

EXERCISE 2.2

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1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) $x^2 - 2x - 8$

$$\Rightarrow x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$$

Therefore, zeroes of polynomial equation $x^2 - 2x - 8$ are (4, -2)

$$\text{Sum of zeroes} = 4 - 2 = 2 = -(-2)/1 = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = (-8)/1 = (\text{Constant term})/(\text{Coefficient of } x^2)$$

(ii) $4s^2 - 4s + 1$

$$\Rightarrow 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$$

Therefore, zeroes of polynomial equation $4s^2 - 4s + 1$ are (1/2, 1/2)

$$\text{Sum of zeroes} = (1/2) + (1/2) = 1 = -(-4)/4 = -(\text{Coefficient of } s)/(\text{Coefficient of } s^2)$$

$$\text{Product of zeroes} = (1/2) \times (1/2) = 1/4 = (1)/4 = (\text{Constant term})/(\text{Coefficient of } s^2)$$

(iii) $6x^2 - 3 - 7x$

$$\Rightarrow 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$$

Therefore, zeroes of polynomial equation $6x^2 - 3 - 7x$ are (-1/3, 3/2)

$$\text{Sum of zeroes} = -(1/3) + (3/2) = (7/6) = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = -(1/3) \times (3/2) = -(3/6) = (\text{Constant term})/(\text{Coefficient of } x^2)$$

(iv) $4u^2 + 8u$

$$\Rightarrow 4u(u+2)$$

Therefore, zeroes of polynomial equation $4u^2 + 8u$ are (0, -2).

$$\text{Sum of zeroes} = 0 + (-2) = -2 = -(8/4) = -(\text{Coefficient of } u)/(\text{Coefficient of } u^2)$$

$$\text{Product of zeroes} = 0 \times -2 = 0 = 0/4 = (\text{Constant term})/(\text{Coefficient of } u^2)$$

(v) $t^2 - 15$

$$\Rightarrow t^2 = 15 \text{ or } t = \pm\sqrt{15}$$

Therefore, zeroes of polynomial equation $t^2 - 15$ are ($\sqrt{15}$, $-\sqrt{15}$)

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = -(0/1) = -(\text{Coefficient of } t)/(\text{Coefficient of } t^2)$$

$$\text{Product of zeroes} = \sqrt{15} \times (-\sqrt{15}) = -15 = -15/1 = (\text{Constant term})/(\text{Coefficient of } t^2)$$

(vi) $3x^2 - x - 4$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4) = (3x-4)(x+1)$$

Therefore, zeroes of polynomial equation $3x^2 - x - 4$ are (4/3, -1)

$$\text{Sum of zeroes} = (4/3) + (-1) = (1/3) = -(-1/3) = -(\text{Coefficient of } x) / (\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = (4/3) \times (-1) = (-4/3) = (\text{Constant term}) / (\text{Coefficient of } x^2)$$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) $1/4, -1$

Solution:

From the formulas of sum and product of zeroes, we know,

$$\text{Sum of zeroes} = \alpha + \beta$$

$$\text{Product of zeroes} = \alpha \beta$$

$$\text{Sum of zeroes} = \alpha + \beta = 1/4$$

$$\text{Product of zeroes} = \alpha \beta = -1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (1/4)x + (-1) = 0$$

$$4x^2 - x - 4 = 0$$

Thus, $4x^2 - x - 4$ is the quadratic polynomial.

(ii) $\sqrt{2}, 1/3$

Solution:

$$\text{Sum of zeroes} = \alpha + \beta = \sqrt{2}$$

$$\text{Product of zeroes} = \alpha \beta = 1/3$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\sqrt{2})x + (1/3) = 0$$

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

Thus, $3x^2 - 3\sqrt{2}x + 1$ is the quadratic polynomial.

(iii) $0, \sqrt{5}$

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 0$$

$$\text{Product of zeroes} = \alpha \beta = \sqrt{5}$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (0)x + \sqrt{5} = 0$$

Thus, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv) 1, 1

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

$$\text{Product of zeroes} = \alpha \beta = 1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - x + 1 = 0$$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(v) $-1/4$, $1/4$

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = -1/4$$

$$\text{Product of zeroes} = \alpha \beta = 1/4$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-1/4)x + (1/4) = 0$$

$$4x^2 + x + 1 = 0$$

Thus, $4x^2 + x + 1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 4$$

$$\text{Product of zeroes} = \alpha\beta = 1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - 4x + 1 = 0$$

Thus, $x^2 - 4x + 1$ is the quadratic polynomial.