EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

represents two lines which are

(A) intersecting at exactly one point (B) intersecting at exactly two points

(C) coincident (D) parallel.

Solution:

(D) Parallel

Explanation:

The given equations are,

6x-3y+10 = 0

dividing by 3

⇒ 2x-y+ 10/3= 0… (i)

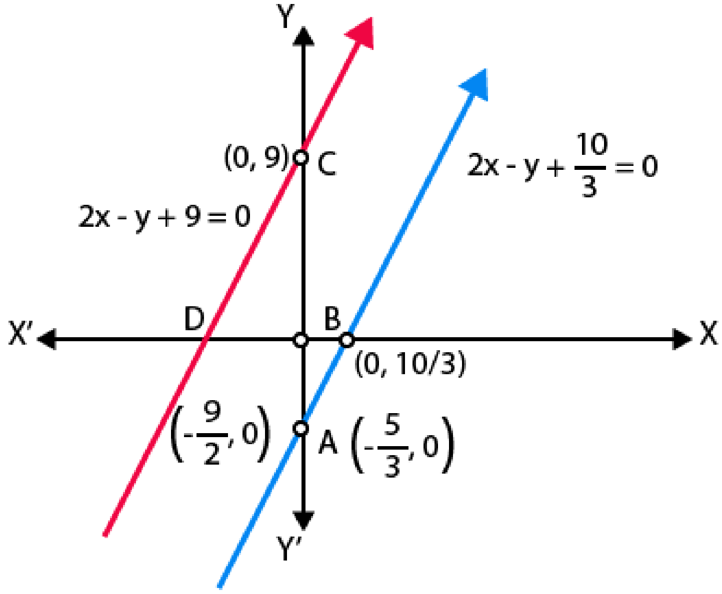
And 2x-y+9=0…(ii)

Table for 2x-y+ 10/3 = 0,

x	0	-5/3
y	10/3	0

Table for 2x-y+9=0

x	0	-9/2
y	9	0



Hence, the pair of equations represents two parallel lines.

2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(A) a unique solution (B) exactly two solutions

(C) infinitely many solutions (D) no solution

Solution:

(D) no solution

Explanation:

The equations are:

$x + 2y + 5 = 0$

$-3x - 6y + 1 = 0$

$a_1 = 1$; $b_1 = 2$; $c_1 = 5$

$a_2 = -3$; $b_2 = -6$; $c_2 = 1$

$a_1/a_2 = -1/3$

$b_1/b_2 = -2/6 = -1/3$

$c_1/c_2 = 5/1 = 5$

Here,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Therefore, the pair of equations has no solution.

3. If a pair of linear equations is consistent, then the lines will be

(A) parallel (B) always coincident

(C) intersecting or coincident (D) always intersecting

Solution:

(C) intersecting or coincident

Explanation:

Conditions for a pair of linear equations to be consistent are:

Intersecting lines, having a unique solution,

$a_1/a_2 \neq b_1/b_2$

Coincident or dependent

$a_1/a_2 = b_1/b_2 = c_1/c_2$

4. The pair of equations *y* = 0 and *y* = –7 has

(A) one solution (B) two solutions

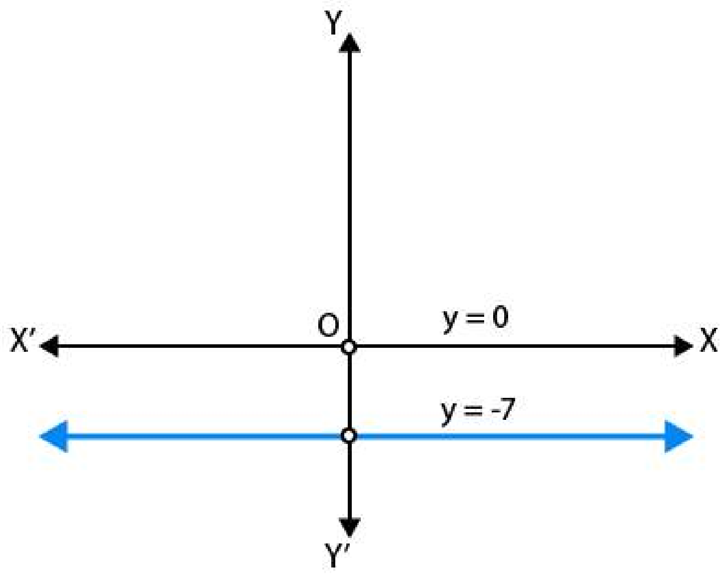
(C) infinitely many solutions (D) no solution

Solution:

(D) no solution

Explanation:

The given pair of equations are y = 0 and y = – 7.



Graphically, both lines are parallel and have no solution

5. The pair of equations *x* = *a* and *y* = *b* graphically represents lines which are

(A) parallel (B) intersecting at (*b*, *a*)

(C) coincident (D) intersecting at (*a*, *b*)

Solution:

(D) intersecting at (*a*, *b*)

Explanation:

Graphically in every condition,

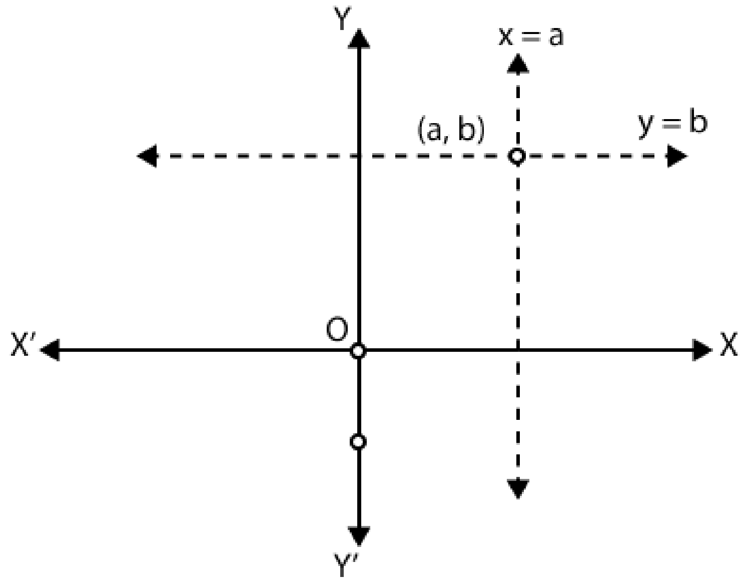
a, b>>0

a, b< 0

a>0, b< 0

a<0, b>0 but a = b≠ 0.

The pair of equations x = a and y = b graphically represents lines which are intersecting at (a, b).



Hence, in this case, two lines intersect at (a, b).

EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$

$12y + 6x = 6$

(ii) $x = 2y$

$y = 2x$

(iii) $3x + y – 3 = 0$

$2x + 2/3y = 2$

Solution:

The Condition for no solution = $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines)

(i) Yes.

Given pair of equations are,

$2x+4y – 3 = 0$ and $6x + 12y – 6 = 0$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 2$, $b_1 = 4$, $c_1 = – 3$;

$a_2 = 6$, $b_2 = 12$, $c_2 = – 6$;

$a_1 /a_2 = 2/6 = 1/3$

$b_1 /b_2 = 4/12 = 1/3$

$c_1 /c_2 = – 3/ – 6 = ½$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e parallel lines

Hence, the given pair of linear equations has no solution.

(ii) No.

Given pair of equations,

$x = 2y$ or $x – 2y = 0$

$y = 2x$ or $2x – y = 0$;

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 1$, $b_1 = – 2$, $c_1 = 0$;

$a_2 = 2$, $b_2 = – 1$, $c_2 = 0$;

$a_1 /a_2 = ½$

$b_1 /b_2 = -2/-1 = 2$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution.

(iii) No.

Given pair of equations,

$3x + y – 3 = 0$

$2x + 2/3\ y = 2$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 3, b_1 = 1, c_1 = – 3$;

$a_2 = 2, b_2 = 2/3, c_2 = – 2$;

$a_1 /a_2 = 2/6 = 3/2$

$b_1 /b_2 = 4/12 = 3/2$

$c_1 /c_2 = – 3/-2 = 3/2$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + 1/7y = 3$

$7x + 3y = 7$

(ii) $–2x – 3y = 1$

$6y + 4x = – 2$

(iii) $x/2 + y + 2/5 = 0$

$4x + 8y + 5/16 = 0$

Solution:

Condition for coincident lines,

$a_1/a_2 = b_1/b_2 = c_1/c_2$;

(i) No.

Given pair of linear equations are:

$3x + 1/7y = 3$

$7x + 3y = 7$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = 3, b_1 = 1/7, c_1 = – 3$;

And $a_2 = 7, b_2 = 3, c_2 = – 7$;

$a_1 /a_2 = 3/7$

$b_1 /b_2 = 1/21$

$c_1/c_2 = -3/-7 = 3/7$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution.

(ii) Yes.

Given pair of linear equations.

$-2x - 3y - 1 = 0$ and $4x + 6y + 2 = 0$;

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = -2$, $b_1 = -3$, $c_1 = -1$;

And $a_2 = 4$, $b_2 = 6$, $c_2 = 2$;

$a_1/a_2 = -2/4 = -½$

$b_1/b_2 = -3/6 = -½$

$c_1/c_2 = -½$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines

Hence, the given pair of linear equations is coincident.

(iii) No.

Given pair of linear equations are

$x/2 + y + 2/5 = 0$

$4x + 8y + 5/16 = 0$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = ½$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;

$a_1/a_2 = 1/8$

$b_1/b_2 = 1/8$

$c_1/c_2 = 32/25$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines

Hence, the given pair of linear equations has no solution.

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$

$4y + 3x = 12$

(ii) $(3/5)x - y = ½$

$(1/5)x - 3y = 1/6$

(iii) $2ax + by = a$

1. ***ax* + 2*by* – 2*a* = 0; *a*, *b* ≠ 0**

(iv) $x + 3y = 11$

$2(2x + 6y) = 22$

Solution:

Conditions for pair of linear equations to be consistent are:

$a_1/a_2 \neq b_1/b_2$. [unique solution]

$a_1/a_2 = b_1/b_2 = c_1/c_2$ [coincident or infinitely many solutions]

(i) No.

The given pair of linear equations

$-3x - 4y - 12 = 0$ and $4y + 3x - 12 = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = -3, b_1 = -4, c_1 = -12$;

$a_2 = 3, b_2 = 4, c_2 = -12$;

$a_1/a_2 = -3/3 = -1$

$b_1/b_2 = -4/4 = -1$

$c_1/c_2 = -12/-12 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Hence, the pair of linear equations has no solution, i.e., inconsistent.

(ii) Yes.

The given pair of linear equations

$(3/5)x - y = ½$

$(1/5)x - 3y = 1/6$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 3/5, b_1 = -1, c_1 = -½$;

$a_2 = 1/5, b_2 = 3, c_2 = -1/6$;

$a_1/a_2 = 3$

$b_1/b_2 = -1/-3 = 1/3$

$c_1/c_2 = 3$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution, i.e., consistent.

(iii) Yes.

The given pair of linear equations –

$2ax + by – a = 0$ and $4ax + 2by – 2a = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 2a$, $b_1 = b$, $c_1 = – a$;

$a_2 = 4a$, $b_2 = 2b$, $c_2 = – 2a$;

$a_1 /a_2 = ½$

$b_1 /b_2 = ½$

$c_1 /c_2 = ½$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$

Hence, the given pair of linear equations has infinitely many solutions, i.e., consistent

(iv) No.

The given pair of linear equations

$x + 3y = 11$ and $2x + 6y = 11$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 1$, $b_1 = 3$, $c_1 = 11$

$a_2 = 2$, $b_2 = 6$, $c_2 = 11$

$a_1 /a_2 = ½$

$b_1 /b_2 = ½$

$c_1 /c_2 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Hence, the given pair of linear equations has no solution.

EXERCISE 3.3

1. For which value(s) of λ , do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given pair of linear equations is

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

The given equations are;

$\lambda x + y - \lambda^2 = 0$

$x + \lambda y - 1 = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$;

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$;

$a_1/a_2 = \lambda/1$

$b_1/b_2 = 1/\lambda$

$c_1/c_2 = \lambda^2$

(i) For no solution,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

i.e. $\lambda = 1/\lambda \neq \lambda^2$

so, $\lambda^2 = 1$;

and $\lambda^2 \neq \lambda$

Here, we take only $\lambda = -1$,

Since the system of linear equations has no solution.

(ii) For infinitely many solutions,

$a_1/a_2 = b_1/b_2 = c_1/c_2$

i.e. $\lambda = 1/\lambda = \lambda^2$

so $\lambda = 1/\lambda$ gives $\lambda = +1$;

$\lambda = \lambda^2$ gives $\lambda = 1,0$;

Hence satisfying both the equations

$\lambda = 1$ is the answer.

(iii) For a unique solution,

$a_1/a_2 \neq b_1/b_2$

so $\lambda \neq 1/\lambda$

hence, $\lambda^2 \neq 1$;

$\lambda \neq + 1$;

So, all real values of λ except +1.

2. For which value(s) of *k* will the pair of equations

***kx* + 3*y* = *k* – 3**

12*x* + *ky* = *k*

have no solution?

Solution:

The given pair of linear equations is

$kx + 3y = k – 3$ …(i)

$12x + ky = k$ …(ii)

On comparing the equations (i) and (ii) with $ax + by = c = 0$,

We get,

$a_1 = k$, $b_1 = 3$, $c_1 = -(k – 3)$

$a_2 = 12$, $b_2 = k$, $c_2 = – k$

Then,

$a_1 /a_2 = k/12$

$b_1 /b_2 = 3/k$

$c_1 /c_2 = (k-3)/k$

For no solution of the pair of linear equations,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$k/12 = 3/k \neq (k-3)/k$

Taking the first two parts, we get

$k/12 = 3/k$

$k^2 = 36$

$k = + 6$

Taking the last two parts, we get

$3/k \neq (k-3)/k$

$3k \neq k(k-3)$

$k^2 - 6k \neq 0$

so, $k \neq 0,6$

Therefore, the value of k for which the given pair of linear equations has no solution is k = – 6.

3. For which values of *a* and *b* will the following pair of linear equations have infinitely many solutions?

$\boldsymbol{x + 2y = 1}$

$\boldsymbol{(a - b)x + (a + b)y = a + b - 2}$

Solution:

The given pair of linear equations are:

$x + 2y = 1$ …(i)

$(a-b)x + (a + b)y = a + b - 2$ …(ii)

On comparing with $ax + by = c = 0$ we get

$a_1 = 1$, $b_1 = 2$, $c_1 = -1$

$a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(a + b - 2)$

$a_1/a_2 = 1/(a-b)$

$b_1/b_2 = 2/(a+b)$

$c_1/c_2 = 1/(a+b-2)$

For infinitely many solutions of the pair of linear equations,

$a_1/a_2 = b_1/b_2 = c_1/c_2$(coincident lines)

so, $1/(a-b) = 2/(a+b) = 1/(a+b-2)$

Taking the first two parts,

$1/(a-b) = 2/(a+b)$

$a + b = 2(a - b)$

$a = 3b$ …(iii)

Taking the last two parts,

$2/(a+b) = 1/(a+b-2)$

$2(a + b - 2) = (a + b)$

$a + b = 4$ …(iv)

Now, put the value of a from Eq. (iii) in Eq. (iv), and we get

$3b + b = 4$

$4b = 4$

$b = 1$

Put the value of b in Eq. (iii), and we get

$a = 3$

So, the values $(a,b) = (3,1)$ satisfy all the parts. Hence, the required values of a and b are 3 and 1, respectively, for which the given pair of linear equations has infinitely many solutions.

4. Find the value(s) of *p* in (i) to (iv) and *p* and *q* in (v) for the following pair of equations:

(i) $3x – y – 5 = 0$ and $6x – 2y – p = 0$, if the lines represented by these equations are parallel.

Solution:

Given pair of linear equations is

$3x – y – 5 = 0$ …(i)

$6x – 2y – p = 0$ …(ii)

On comparing with $ax + by + c = 0$ we get

We get,

$a_1 = 3$, $b_1 = – 1$, $c_1 = – 5$;

$a_2 = 6$, $b_2 = – 2$, $c_2 = – p$;

$a_1 /a_2 = 3/6 = ½$

$b_1 /b_2 = ½$

$c_1 /c_2 = 5/p$

Since the lines represented by these equations are parallel, then

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Taking the last two parts, we get $½ \neq 5/p$

So, $p \neq 10$

Hence, the given pair of linear equations are parallel for all real values of p except 10.

(ii) $– x + py = 1$ and $px – y = 1$, if the pair of equations has no solution.

Solution:

Given pair of linear equations is

$– x + py = 1$ …(i)

$px – y – 1 = 0$ …(ii)

On comparing with $ax + by + c = 0$,

We get,

$a_1 = -1$, $b_1 = p$, $c_1 =- 1$;

$a_2 = p$, $b_2 = – 1$, $c_2 =- 1$;

$a_1 /a_2 = -1/p$

$b_1 /b_2 = – p$

$c_1/c_2 = 1$

Since the lines equations have no solution, i.e., both lines are parallel to each other,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$-1/p = -p \neq 1$

Taking the last two parts, we get

$p \neq -1$

Taking the first two parts, we get

$p^2 = 1$

$p = +1$

Hence, the given pair of linear equations has no solution for p = 1.

(iii) – 3*x* + 5*y* = 7 and 2*px* – 3*y* = 1, if the lines represented by these equations are intersecting at a unique point.

Solution:

Given, pair of linear equations is

$-3x + 5y = 7$

$2px - 3y = 1$

On comparing with ax + by + c = 0, we get

Here, $a_1 = -3$, $b_1 = 5$, $c_1 = -7$;

And $a_2 = 2p$, $b_2 = -3$, $c_2 = -1$;

$a_1/a_2 = -3/2p$

$b_1/b_2 = -5/3$

$c_1/c_2 = 7$

Since the lines intersect at a unique point, i.e., it has a unique solution

$a_1/a_2 \neq b_1/b_2$

$-3/2p \neq -5/3$

$p \neq 9/10$

Hence, the lines represented by these equations intersect at a unique point for all real values of p except 9/10.

(iv) 2*x* + 3*y* – 5 = 0 and *px* – 6*y* – 8 = 0, if the pair of equations has a unique solution.

Solution:

Given, pair of linear equations is

$2x + 3y - 5 = 0$

$px - 6y - 8 = 0$

On comparing with ax + by + c = 0 we get

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$;

And $a_2 = p$, $b_2 = -6$, $c_2 = -8$;

$a_1/a_2 = 2/p$

$b_1/b_2 = -3/6 = -½$

$c_1/c_2 = 5/8$

Since the pair of linear equations has a unique solution,

$a_1/a_2 \neq b_1/b_2$

so $2/p \neq -½$

$p \neq -4$

Hence, the pair of linear equations has a unique solution for all values of p except – 4.

(v) 2*x* + 3*y* = 7 and 2*px* + *py* = 28 – *qy*, if the pair of equations has infinitely many solutions.

Solution:

Given pair of linear equations is

$2x + 3y = 7$

$2px + py = 28 - qy$

or $2px + (p + q)y - 28 = 0$

On comparing with $ax + by + c = 0$,

We get,

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$;

And $a_2 = 2p$, $b_2 = (p + q)$, $c_2 = -28$;

$a_1/a_2 = 2/2p$

$b_1/b_2 = 3/(p+q)$

$c_1/c_2 = ¼$

Since the pair of equations has infinitely many solutions i.e., both lines are coincident.

$a_1/a_2 = b_1/b_2 = c_1/c_2$

$1/p = 3/(p+q) = ¼$

Taking the first and third parts, we get

$p = 4$

Again, taking the last two parts, we get

$3/(p+q) = ¼$

$p + q = 12$

Since $p = 4$

So, $q = 8$

Here, we see that the values of p = 4 and q = 8 satisfy all three parts.

Hence, the pair of equations has infinitely many solutions for all values of p = 4 and q = 8.

5. Two straight paths are represented by the equations $x – 3y = 2$ and $–2x + 6y = 5$.Check whether the paths cross each other or not.

Solution:

Given linear equations are

$x – 3y – 2 = 0$ …(i)

$-2x + 6y – 5 = 0$ …(ii)

On comparing with ax + by c=0,

We get

$a_1 = 1$, $b_1 = -3$, $c_1 = - 2$;

$a_2 = -2$, $b_2 = 6$, $c_2 = - 5$;

$a_1/a_2 = – ½$

$b_1/b_2 = – 3/6 = – ½$

$c_1/c_2 = 2/5$

i.e., $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ [parallel lines]

Hence, two straight paths represented by the given equations never cross each other because they are parallel to each other.

6. Write a pair of linear equations which has the unique solution $x = – 1$, $y = 3$. How many such pairs can you write?

Solution:

Condition for the pair of system to have a unique solution

$a_1/a_2 \neq b_1/b_2$

Let the equations be,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Since, x = – 1 and y = 3 is the unique solution of these two equations, then

It must satisfy the equations –

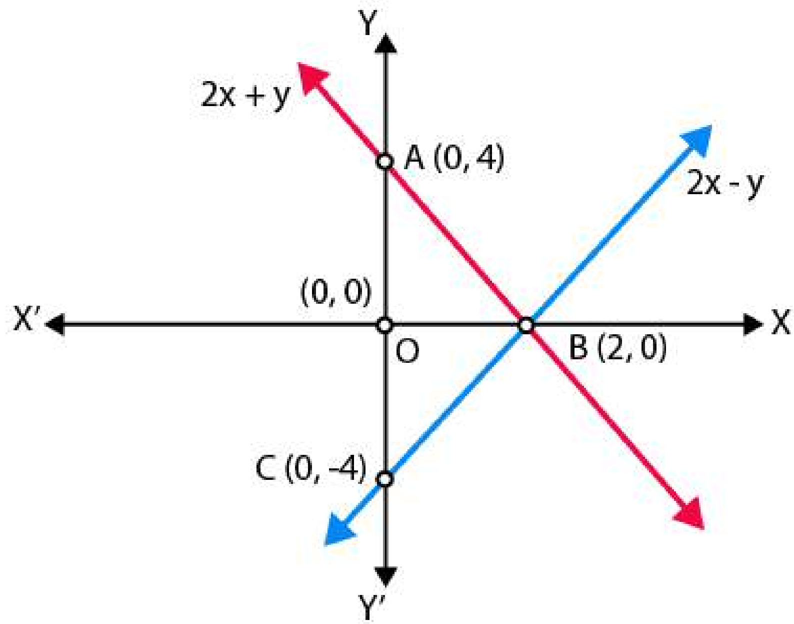
$a_1(-1) + b_1(3) + c_1 = 0$

$– a_1 + 3b_1 + c_1 = 0$ …(i)

and $a_2(- 1) + b_2(3) + c_2 = 0$

$– a_2 + 3b_2 + c_2 = 0$ …(ii)

Since for the different values of a_1, b_1, c_1 and a_2, b_2, c_2 satisfy the Eqs. (i) and (ii),



Hence, infinitely many pairs of linear equations are possible.

7. If 2*x* + *y* = 23 and 4*x* – *y* = 19, find the values of 5*y* – 2*x* and y/x – 2.

Solution:

Given equations are

2x + y = 23 …(i)

4x – y = 19 …(ii)

On adding both equations, we get

6x = 42

So, x = 7

Put the value of x in Eq. (i), and we get

2(7) + y = 23

y = 23 – 14

so, y = 9

Hence 5y – 2x = 5(9) – 2(7) = 45 – 14 = 31

y/x – 2 = 9/7 -2 = -5/7

Hence, the values of (5y – 2x) and y/x – 2 are 31 and -5/7 respectively.

8. Find the values of *x* and *y* in the following rectangle [see Fig. 3.2].

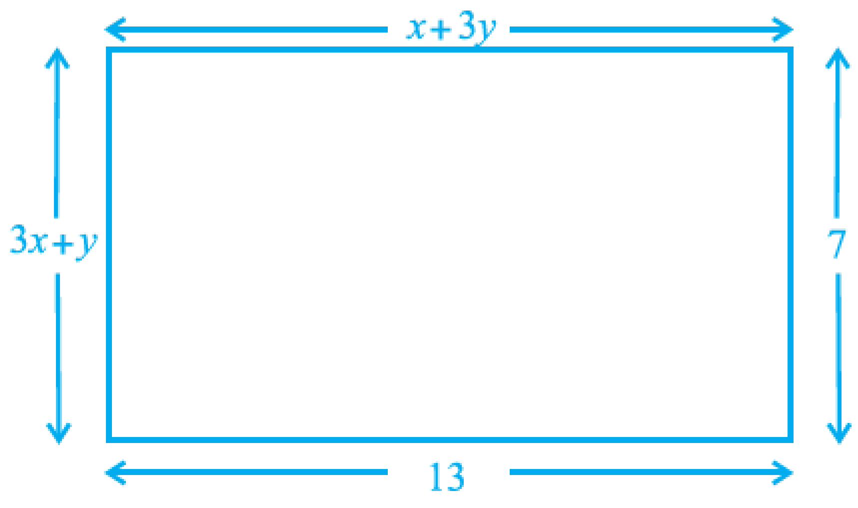


Fig. 3.2

Solution:

Using the property of a rectangle,

We know that,

Lengths are equal,

i.e., CD = AB

Hence, $x + 3y = 13$ …(i)

Breadth are equal,

i.e., AD = BC

Hence, $3x + y = 7$ …(ii)

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),

We get,

$8x = 8$

So, $x = 1$

On substituting $x = 1$ in Eq. (i),

We get,

$y = 4$

Therefore, the required values of x and y are 1 and 4, respectively.

EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$2x + y = 6$

$2x – y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the *x*-axis and the lines with the *y*-axis.

Solution:

Given equations are $2x + y = 6$ and $2x – y + 2 = 0$

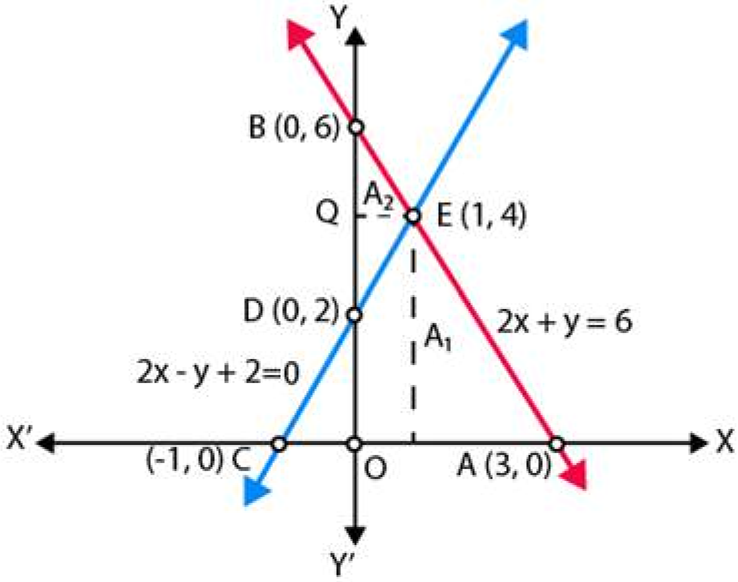
Table for equation $2x + y – 6 = 0$, for $x = 0$, $y = 6$, for $y = 0$, $x = 3$.

x	0	3
y	6	0

Table for equation $2x – y + 2 = 0$, for $x = 0$, $y = 2$, for $y = 0$, $x = – 1$

x	0	-1
y	2	0

Let A_1 and A_2 represent the areas of triangles ACE and BDE, respectively.



Let the area of triangle formed with x-axis = T_1

T_1 = Area of △ACE = ½ × AC × PE

T_1 = ½ × 4 × 4 = 8

And area of triangle formed with y-axis = T_2

T_1 = Area of △BDE = ½ × BD × QE

T_1 = ½ × 4 × 1 = 2

T_1:T_2 = 8:2 = 4:1

Hence, the pair of equations intersect graphically at point E(1,4)

i.e., x = 1 and y = 4.

2. Determine, graphically, the vertices of the triangle formed by the lines

y = x, 3y = x, x + y = 8

Solution:

Given linear equations are

y = x …(i)

3y = x …(ii)

and x + y = 8 …(iii)

Table for line y = x,

x	0	1	2
y	0	1	2

Table for line x = 3y,

x	0	3	6
y	0	1	2

Table for line x + y = 8

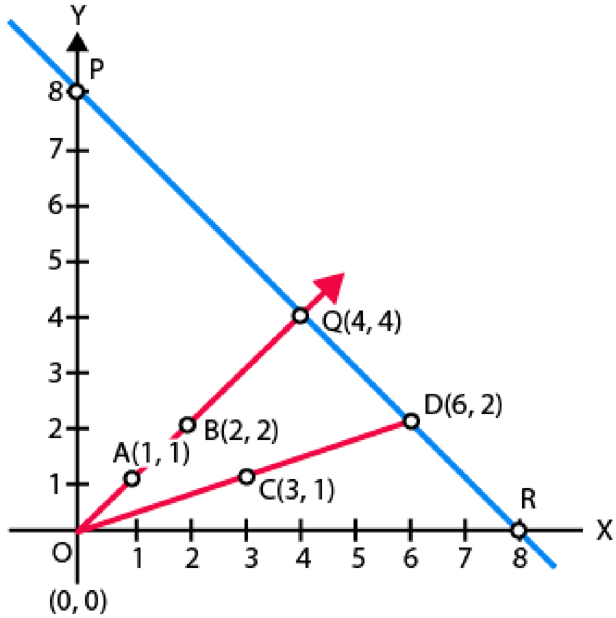
x	0	4	8

y	8	4	0

Plotting the points A (1, 1), B(2,2), C (3, 1), D (6, 2), we get the straight lines AB and CD.

Similarly, plotting the points P (0, 8), Q(4, 4) and R(8, 0), we get the straight line PQR.

AB and CD intersect the line PR on Q and D, respectively.



So, △OQD is formed by these lines. Hence, the vertices of the △OQD formed by the given lines are O(0, 0), Q(4, 4) and D(6,2).

3. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x–axis.

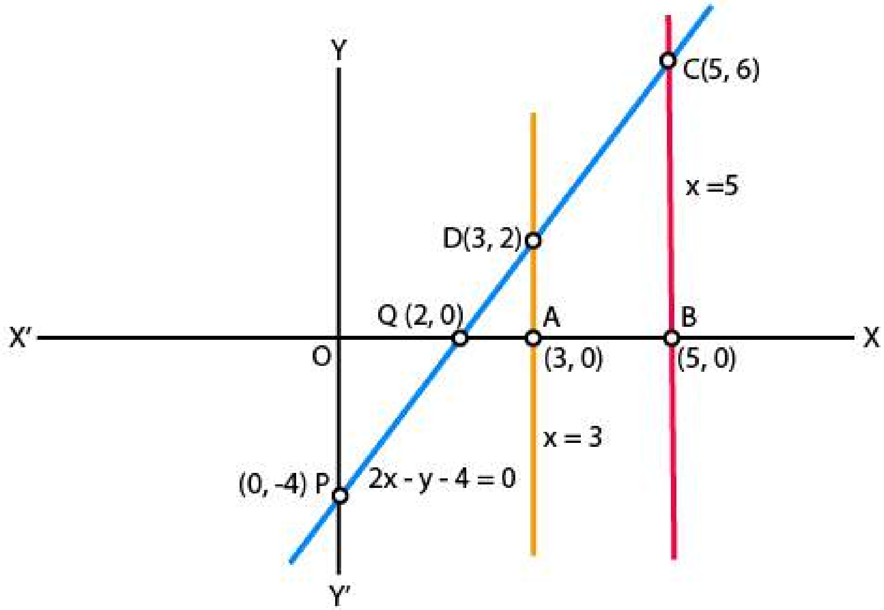
Solution:

Given equation of lines x = 3, x = 5 and 2x-y-4 = 0.

Table for line 2x – y – 4 = 0

x	0	2
y	-4	0

Plotting the graph, we get,



From the graph, we get,

AB = OB-OA = 5-3 = 2

AD = 2

BC = 6

Thus, quadrilateral ABCD is a trapezium, then,

Area of quadrilateral ABCD = ½ × (distance between parallel lines) = ½ × (AB) × (AD + BC)

= 8 sq units

4. The cost of 4 pens and 4 pencil boxes is Rs. 100. Three times the cost of a pen is Rs. 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Let the cost of a pen and a pencil box be Rs x and Rs y, respectively.

According to the question,

$4x + 4y = 100$

Or $x + y = 25$ …(i)

$3x = y + 15$

Or $3x-y = 15$ …(ii)

On adding Equation (i) and (ii), we get,

$4x = 40$

So, $x = 10$

Substituting $x = 10$, in Eq. (i), we get

$y = 25-10 = 15$

Hence, the cost of a pen = Rs. 10

The cost of a pencil box = Rs. 15

5. Determine, algebraically, the vertices of the triangle formed by the lines

$3x - y = 3$

$2x - 3y = 2$

$x + 2y = 8$

Solution:

$3x - y = 2$ …(i)

$2x - 3y = 2$ …(ii)

$x + 2y = 8$ …(iii)

Let the equations of the line (i), (ii) and (iii) represent the side of a ΔABC.

On solving (i) and (ii),

We get,

[First, multiply Eq. (i) by 3 in Eq. (i) and then subtract]
$(9x-3y)-(2x-3y) = 9-2$

$7x = 7$

$x = 1$

Substituting x=1 in Eq. (i), we get

$3\times1-y = 3$

$y = 0$

So, the coordinate of point B is (1, 0)

On solving lines (ii) and (iii),

We get,

[First, multiply Eq. (iii) by 2 and then subtract]
$(2x + 4y)-(2x-3y) = 16-2$

$7y = 14$

$y = 2$

Substituting y=2 in Eq. (iii), we get

$x + 2 \times 2 = 8$

$x + 4 = 8$

$x = 4$

Hence, the coordinate of point C is (4, 2).

On solving lines (iii) and (i),

We get,

[First, multiply in Eq. (i) by 2 and then add]
$(6x-2y) + (x + 2y) = 6 + 8$

$7x = 14$

$x = 2$

Substituting $x=2$ in Eq. (i), we get

$3 \times 2 - y = 3$

$y = 3$

So, the coordinate of point A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are as follows,

A (2, 3), B (1, 0) and C (4, 2).

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

Solution:

Let the speed of the rickshaw and the bus be x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw, $t_1 = (2/x)$ hr

Speed = distance/ time

She has taken time to travel the remaining distance i.e., $(14 - 2) = 12$km

By bus $t_2 = (12/y)$ hr

By the first condition,

$t_1 + t_2 = ½ = (2/x) + (12/y)$ … (i)

Now, she has taken time to travel 4 km by rickshaw, $t_3 = (4/x)$ hr

and she has taken time to travel the remaining distance i.e., $(14 - 4) = 10$km, by bus = $t_4 = (10/y)$ hr

By the second condition,

$t_3 + t_4 = ½ + 9/60 = ½ + 3/20$

$(4/x) + (10/y) = (13/20)$ …(ii)

Let $(1/x) = u$ and $(1/y) = v$

Then equations (i) and (ii) become

$2u + 12v = ½$ …(iii)

$4u + 10v = 13/20$…(iv)

[First, multiply Eq. (iii) by 2 and then subtract]
$(4u + 24v) - (4u + 10v) = 1 - 13/20$

$14v = 7/20$

$v = 1/40$

Substituting the value of v in Eq. (iii),

$2u + 12(1/40) = ½$

$2u = 2/10$

$u = 1/10$

$x = 1/u = 10$km/hr

$y = 1/v = 40$km/hr

Hence, the speed of rickshaw = 10 km/h

And the speed of the bus = 40 km/h.