are False

(2) Statement I is incorrect but Statement II is true

(3) Statement I is correct but Statement II is flase

(4) Both Statement I and Statement II are true

Answer (2)

Sol. Diamagnetic materials oppose external field.

37. A wire of resistance 160Ω is melted and drawn in a wire of one-fourth of its length. The new resistance of the wire will be

(1) 16 Ω (2) 10 Ω

(3) 640 Ω (4) 40 Ω

Answer (2)

Sol. $R' = R/16$

$R' = 10\Omega$

38. A body cools from 80°C to 60° in 5 minutes. The temperature for the surrounding is 20°C. The time it takes to cool from 60°C to 40°C is

(1) 450 s (2) 420 s

(3) 500 s (4) $\frac{25}{3}$s

Answer (3)

Sol. $\frac{20}{5} = c(50)$

$\frac{20}{x} = c(30)$

$\Rightarrow \quad x = \frac{25}{3}$ minutes

= 500 s

39. Match List I with List II

	List-I		List-II
A.	Spring constant	I.	$[T^{-1}]$
B.	Angular speed	II.	$[MT^{-2}]$
C.	Angular momentum	III.	$[ML^2]$
D.	Moment of Inertia	IV.	$[ML^2T^{-1}]$

Choose the correct answer from the options given below:

(1) A-I, B-III, C-II, D-IV

(2) A-IV, B-I, C-III, D-II

(3) A-II, B-I, C-IV, D-III

(4) A-II, B-III, C-I, D-IV

Answer (3)

Sol. $[k] = [M^1L^0T^{-2}]$

$\omega = [T^{-1}]$

40. In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from 10 μA and 200 μA, respectively. The current gain of transistor is ______

(1) 110 (2) 210

(3) 0.9 (4) 9

Answer (1)

Sol. $\Delta I_c = 11$ mA

$\Delta I_b = 100\ \mu A$

$\beta = \frac{\Delta I_c}{\Delta I_b} = 110$

41. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$

(3) $\frac{6}{5}$ (4) $\frac{4}{3}$

Answer (1)

Sol. $\frac{x}{\mu} = 12$

$\frac{a-x}{\mu} = 4$

$24 - 12\ \mu = 4\ \mu$

$\mu = \frac{24}{16} = 1.5$

42. The amplitude of 15 sin(1000 πt) is modulated by 10 sin(4 πt) signal. The amplitude modulated signal contains frequencies of

A. 500 Hz B. 2 Hz

C. 250 Hz D. 498 Hz

E. 502 Hz

Choose the correct answer from the options given below

(1) A and B only (2) A and C only

(3) A and D only (4) A, D and E only

Answer (4)

Sol. Frequencies in the system

$= \frac{1000\pi}{2\pi}, \frac{996\pi}{2\pi}, \frac{1004\pi}{2\pi}$

= 500 Hz, 498 Hz, 502 Hz

43. An engine operating between the boiling and freezing points of water will have

A. Efficiency more than 27%

B. Efficiency less than the efficiency of a Carnot engine operating between the same two temperatures.

C. Efficiency equal to 27%.

D. Efficiency less than 27%.

Choose the correct answer from the options given below

(1) B, C and D only (2) A and B only

(3) B and D only (4) B and C only

Answer (3)

Sol. $\eta_C = 1 - \frac{273}{373} = 26.8\%$

$\Rightarrow \quad \eta_{engine} < \eta_{carnot} = \eta_C$

44. Three forces $F_1 = 10$ N, $F_2 = 8$ N, $F_3 = 6$ N are acting on a particle of mass 5 kg. The forces F_2 and F_3 are applied perpendicularly so that particle remains at rest. If the force F_1 is removed, then the acceleration of the particle is

(1) 7 ms^{-2} (2) 0.5 ms^{-2}

(3) 4.8 ms^{-2} (4) 2 ms^{-2}

Answer (4)

Sol. $\sqrt{F_2^2 + F_3^2} = 10$ N

$\Rightarrow \quad a = \frac{10}{5} = 2$ m/s^2

45. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : If an electric dipole of dipole moment 30×10^{-5} Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.

Reason R : Electric dipole consists of two equal and opposite charges.

In the light of above, statements, choose the **correct** answer from the options given below.

(1) Both A and R are true and R is the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is NOT the correct explanation of A

Answer (1)

Sol. $\phi = \frac{q_{in}}{\varepsilon_0} = 0$

46. Given below are two statements:

Statement I : When the frequency of an a.c source in a series LCR circuit increases, the current the circuit first increases, attains a maximum value and then decreases.

Statement II : In a series LCR circuit, the value of power factor at resonance is one.

In the light of given statements, choose the most appropriate answer from the options given below.

(1) Statement I is incorrect but Statement II is true.

(2) Both Statement I and Statement II are false.

(3) Both Statement I and Statement II are true.(4) Statement I is correct but Statement II is false.

Answer (3)

Sol. Z first decreases and then increases

$\phi_{resonance} = 1$ as $Z = R$

47. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:

(1) 1 (2) 4

(3) 2 (4) 3

Answer (4)

Sol. $h\nu_{max} = 12.5$ eV

$\Rightarrow$ maximum excitation level = 3

$\Rightarrow$ spectral lines possible = $3C_2$

= 3

48. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5 s is $\frac{x+1}{x}$. The value of x is __________.

{take, g = 10 m/s^2}

(1) 10 (2) –5

(3) 6 (4) 5

Answer (4)

Sol. $\frac{v_3}{v_5} = \frac{150-30}{150-50} = \frac{120}{100}$

$= \frac{6}{5}$

$x = 5$

49. Given below are two statements:

Statement I : A truck and a car moving with same kinetic energy are brought to rest by applying breaks which provide equal retarding forces. Both come to rest in equal distance.

Statement II : A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In the light of given statements, choose the most appropriate answer from the options given below

(1) Statement I is correct but statement II is incorrect.

(2) Statement I is incorrect but statement II is correct.

(3) Both statement I and Statement II are correct.

(4) Both statement I and statement II are incorrect.

Answer (1)

Sol. $WD = Fx = \Delta$KE

$x_1 = x_2$

50. The ratio of escape velocity of a planet to the escape velocity of earth will be:-

Given: Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.

(1) 4 : 1 (2) 1 : 4

(3) $1:\sqrt{2}$ (4) 2 : 1

Answer (4)

Sol. $v = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e}} \times \sqrt{\frac{R_e}{R_p}} = 4 \times \frac{1}{2}$$

= 2 : 1

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. 64 identical drops each charged upto potential of 10 mV are combined to form a bigger drop. The potential of the bigger drop will be _______ mV.

Answer (160)

Sol. $V' = \dfrac{kQ'}{R'}$

$V' = \dfrac{k \times 64 \times Q}{4R}$ = 16 × V = 160 mV

52. A common example of alpha decay is

$${}^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}_2\text{He}^4 + Q$$

Given :

${}^{238}_{92}\text{U} = 238.05060$ u

${}^{234}_{90}\text{Th} = 234.04360$ u

${}^{4}_{2}\text{He} = 4.00260$ u and $1\text{u} = 931.5\dfrac{\text{MeV}}{c^2}$

The energy released (Q) during the alpha decay of ${}^{238}_{92}\text{U}$ is _______ MeV.

Answer (4)

Sol. Q = 931.5 × 00.0044 = 4.0986

53. Glycerin of density 1.25 × 10^3 kg m^{-3} is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends are 10 cm^2 and 5 cm^2 and pressure drop across its length is 3 Nm^{-2}. The rate of flow of glycerin through the pipe is $x \times 10^{-5}$ m^3s^{-1}. The value of x is _______.

Answer (4)

Sol. A_1 = 10 cm^2; A_2 = 5 cm^2

ΔP = –3 N/m^2

$$-\frac{1}{2}\rho v_1^2 + \frac{1}{2}\rho v_2^2 = \Delta P$$

$$= v_1^2(3) = \frac{3 \times 2}{1.25 \times 10^3}$$

$$\Rightarrow v_1 = \frac{\sqrt{2}}{\sqrt{1250}}$$

$$Q = A_2 v_2 = \frac{10 \times \sqrt{2}}{\sqrt{1250}} = 4 \times 10^{-5}$$

54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be _______ μV.

Answer (50)

Sol. $\phi = 2\pi R \dfrac{dR}{dt} \times B$

$$= 2 \times \frac{22}{7} \times 2 \times 10^{-2} \times 10^{-3} \times 0.4$$

= 5 × 10^{-5} V = 50.28 μV

55. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two places respectively is $\dfrac{4}{\sqrt{x}}$. The value of x is

Answer (243)

Sol. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$$T = 2\pi\sqrt{\frac{I}{MB\cos\theta}}$$

$$\frac{20}{30}=\sqrt{\frac{B_1}{B_2}\frac{\sqrt{3}/2}{1/2}}$$

$$\frac{4}{9}=\frac{B_1}{B_2}\sqrt{3}$$

$$\frac{B_1}{B_2}=\frac{4}{\sqrt{243}}$$

56. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at __________ cm from the first lens.

Answer (100)

Sol. $u_1 = \infty$, f_1 = 20 cm, v_1 = 20 cm

u_2 = 40 cm, f_2 = 20 cm, v_2 = 40 cm

$\Rightarrow x$ = 40 cm + 60 cm

= 100 cm

57. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be ________ kJ. [The coefficient of friction between tyre of bus and road is 0.04]

Answer (784)

Sol. v = 80 kph

m = 500

S = 4 km

$P_{friction}$ = (0.04) (500g) × 80 × $\frac{5}{18}$ watts

$$P=\frac{20\times 80\times 5\times 9.8}{18}$$

$$W=\frac{20\times 80\times 5}{18}\times\frac{4\times 10^3}{80\times\frac{5}{18}}\times 9.8$$

= 784 kJ

58. For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is 324 ms^{-1} the length of the organ pipe is ________ m.

Answer (1)

Sol. f_5 = 405

$$\Rightarrow \frac{5v}{4l}=405$$

$$\Rightarrow l=\frac{405\times 4}{5\times 324}$$

= 1 m

59. For rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$.The value of x is ________.

Answer (2)

Sol. $$\frac{K\cdot E_R}{K\cdot E_T}=\frac{\frac{2}{3}MR^2\omega^2}{\frac{5}{3}MR^2\omega^2}=\frac{2}{5}$$

60. The current flowing through a conductor connected across a source is 2 A and 1.2 A at 0ºC and 100ºC respectively. The current flowing through the conductor at 50ºC will be _________ × 10^2 mA.

Answer (15)

Sol. $$\frac{R_1}{R_2}=\frac{1\cdot 2}{2}=\frac{1}{1+\alpha 100}$$

$$1+\alpha 100=\frac{2}{1.2}$$

$$\alpha 100=\frac{0.8}{1.2}=\frac{2}{3}$$

$$\alpha 50=\frac{1}{3}$$

$$1+\alpha 50=\frac{4}{3}$$

$$\Rightarrow \frac{I_3}{I_1}=\frac{3}{4}\times 2\text{ A}=1.5\text{ A}$$

= 15 × 10^2 mA

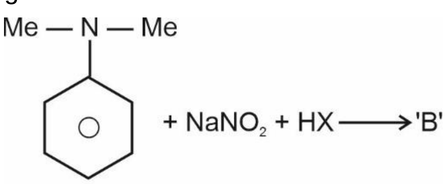
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

61. The incorrect statement regarding the reaction given below is



'A'

(1) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

(2) 'B' is N-nitroso ammonium compound

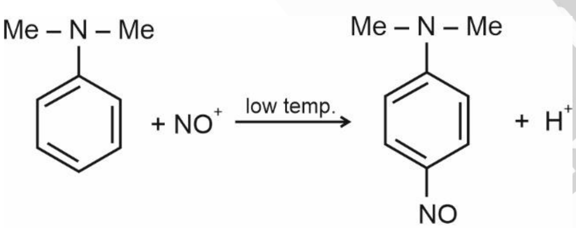
(3) The reaction occurs at low temperature

(4) The electrophile involved in the reaction is NO^+

Answer (2)

Sol. $NaNO_2 + HX \longrightarrow HNO_2 + NaX$

$2HNO_2 \longrightarrow NO^+ + NO_2^- + H_2O$



(B)

∴ Statement (2) is incorrect

62. The bond order and magnetic property of acetylide ion are same as that of

(1) O_2^+ (2) N_2^+

(3) NO^+ (4) O_2^-

Answer (3)

Sol.

Species	Bond Order	Magnetic moment
$HC\equiv C^-$	3	0
O_2^+	2.5	$\sqrt{3}$ B.M
N_2^+	2.5	$\sqrt{3}$ B.M
NO^+	3	0
O_2^-	1.5	$\sqrt{3}$ B.M

63. Match List I with List II

	List I Complex		List II CFSE (Δ_0)
A.	$[Cu(NH_3)_6]^{2+}$	I.	–0.6
B.	$[Ti(H_2O)_6]^{3+}$	II.	–2.0
C.	$[Fe(CN)_6]^{3-}$	III.	–1.2
D.	$[NiF_6]^{4-}$	IV.	–0.4

Choose the correct answer from the options given below:

(1) A(III), B(IV), C(I), D(II)

(2) A(I), B(IV), C(II), D(III)

(3) A(I), B(II), C(IV), D(III)

(4) A(II), B(III), C(I), D(IV)

Answer (2)

Sol.

(A) $[Cu(NH_3)_6]^{2+}$ Cu^{2+}: $3d^6$, $t_{2g}^6\ e_g^3$	CFSE $= (-6 \times 0.4 + 3 \times 0.6)\Delta_0$ $= -0.6\ \Delta_0$
(B) $[Ti(H_2O)_6]^{3+}$ Ti^{3+} : $3d^1$, $t_{2g}^1\ e_g^0$	CFSE $= -1 \times 0.4\Delta_0$ $= -0.4\ \Delta_0$
(C) $[Fe(CN)_6]^{3-}$ Fe^{3+} : $3d^5$, $t_{2g}^5\ e_g^0$	CFSE $= -5 \times 0.4\Delta_0$ $= -2.0\ \Delta_0$
(D) $[NiF_6]^{4-}$ Ni^{2+} : $3d^8$, $t_{2g}^6\ e_g^2$	CFSE $= (-6 \times 0.4 + 2 \times 0.6)\Delta_0$ $= -1.2\ \Delta_0$

A – I; B – IV; C – II; D – III

64. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: In the Ellingham diagram, a sharp change in slope of the line is observed from $Mg \rightarrow MgO$ at ~ 1120°C

Reason R: There is a large change of entropy associated with the change of state

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) Both A and R are true and R is the correct explanation of A

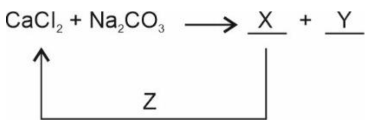
(4) A is true but R is false

Answer (3)

Sol. In the Ellingham diagram, a sharp change in slope of the line is observed for Mg – MgO at ~1120°C because that is the boiling point of magnesium.

There is a large increase in entropy associated with the change of state of magnesium. So, both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).

65. In the given reaction cycle



X, Y and Z respectively are

	X	Y	Z
(1)	$CaCO_3$	NaCl	KCl
(2)	$CaCO_3$	NaCl	HCl
(3)	CaO	NaCl + CO_2	NaCl
(4)	CaO	NaCl + CO_2	KCl

Answer (2)

Sol. $CaCl_2 + Na_2CO_3 \longrightarrow \underset{(X)}{CaCO_3} + \underset{(Y)}{2NaCl}$

$$\underset{(X)}{CaCO_3} + \underset{(Z)}{2HCl} \longrightarrow CaCl_2 + CO_2(g) + H_2O(l)$$

∴ (X) $CaCO_3$

(Y) NaCl

(Z) HCl

66. Match List I with List II

	List I (Example)		List II (Type)
A.	2-chloro-1, 3-butadiene	I.	Biodegradable polymer
B.	Nylon 2-nylon 6	II.	Synthetic Rubber
C.	Polyacrylonitrile	III.	Polyester
D.	Dacron	IV.	Addition Polymer

Choose the correct answer from the options given below:

(1) A(II), B(IV), C(I), D(III)

(2) A(II), B(I), C(IV), D(III)

(3) A(IV), B(I), C(III), D(II)

(4) A(IV), B(III), C(I), D(II)

Answer (2)

Sol.

A $\quad CH_2 = \overset{\overset{Cl}{|}}{C} - CH = CH_2 \xrightarrow{\text{Polymerisation}} \left[CH_2 - \overset{\overset{Cl}{|}}{C} = CH - CH_2 \right]_n$

2-chloro-1, 3-butadiene — Synthetic rubber

B	Nylon-2-nylon-6	Biodegradable polymer
C	Polyacrylonitrile	Addition polymer
D	Dacron	Polyester

A-II; B-I; C-IV; D-III

67. For lead storage battery pick the correct statements

A. During charging of battery, $PbSO_4$ on anode is converted into PbO_2

B. During charging of battery, $PbSO_4$ on cathode is converted into PbO_2

C. Lead storage battery consists of grid of lead packed with PbO_2 as anode

D. Lead storage battery has ~ 38% solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below:

(1) A, B, D only

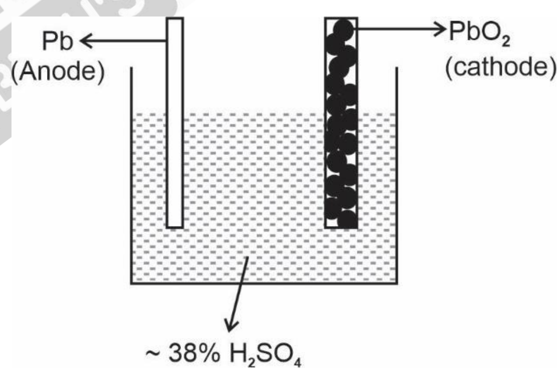
(2) B, C, D only

(3) B, C only

(4) B, D only

Answer (4)

Sol.



The lead storage battery is as shown in diagram. During charging of battery $PbSO_4$ deposited on cathode is converted in PbO_2 and $PbSO_4$ deposited on anode is converted into Pb. The electrolyte used in battery is H_2SO_4 which is about 38% by mass.

68. Given below are two statements:

Statement I: $SbCl_5$ is more covalent than $SbCl_3$

Statement II: The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is incorrect but statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Both Statement I and Statement II are correct

(4) Statement I is correct but statement II is incorrect

Answer (3)

Sol. $SbCl_5$ is more covalent than $SbCl_3$ due to higher polarisation power of Sb^V than that of Sb^{III}. Higher oxides of halogens are more stable than lower ones due to the formation of more number of bonds, which results in higher release of energy. So, both the statements I and II are correct.

69. Four gases, A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively

For their adsorption on a fixed amount of charcoal, the correct order is :

(1) C > D > B > A

(2) C > B > D > A

(3) D > C > B > A

(4) D > C > A > B

Answer (3)

Sol. The extent of adsorption of different gases on the surface of a fixed amount of charcoal is directly proportional to their critical temperatures.

Order of T_c D > C > B > A

Order of adsorption D > C > B > A

70. Match List I with List II

List I		List II	
A.	Nitrogen oxides in air	I.	Eutrophication
B.	Methane in air	II.	pH of rain water becomes 5.6
C.	Carbon dioxide	III.	Global warming
D.	Phosphate fertilisers in water	IV.	Acid rain

Choose the correct answer from the options given below :

(1) A-II, B-III, C-I, D-IV

(2) A-I, B-II, C-III, D-IV

(3) A-IV, B-III, C-II, D-I

(4) A-IV, B-II, C-III, D-I

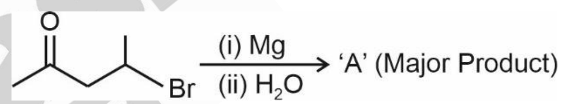
Answer (3)

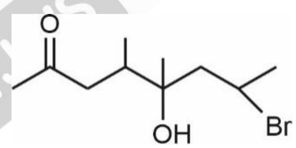
Sol.

A.	Nitrogen oxides in air	IV	Acid rain
B.	Methane in air	III	Global warming
C.	Carbon dioxide	II	pH of rain water becomes 5.6
D.	Phosphate fertilisers in water	I	Eutrophication

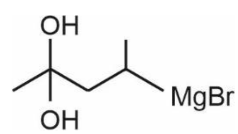
∴ A-IV; B-III; C-II; D-I

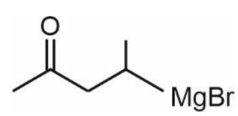
71. In the following reaction



(1) 

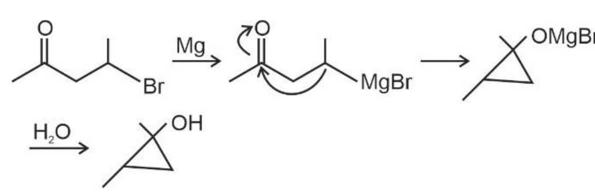
(2) 

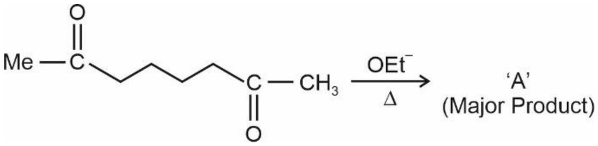
(3) 

(4) 

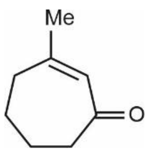
Answer (2)

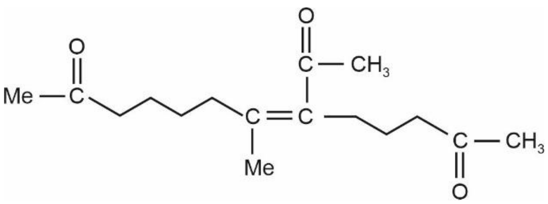
Sol.

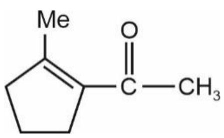


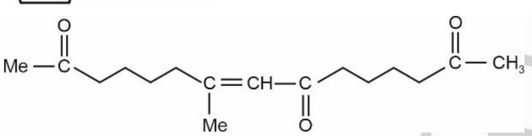
72. 

A in the above reaction is :

(1) 

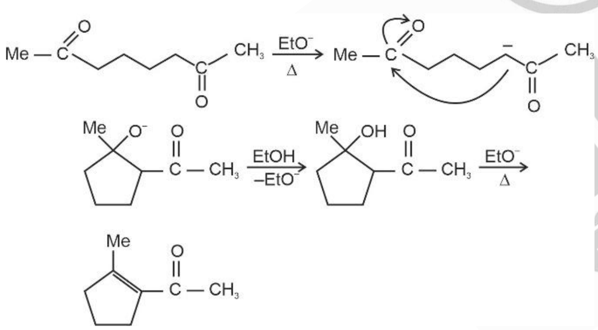
(2) 

(3) 

(4) 

Answer (3)

Sol.



73. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : 5f electron can participate in bonding to a far greater extent than 4f electrons

Reason R : 5f orbitals are not as buried as 4f orbitals

In the light of the above statements, choose the correct answer from the options given below

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is **NOT** the correct explanation of A

Answer (2)

Sol. 5f electron can participate in bonding to a far greater extent than 4f electron because 5f orbitals are more exposed than 4f orbitals.

So, Assertions (A) is true

5f orbitals are not as buried as 4f orbitals.

So, reason (R) is also true and (R) is the correct explanation of (A)

74. 2-hexene $\xrightarrow[\text{(ii) } H_2O]{\text{(i) } O_3}$ Products

The two products formed in above reaction are-

(1) Butanal and acetaldehyde

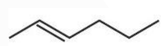
(2) Butanoic acid and acetaldehyde

(3) Butanal and acetic acid

(4) Butanoic acid and acetic acid

Answer (4)

Sol.

 $\xrightarrow[\text{2. } H_2O]{\text{1. } O_3}$ $CH_3COOH + CH_3–CH_2–CH_2–COOH$
(Acetic acid) (Butanoic acid)

75. Match List I with List II

List I Type of Hydride		List II Example	
A.	Electron deficient hydride	I.	MgH_2
B.	Electron rich hydride	II.	HF
C.	Electron precise hydride	III.	B_2H_6
D.	Saline hydride	IV.	CH_4

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-III, C-I, D-IV

(3) A-II, B-III, C-IV, D-I

(4) A-III, B-II, C-I, D-IV

Answer (1)

Sol.

A	Electron deficient hydride	III	B_2H_6
B	Electron rich hydride	II	HF
C	Electron precise hydride	IV	CH_4
D	Saline hydride	I	MgH_2

76. The density of alkali metals is in the order

(1) K < Cs < Na < Rb

(2) Na < Rb < K < Cs

(3) Na < K < Cs < Rb

(4) K < Na < Rb < Cs

Answer (4)

Sol. Density of alkali metals increases down the group with the exception of potassium. Therefore, correct order of density of given alkali metals is

K < Na < Rb < Cs

77. Given below are two statements:

Statement I: Boron is extremely hard indicating its high lattice energy

Statement II: Boron has highest melting and boiling point compared to its other group members.

In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) Both Statement I and Statement II are incorrect

(2) Statement I is correct but Statement II is incorrect

(3) Both statement I and Statement II are correct

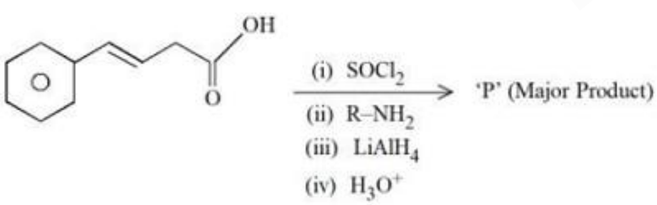
(4) Statement I is incorrect but Statement II is correct

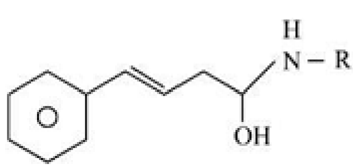
Answer (3)

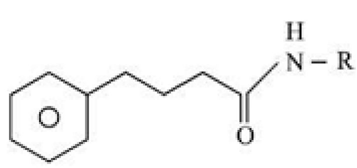
Sol. Boron is extremely hard as it exists as B_{12} icosahedral molecular units which accounts for its high lattice energy. So, statement-I is correct.

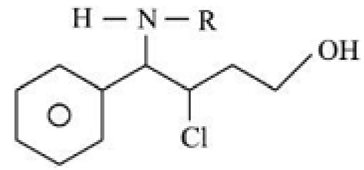
Boron has the highest melting point and boiling point than other members of its group due to its network structure. So, statement-II is also correct.

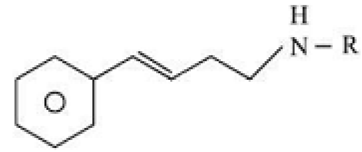
78. The major product 'P' formed in the following sequence of reactions is



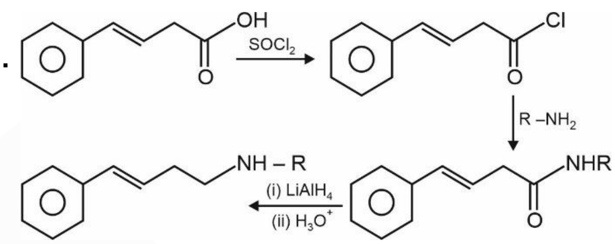
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

79. A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is

(Given: Atomic mass of chlorine is 35.5 u)

(1) MCl_4

(2) MCl_3

(3) MCl_2

(4) MCl

Answer (3)

Sol. Let the formula of metal chloride be MCl_x

Mass of 100 mL of vapours of MCl_x at

STP = 0.57 g

$$\therefore \frac{100 \times M}{22400} = 0.57$$

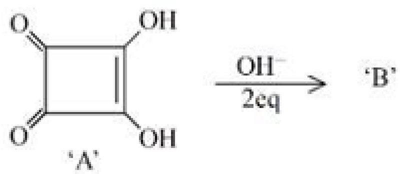
$\therefore$ Molar mass of MCl_x = 127.68 g mol^{-1}

$$\% \text{ of Cl} = \frac{35.5x \times 100}{127.68} = 55$$

$\Rightarrow$ x = 2

$\therefore$ Formula of metal chloride is MCl_2

80. Correct statements for the given reaction are:



A. Compound 'B' is aromatic

B. The completion of above reaction is very slow

C. 'A' shows tautomerism

D. The bond lengths of C-C in compound B are found to be same

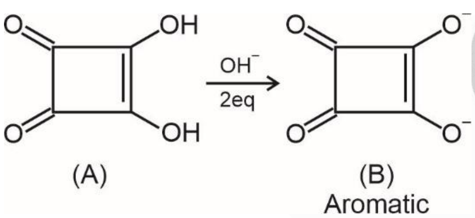
Choose the correct answer from the options given below.

(1) B, C and D only (2) A, B and C only

(3) A, C and D only (4) A, B and D only

Answer (3)

Sol.



Compound (A), also called squaric acid is a strong acid. So, above reaction proceeds very fast. All C – C bond lengths in (B) are same due to resonance. (A) shows tautomerism.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. At 600 K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90 K. The molar mass of the gas Y is ________ g mol^{-1}. (Nearest integer)

Answer (4)

Sol. $\frac{V_{rms} \text{ of } X}{V_{rmp} \text{ of } Y} = \sqrt{\frac{3\,T_X\,M_Y}{2\,T_Y\,M_X}}$

$$1 = \sqrt{\frac{3 \times 600 \times M_Y}{2 \times 90 \times 40}} = \sqrt{\frac{M_Y}{4}}$$

$M_Y = 4$ gm mol^{-1}

82. The reaction $2NO + Br_2 \rightarrow 2NOBr$

takes place through the mechanism given below

$NO + Br_2 \rightleftharpoons NOBr_2$ (fast)

$NOBr_2 + NO \rightarrow 2NOBr$ (slow)

The overall order of the reaction is ________.

Answer (3)

Sol. On applying LOMA over slowest step (r.d.s.)

$$R = k[NOBr_2][NO]$$

On applying LOMA on equilibrium of Step I

$$k = \frac{[NOBr_2]}{[NO]\,[Br_2]}$$

$[NOBr_2] = k\,[NO][Br_2]$

Hence overall rate equation will be

$$R = k\,[NO]^2\,[Br_2]$$

$$= k'\,[NO]^2\,[Br_2]$$

Hence, overall order will be 3.

83. 80 mole percent of $MgCl_2$ is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of $MgCl_2$ at 38°C is ________ mm Hg. (Nearest integer)

Given: Vapour pressure of water at 38°C is 50 mm Hg

Answer (48)

$$MgCl_2 \rightleftharpoons Mg^{2+} + 2\,Cl^-$$

Sol.

	$MgCl_2$	Mg^{2+}	$2\,Cl^-$
	1	0	0
	$1-0.8$	0.8	1.6

Hence overall molality will be equal to = 2.6

$$\frac{p^\circ - p}{p^\circ} = \frac{2.6}{\frac{1000}{18} + 2.6}$$

For dil solution

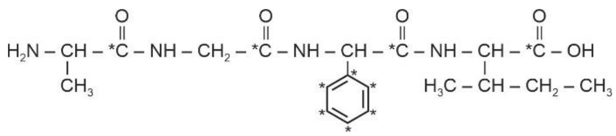
$$\frac{p^\circ - p}{p^\circ} = \frac{2.6}{\frac{1000}{18}}$$

$p = 47.66 \simeq 48$ mm Hg

84. In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of sp^2 hybridised carbons is _________.

Answer (10)

Sol. The given Oligopeptide has the following structure



It has 10 sp^2 hybridised C atoms given with star in the above structure.

85. One mole of an ideal gas at 350 K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0 L to 3.0 L against a constant pressure of 4 atm. The change in entropy of the surroundings (ΔS) is _____ $J\ K^{-1}$ (Nearest integer)

Given: R = 8.314 $J\ K^{-1}\ mol^{-1}$.

Answer (03)

Sol. For isothermal process Q = – W

The W for isothermal reversible process is

$$W = -nRT \ln\frac{V_2}{V_1}$$

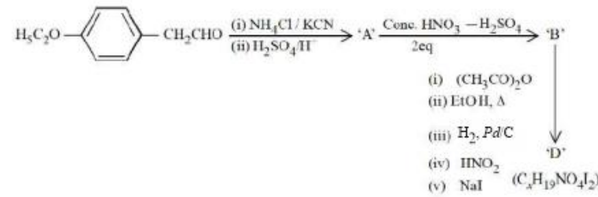
$$\Delta S_{surrounding} = -nR\ln\frac{V_2}{V_1}$$

$$= -1 \times 8.314 \times 2.303 \times \log\frac{3}{2}$$

$$= -3.37\ JK^{-1}$$

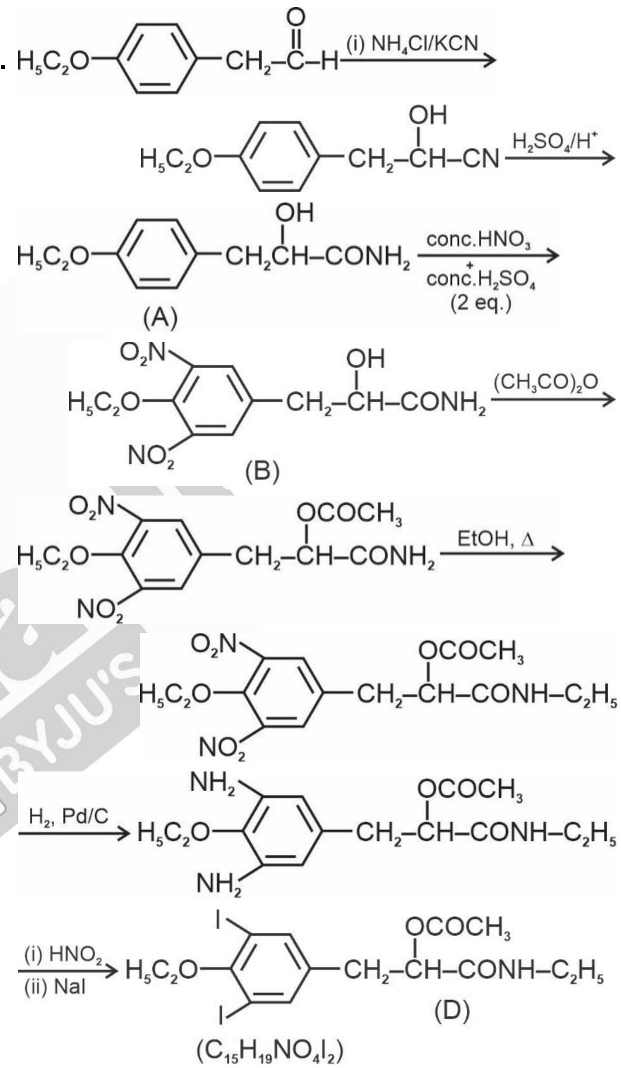
$$\simeq -3\ JK^{-1}$$

Note : The given process is mentioned as isothermal reversible process hence the $\Delta S_{surrounding}$ is calculated accordingly.

86. 

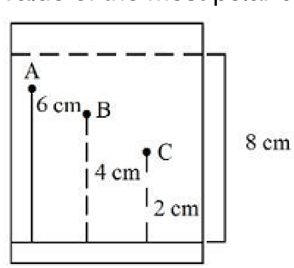
The value of x in compound 'D' is _______

Answer (15)

Sol. 

$\therefore$ x = 15

87. Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R_f value of the most polar compound is _____ × 10^{-2}



Answer (25)

Sol. In thin layer chromatography using hexane, the least polar compound will rise to maximum height and most polar compound will rise to minimum height.

$\therefore$ R_f value for most polar compound

$$= \frac{2}{8} = 0.25$$

$$= 25 \times 10^{-2}$$

88. Values of work function (W_0) for a few metals are given below

Metal	Li	Na	K	Mg	Cu	Ag
W_0/eV	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength 400 nm falls on it is ________

Given: h = 6.6×10^{-34} J s

c = 3×10^{8} m s^{-1}

e = 1.6×10^{-19} C

Answer (3)

Sol. Energy of incident photon = $\frac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}} J$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9} \times 1.6 \times 10^{-19}} eV$$

= 3.1 eV

$\therefore$ Li, Na and K will show photoelectric effect.

89. The mass of NH_3 produced when 131.8 kg of cyclohexane carbaldehyde undergoes Tollen's test is ________ kg. (Nearest Integer)

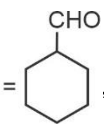
Molar mass of C = 12 g/mol

N = 14 g/mol

O = 16 g/mol

Answer (60)

Sol. $RCHO + 2[Ag(NH_3)_2]OH \rightarrow RCOONH_4 + 2Ag + 3NH_3 + H_2O$

RCHO =  , moles $= \frac{131.8 \times 10^3}{112} = 1.176 \times 10^3$

1 mole of aldehyde produces 3 moles of NH_3

1.176×10^3 mol aldehyde will produce

$= 3 \times 1.176 \times 10^3$ moles of NH_3

Mass of NH_3 produced = $3 \times 1.176 \times 10^3 \times 17$

$= 59.97 \times 10^3$ g

$\approx$ 60 kg

90. An analyst wants to convert 1 L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is ______ mL. (Nearest integer)

Answer (9000)

Sol. The concentration of H^+ or say HCl in both the given solution is 10^{-1} and 10^{-2} mole L^{-1} respectively.

For the given amount of HCl the concentration depends over dilution hence

$$\frac{C_1}{C_2} = \frac{V_2}{V_1}$$

$$\frac{10^{-1}}{10^{-2}} = \frac{V_2}{1L}$$

V_2 = 10 L = 10000 mL

ΔV = 10000 – 1000 mL = 9000 mL

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