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1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3-3x^2+5x-3$, $g(x) = x^2-2$

Solution:

Given,

Dividend = $p(x) = x^3-3x^2+5x-3$

Divisor = $g(x) = x^2-2$

$$
\begin{array}{r|llll}
 & x & -3 & & \\
\hline
x^2-2 & x^3 & -3x^2 & +5x & -3 \\
- & & & & \\
 & x^3 & +0x^2 & -2x & \\
\hline
 & & -3x^2 & +7x & -3 \\
 & & - & & \\
 & & -3x^2 & +0x & +6 \\
\hline
 & & & 7x & -9
\end{array}
$$

Therefore, upon division we get,

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x) = x^4-3x^2+4x+5$, $g(x) = x^2+1-x$

Solution:

Given,

Dividend = $p(x) = x^4-3x^2+4x+5$

Divisor = $g(x) = x^2+1-x$

$$
\begin{array}{r|rrrrr}
 & x^2 & +x & -3 & & \\
\hline
x^2 - x + 1 & x^4 & +0x^3 & -3x^2 & +4x & +5 \\
- & & & & & \\
 & x^4 & -x^3 & +x^2 & & \\
\hline
 & & x^3 & -4x^2 & +4x & +5 \\
 & & - & & & \\
 & & x^3 & -x^2 & +x & \\
\hline
 & & & -3x^2 & +3x & +5 \\
 & & & - & & \\
 & & & -3x^2 & +3x & -3 \\
\hline
 & & & & & 8
\end{array}
$$

Therefore, upon division we get,

Quotient = $x^2 + x–3$

Remainder = 8

(iii) p(x) =x^4–5x+6, g(x) = 2–x^2

Solution:

Given,

Dividend = p(x) =x^4 – 5x + 6 = x^4+0x^2–5x+6

Divisor = g(x) = 2–x^2 = –x^2+2

$$
\begin{array}{r|rrrrr}
 & -x^2 & -2 & & & \\
\hline
-x^2 + 2 & x^4 & +0x^3 & +0x^2 & -5x & +6 \\
- & & & & & \\
 & x^4 & +0x^3 & -2x^2 & & \\
\hline
 & & & 2x^2 & -5x & +6 \\
 & & & - & & \\
 & & & 2x^2 & +0x & -4 \\
\hline
 & & & & -5x & +10
\end{array}
$$

Therefore, upon division we get,

Quotient = $-x^2-2$

Remainder = $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) t^2-3, $2t^4+3t^3-2t^2-9t-12$

Solutions:

Given,

First polynomial = t^2-3

Second polynomial = $2t^4+3t^3-2t^2-9t-12$

$$
\begin{array}{r|rrrrr}
 & 2t^2 & +3t & +4 & & \\
\hline
t^2-3 & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 & - & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\hline
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & & - & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & 4t^2 & +0t & -12 \\
 & & & - & & \\
 & & & 4t^2 & +0t & -12 \\
\hline
 & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii)x^2+3x+1 , $3x^4+5x^3-7x^2+2x+2$

Solutions:

Given,

First polynomial = x^2+3x+1

Second polynomial = $3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|rrrrr}
 & 3x^2 & -4x & +2 & & \\
\hline
x^2+3x+1 & 3x^4 & +5x^3 & -7x^2 & +2x & +2 \\
 & - & & & & \\
 & 3x^4 & +9x^3 & +3x^2 & & \\
\hline
 & & -4x^3 & -10x^2 & +2x & +2 \\
 & & - & & & \\
 & & -4x^3 & -12x^2 & -4x & \\
\hline
 & & & 2x^2 & +6x & +2 \\
 & & & - & & \\
 & & & 2x^2 & +6x & +2 \\
\hline
 & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) x^3-3x+1, $x^5-4x^3+x^2+3x+1$

Solutions:

Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|rrrrrr}
 & x^2 & -1 & & & & \\
\hline
x^3-3x+1 & x^5 & +0x^4 & -4x^3 & +x^2 & +3x & +1 \\
 & - & & & & & \\
 & x^5 & +0x^4 & -3x^3 & +x^2 & & \\
\hline
 & & & -x^3 & +0x^2 & +3x & +1 \\
 & & & - & & & \\
 & & & -x^3 & +0x^2 & +3x & -1 \\
\hline
 & & & & & & 2
\end{array}
$$

As we can see, the remainder is not equal to 0. Therefore, we say that, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{(5/3)}$ and – $\sqrt{(5/3)}$.

Solutions:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{(5/3)}$ and – $\sqrt{(5/3)}$ are zeroes of polynomial f(x).

$\therefore$ $(x - \sqrt{(5/3)})(x+\sqrt{(5/3)}) = x^2-(5/3) = 0$

$(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2-5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad - 5x^2 \\
 & (-) \qquad\quad (+) \\
\hline
 & \quad +6x^3 + 3x^2 - 10x - 5 \\
 & \quad -6x^3 \qquad - 10x \\
 & \quad (+) \qquad\quad (-) \\
\hline
 & \qquad 3x^2 \qquad -5 \\
 & \qquad 3x^2 \qquad -5 \\
 & \qquad (-) \qquad (+) \\
\hline
 & \qquad\quad 0
\end{array}
$$

Therefore, $3x^4+6x^3-2x^2-10x-5 = (3x^2-5)$**$(x^2+2x+1)$**

Now, on further factorizing (x^2+2x+1) we get,

x^2+2x+1 $= x^2+x+x+1 = 0$

$x(x+1)+1(x+1) = 0$

$(x+1)(x+1) = 0$

So, its zeroes are given by: **$x= -1$** and **$x = -1$.**

Therefore, all four zeroes of given polynomial equation are:

$\sqrt{(5/3)}$,- $\sqrt{(5/3)}$, -1 and -1.

Hence, is the answer.

4. On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were x–2 and –2x+4, respectively. Find g(x).

Solution:

Given,

Dividend, $p(x) = x^3-3x^2+x+2$

Quotient = $x-2$

Remainder = $-2x+4$

We have to find the value of Divisor, $g(x)$ =?

As we know,

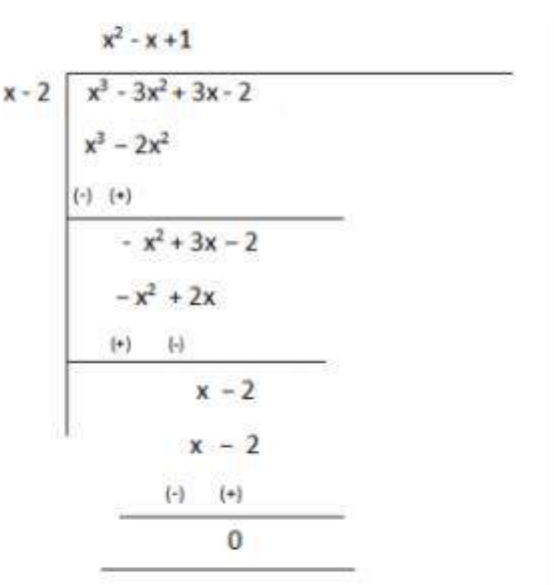
Dividend = Divisor × Quotient + Remainder

$\therefore x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$

$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$

Therefore, $g(x) \times (x-2) = x^3-3x^2+3x-2$

Now, for finding $g(x)$ we will divide x^3-3x^2+3x-2 with $(x-2)$



Therefore, $\mathbf{g(x) = (x^2-x+1)}$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Solutions:

According to the division algorithm, dividend $p(x)$ and divisor $g(x)$ are two polynomials, where $g(x)\neq 0$. Then we can find the value of quotient $q(x)$ and remainder $r(x)$, with the help of below given formula;

Dividend = Divisor × Quotient + Remainder

$\therefore p(x) = g(x)\times q(x)+r(x)$

Where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

Now let us proof the three given cases as per division algorithm by taking examples for each.

(i) deg p(x) = deg q(x)

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example, $p(x) = 3x^2+3x+3$ is a polynomial to be divided by $g(x) = 3$.

So, $(3x^2+3x+3)/3 = x^2+x+1 = q(x)$

Thus, you can see, the degree of quotient $q(x) = 2$, which also equal to the degree of dividend $p(x)$.

Hence, division algorithm is satisfied here.

(ii) deg q(x) = deg r(x)

Let us take an example, $p(x) = x^2 + 3$ is a polynomial to be divided by $g(x) = x – 1$.

So, $x^2 + 3 = (x – 1)\times(x) + (x + 3)$

Hence, quotient $q(x) = x$

Also, remainder $r(x) = x + 3$

Thus, you can see, the degree of quotient $q(x) = 1$, which is also equal to the degree of remainder $r(x)$.

Hence, division algorithm is satisfied here.

(iii) deg r(x) = 0

The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example, $p(x) = x^2 + 1$ is a polynomial to be divided by $g(x) = x$.

So, $x^2 + 1 = (x)\times(x) + 1$

Hence, quotient $q(x) = x$

And, remainder $r(x) = 1$

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.