EXERCISE 3.7

1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Solution:

The age difference between Ani and Biju is 3 yrs.

Either Biju is 3 years older than Ani or Ani is 3 years older than Biju.

From both cases, we find out that Ani's father's age is 30 yrs more than that of Cathy's age.

Let the ages of Ani and Biju be A and B, respectively.

Therefore, the age of Dharam = 2 x A = 2A yrs.

And the age of Biju's sister Cathy is B/2 yrs.

By using the information that is given,

Case (i)

When Ani is older than Biju by 3 yrs, then A – B = 3 …..(1)

2A − B/2 = 30

4A – B = 60 ….(2)

By subtracting the equation (1) from (2), we get;

3A = 60 – 3 = 57

A = 57/3 = 19

Therefore, the age of Ani = 19 yrs

And the age of Biju is 19 – 3 = 16 yrs.

Case (ii)

When Biju is older than Ani,

B – A = 3 ….(1)

2A − B/2 = 30

4A – B = 60 ….(2)

Adding the equations (1) and (2), we get;

3A = 63

A = 21

Therefore, the age of Ani is 21 yrs

And the age of Biju is 21 + 3 = 24 yrs.

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II] [Hint : x + 100 = 2(y – 100), y + 10 = 6(x – 10)].

Solution:

Let the capital amount with two friends be Rs. x and Rs. y, respectively.

As per the given,

$x + 100 = 2(y - 100)$…..(i)

And

$6(x - 10) = (y + 10)$….(ii)

Consider the equation (i),

$x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$…..(iii)

From equation (ii),

$6x - 60 = y + 10$

$6x - y = 70$…..(iv)

(iv) × 2 – (iii)

$12x - 2y - (x - 2y) = 140 - (-300)$

$11x = 440$

$x = 40$

Substituting x = 40 in equation (iii), we get;

$40 - 2y = -300$

$2y = 340$

$y = 170$

Therefore, the two friends had Rs. 40 and Rs. 170 with them.

3. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution:

Let the speed of the train be x km/hr and the time taken by the train to travel a distance be t hours, and the d km be the distance.

Speed of the train = Distance travelled by train / Time taken to travel that distance

$x = d/t$

$d = xt$ …..(i)

Case 1: When the speed of the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time.

$(x + 10) = d/(t - 2)$

$(x + 10)(t – 2) = d$

$xt + 10t – 2x – 20 = d$

$d + 10t – 2x = 20 + d$ [From (i)]

$10t – 2x = 20$…..(ii)

Case 2: When the train was slower by 10 km/h, it would have taken 3 hours more than the scheduled time.

So, $(x – 10) = d/(t + 3)$

$(x – 10)(t + 3) = d$

$xt – 10t + 3x – 30 = d$

$d – 10t + 3x = 30 + d$ [From (i)]

$-10t + 3x = 30$…..(iii)

Adding (ii) and (iii), we get;

$x = 50$

Thus, the speed of the train is 50 km/h.

Substituting $x = 50$ in equation (ii), we get;

$10t – 100 = 20$

$10t = 120$

$t = 12$ hours

Distance travelled by train, $d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Solution:

Let x be the number of rows and y be the number of students in a row.

Total students in the class = Number of rows × Number of students in a row

$= xy$

Case 1:

Total number of students $= (x − 1) (y + 3)$

$xy = (x − 1) (y + 3)$

$xy = xy − y + 3x − 3$

$3x − y − 3 = 0$

$3x − y = 3$…..(i)

Case 2:

Total number of students = $(x + 2) (y - 3)$

$xy = xy + 2y - 3x - 6$

$3x - 2y = -6$…..(ii)

Subtracting equation (ii) from (i), we get;

$(3x - y) - (3x - 2y) = 3 - (-6)$

$- y + 2y = 9$

$y = 9$

Substituting $y = 9$ in equation (i), we get;

$3x - 9 = 3$

$3x = 12$

$x = 4$

Therefore, the total number of students in a class = $xy = 4 \times 9 = 36$

5. In a $\triangle ABC$, $\angle C = 3 \angle B = 2 (\angle A + \angle B)$. Find the three angles.

Solution:

Given,

$\angle C = 3 \angle B = 2(\angle B + \angle A)$

$3\angle B = 2 \angle A + 2 \angle B$

$\angle B = 2 \angle A$

$2\angle A - \angle B = 0$- – – – – – – – – – – – (i)

We know that the sum of a triangle's interior angles is 180°.

Thus, $\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3 \angle B = 180°$

$\angle A + 4 \angle B = 180°$– – – – – – – – – – – – – – – -(ii)

Multiplying equation (i) by 4, we get;

$8 \angle A - 4 \angle B = 0$- – – – – – – – – – – – (iii)

Adding equations (iii) and (ii), we get;

$9 \angle A = 180°$

$\angle A = 20°$

Using this in equation (ii), we get;

$20° + 4\angle B = 180°$

$\angle B = 40°$

And

$\angle C = 3\angle B = 3 \times 40 = 120°$

Therefore, $\angle A = 20°$, $\angle B = 40°$, and $\angle C = 120°$.

6. Draw the graphs of the equations 5x – y = 5 and 3x – y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Solutions:

Given,

$5x - y = 5$

$\Rightarrow y = 5x - 5$

Its solution table will be.

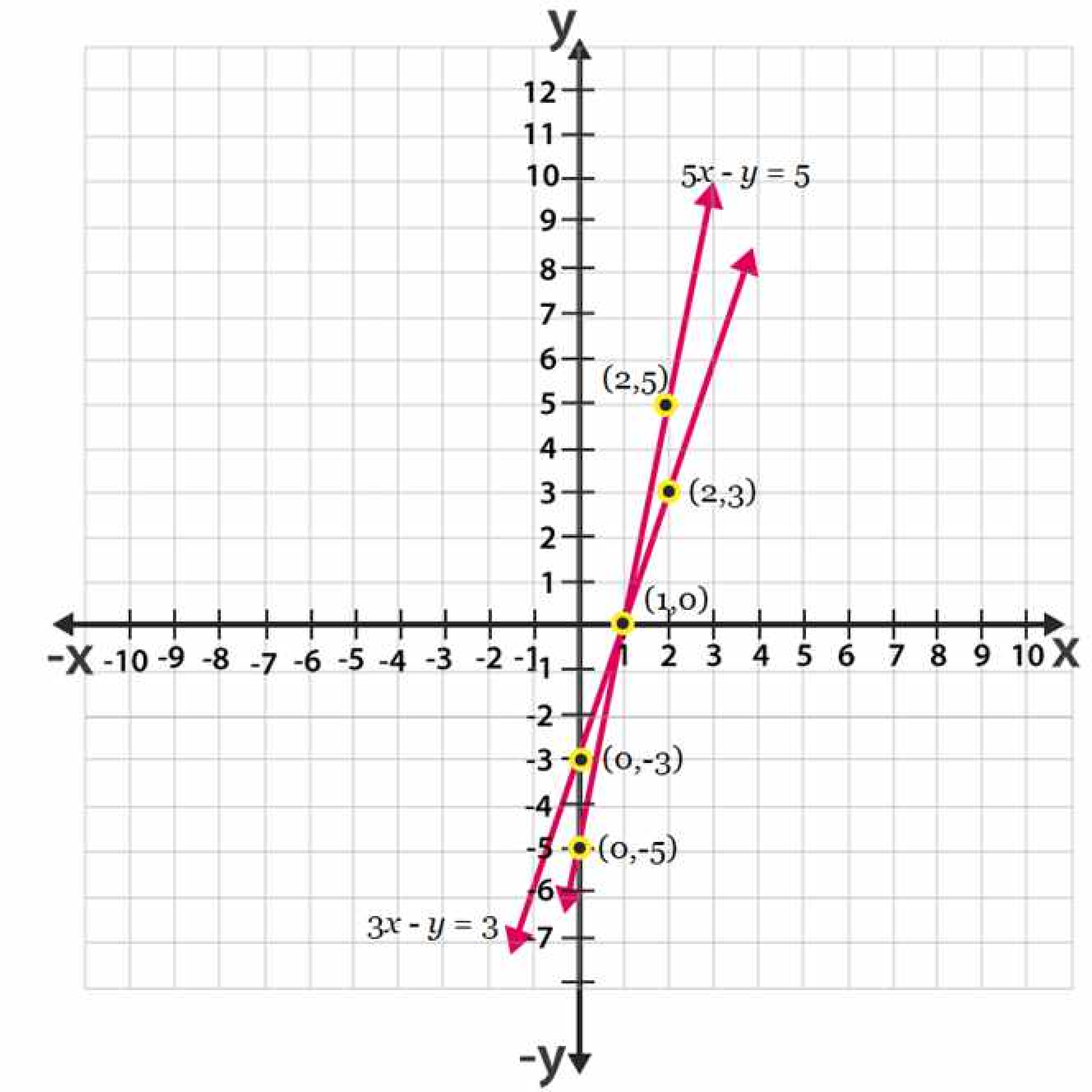
x	2	1	0
y	5	0	-5

Also given, $3x - y = 3$

$y = 3x - 3$

x	2	1	0
y	3	0	-3

The graphical representation of these lines will be as follows:



From the above graph, we can see that the coordinates of the vertices of the triangle formed by the lines and the y-axis are (1, 0), (0, -5) and (0, -3).

7. Solve the following pair of linear equations:

(i) px + qy = p – q

qx – py = p + q

(ii) ax + by = c

$bx + ay = 1 + c$

(iii) $x/a – y/b = 0$

$ax + by = a^2 + b^2$

(iv) $(a – b)x + (a + b) y = a^2 – 2ab – b^2$

$(a + b)(x + y) = a^2 + b^2$

(v) $152x – 378y = – 74$

$–378x + 152y = – 604$

Solutions:

(i) $px + qy = p – q$……………(i)

$qx – py = p + q$……………….(ii)

Multiplying equation (i) by p and equation (ii) by q, we get;

$p^2x + pqy = p^2 − pq$ ………… (iii)

$q^2x − pqy = pq + q^2$ ………… (iv)

Adding equations (iii) and (iv), we get;

$p^2x + q^2 x = p^2 + q^2$

$(p^2 + q^2) x = p^2 + q^2$

$x = (p^2 + q^2)/ (p^2 + q^2) = 1$

Substituting x = 1 in equation (i), we have;

$p(1) + qy = p – q$

$qy = p – q – p$

$qy = -q$

$y = -1$

(ii) $ax + by= c$…………………(i)

$bx + ay = 1+ c$………… ..(ii)

Multiplying equation (i) by a and equation (ii) by b, we get;

$a^2x + aby = ac$ ………………… (iii)

$b^2x + aby = b + bc$…………… (iv)

Subtracting equation (iv) from equation (iii),

$(a^2 – b^2) x = ac − bc– b$

$x = (ac − bc – b)/ (a^2 – b^2)$

$x = c(a – b) – b / (a^2 – b^2)$

From equation (i), we obtain

$ax + by = c$

$a\{c(a-b)-b)/(a^2-b^2)\} + by = c$

$\{[ac(a-b)-ab]/(a^2-b^2)\} + by = c$

$by = c - \{[ac(a-b)-ab]/(a^2-b^2)\}$

$by = (a^2c - b^2c - a^2c + abc + ab)/(a^2-b^2)$

$by = [abc - b^2c + ab]/(a^2-b^2)$

$by = b(ac - bc + a)/(a^2-b^2)$

$y = [c(a-b) + a]/(a^2-b^2)$

(iii) $x/a - y/b = 0$

$ax + by = a^2 + b^2$

$x/a - y/b = 0$

$\Rightarrow bx - ay = 0$ ……. (i)

And

$ax + by = a^2 + b^2$…….. (ii)

Multiplying equations (i) and (ii) by b and a, respectively, we get;

$b^2x - aby = 0$ …………… (iii)

$a^2x + aby = a^3 + ab^2$ …… (iv)

Adding equations (iii) and (iv), we get;

$b^2x + a^2x = a^3 + ab^2$

$x(b^2 + a^2) = a(a^2 + b^2)$

$\Rightarrow x = a$

Substituting x = 1 in equation (i), we get;

$b(a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$\Rightarrow y = b$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

$(a+b)y + (a-b)x = a^2 - 2ab - b^2$ …………… (i)

$(x+y)(a+b) = a^2 + b^2$

$(a+b)y + (a+b)x = a^2 + b^2$ ………………… (ii)

Subtracting equation (ii) from equation (i), we get;

$(a-b)x - (a+b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$x(a - b - a - b) = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$x = a + b$

Substituting $x = a + b$ in equation (i), we get;

$y(a + b) + (a + b)(a - b) = a^2 - 2ab - b^2$

$a^2 - b^2 + y(a + b) = a^2 - 2ab - b^2$

$(a + b)y = -2ab$

$y = -2ab/(a + b)$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

$152x - 378y = -74$ ….(i)

$-378x + 152y = -604$….(ii)

From equation (i),

$152x + 74 = 378y$

$y = (152x + 74)/378$

Or

$y = (76x + 37)/189$…..(iii)

Substituting the value of y in equation (ii), we get;

$-378x + 152[(76x + 37)/189] = -604$

$(-378x)189 + [152(76x) + 152(37)] = (-604)(189)$

$-71442x + 11552x + 5624 = -114156$

$-59890x = -114156 - 5624 = -119780$

$x = -119780/-59890$

$x = 2$

Substituting $x = 2$ in equation (iii), we get;

$y = [76(2) + 37]/189$

$= (152 + 37)/189$

$= 189/189$

$= 1$

Therefore, $x = 2$ and $y = 1$

8. ABCD is a cyclic quadrilateral (see Fig. 3.7). Find the angles of the cyclic quadrilateral.

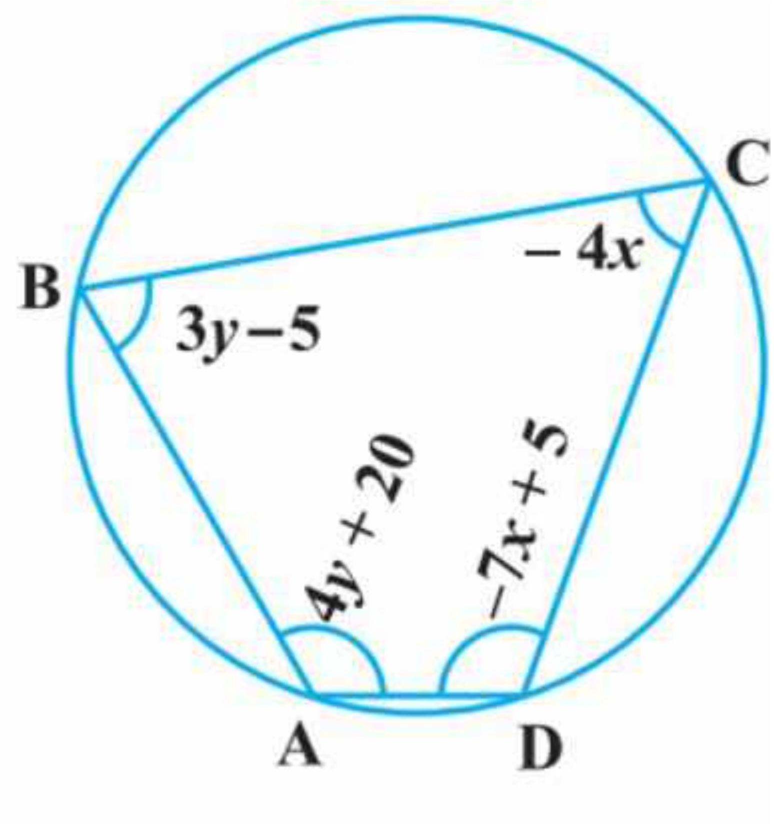


Fig. 3.7

Solution:

Given that ABCD is a cyclic quadrilateral.

As we know, the opposite angles of a cyclic quadrilateral are supplementary.

So,

$\angle A + \angle C = 180$

$4y + 20 + (-4x) = 180$

$-4x + 4y = 160$

$\Rightarrow -x + y = 40$….(i)

And

$\angle B + \angle D = 180$

$3y – 5 + (-7x + 5) = 180$

$\Rightarrow -7x + 3y = 180$…..(ii)

Equation (ii) – 3 × (i),

$-7x + 3y – (-3x + 3y) = 180 – 120$

$-4x = 60$

$x = -15$

Substituting $x = -15$ in equation (i), we get;

$-(-15) + y = 40$

$y = 40 – 15 = 25$

Therefore, $x = -15$ and $y = 25$.