EXERCISE 2.4 PAGE: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solution:

Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for p(x) are = 1/2, 1, -2

$\therefore p(1/2) = 2(1/2)^3+(1/2)^2-5(1/2)+2 = (1/4)+(1/4)-(5/2)+2 = 0$

$p(1) = 2(1)^3+(1)^2-5(1)+2 = 0$

$p(-2) = 2(-2)^3+(-2)^2-5(-2)+2 = 0$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$

a=2, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha +\beta+\gamma = –b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a$

$\alpha \beta\gamma = – d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = ½+1+(-2) = -1/2 = –b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = (1/2\times1)+(1 \times-2)+(-2\times1/2) = -5/2 = c/a$

$\alpha \beta \gamma = ½\times1\times(-2) = -2/2 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) x^3-4x^2+5x-2 ;2, 1, 1

Solution:

Given, $p(x) = x^3-4x^2+5x-2$

And zeroes for p(x) are 2,1,1.

$\therefore p(2)= 2^3-4(2)^2+5(2)-2 = 0$

$p(1) = 1^3-(4\times1^2)+(5\times1)-2 = 0$

Hence proved, 2, 1, 1 are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3+bx^2+cx+d = x^3-4x^2+5x-2$

a = 1, b = -4, c = 5 and d = -2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a.$

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = 2+1+1 = 4 = -(-4)/1 = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2\times1+1\times1+1\times2 = 5 = 5/1= c/a$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -(-2)/1 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solution:

Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

$\alpha+\beta+\gamma = -b/a = 2/1$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a = -7/1$

$\alpha\beta\gamma = -d/a = -14/1$

Thus, from above three expressions we get the values of coefficient of polynomial.

a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b.

Solution:

We are given with the polynomial here,

$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

$-(-3)/1 = 3a$

$a=1$

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$-s/p = 1-b^2$

$-1/1 = 1-b^2$

$b^2 = 1+1 = 2$

$b = \pm\sqrt{2}$

Hence, $1-\sqrt{2}$, 1, $1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1.

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2 \pm\sqrt{3}$, find other zeroes.

Solution:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let $f(x) = x^4-6x^3-26x^2+138x-35$

Since $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of given polynomial f(x).

$\therefore [x-(2+\sqrt{3})]\,[x-(2-\sqrt{3})] = 0$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

x^2-4x+1, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & (-)\ (+)\ (-) \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-2x \\
 & (+)\ (-)\ (+) \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & (+)\ (-)\ (+) \\
\hline
 & 0
\end{array}
$$

So, $x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$

Now, on further factorizing (x^2–2x−35) we get,

x^2**–(7−5)x −35** = x^2– 7x+5x+35 = 0

x(x −7)+5(x−7) = 0

(x+5)(x−7) = 0

So, its zeroes are given by:

x= −5 and x = 7.

Therefore, all four zeroes of given polynomial equation are: 2+√**3** , 2-√**3**, **−5 and 7.**

Q.5: If the polynomial $x^4 – 6x^3 + 16x^2 – 25x + 10$ is divided by another polynomial $x^2 – 2x + k$, the remainder comes out to be x + a, find k and a.

Solution:

Let's divide $x^4 – 6x^3 + 16x^2 – 25x + 10$ by $x^2 – 2x + k$.

$$x^2 - 2x + k\ )\ x^4 - 6x^3 + 16x^2 - 25x + 10\ (x^2 - 4x + (8-k)$$

$$\underset{-}{x^4}\ \underset{+}{-}\ 2x^3\ \underset{-}{+}\ kx^2$$

$$-4x^3 + (16-k)x^2 - 25x$$

$$\underset{+}{-4x^3}\ \underset{-}{+\ 8x^2}\ \underset{+}{-\ 4kx}$$

$$(8-k)x^2 + (4k-25)x + 10$$

$$\underset{-}{(8-k)x^2}\ \underset{+}{-\ 2(8-k)x}\ \underset{-}{+\ k(8-k)}$$

$$(4k - 25 + 16 - 2k)x + [10 - k(8-k)]$$

Given that the remainder of the polynomial division is x + a.

(4k – 25 + 16 – 2k)x + [10 – k(8 – k)] = x + a

(2k – 9)x + (10 – 8k + k^2) = x + a

Comparing the coefficients of the above equation, we get;

2k – 9 = 1

2k = 9 + 1 = 10

k = 10/2 = 5

And

10 – 8k + k^2 = a

10 – 8(5) + $(5)^2$ = a [since k = 5]

$10 - 40 + 25 = a$

$a = -5$

Therefore, $k = 5$ and $a = -5$.